# Multi-sensor Spatial Association using Joint Range-Doppler Features

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Abstract—We investigate the problem of localizing multiple targets using a single set of measurements from a network of radar sensors. Such "single snapshot imaging" provides timely situational awareness, but can utilize neither platform motion, as in synthetic aperture radar, nor track targets across time, as in Kalman filtering and its variants. Associating measurements with targets becomes a fundamental bottleneck in this setting. In this paper, we present a computationally efficient method to extract 2D position and velocity of multiple targets using a linear array of FMCW radar sensors by identifying and exploiting inherent geometric features to drastically reduce the complexity of spatial association. The proposed framework is robust to detection anomalies, and achieves order of magnitude lower complexity compared to conventional methods. While our approach is compatible with conventional FFT-based range-Doppler processing, we show that more sophisticated techniques for range-Doppler estimation lead to reduced data association complexity as well as higher accuracy estimates of target positions and velocities.

*Index Terms*—Sensor Networks, Aggregation, Approximation Algorithms, Single Snapshot Localization

## I. INTRODUCTION

**R** ECENT advances in low-cost design and fabrication enable the potential application of high-accuracy millimeter wave (mmWave) radar sensors to a variety of commercial sectors, including automotive, drones and robotics [1], [2]. The large available bandwidths enable high range resolution, while the small wavelength enhances Doppler and microDoppler resolution. In this paper, we explore the utility of a network of such sensors in providing timely situational awareness for highly dynamic environments, by considering estimation of the kinematic state of the scene (i.e., the positions and velocities of targets) via a single set of measurements obtained by a network of sensors. We do not rely on tracking targets across time, or on platform motion to synthesize larger apertures.

The specific problem we consider is that of localizing multiple targets in a 2D scene using a linear array of radar sensors. Figure 1 shows a scenario with two targets being observed with a linear array of four spatially separated sensors positioned along x-axis. Each sensor collects the relative range and Doppler observations for the targets in the scene. Since these observations are not ordered *a priori*, each range-Doppler measurement must first be associated with a target, and then the measurements associated with a given target from multiple sensors can be used to estimate its position and velocity. Since the number of possible associations grows exponentially in the number of sensors, it is critical to develop efficient algorithms for spatial association. It is also important to build in robustness to missed detections, since millimeter waves can be easily occluded by objects in the scene.



Fig. 1. 2D System model with a linear array of radar sensors placed on x-coordinates,  $[l_1, l_2, l_3, l_4]$ . The kinematic states  $z_1, z_2$  of two targets are to be estimated using the unordered range and doppler observations from the sensors.

## A. Contributions

Our goal is to develop robust and computationally efficient algorithms for single snapshot spatial data association. The main contributions of our study are as follows:

(1) We examine the geometric relations between instantaneous range, Doppler, and sensor locations, and show that features obtained via those geometric relations simplify the association problem. Specifically, we observe and exploit linear relationships between functions of the range-Doppler observations for a target across the linear array of sensors.

(2) We provide a low-complexity solution for the association problem by introducing a new graph-search based algorithm which prunes the set of feasible associations based on geometric relationships. In particular, our proposed algorithm considers a cost function based on the linear geometric relationships together with the triangle inequality constraint for the range observations at pairs of sensors and eliminates a significant number of possible associations. In addition, our approach

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accounts for detection anomalies such as missed detections and false alarms while reducing the complexity.

(3) We compare our proposed algorithm against conventional algorithms in the literature and evaluate performance in terms of localization accuracy, cardinality errors, robustness, and complexity. Also, we show that using an enhanced accuracy estimation algorithm (i.e., NOMP [3]) instead of conventional FFT-based approach improves localization accuracy and reduces association complexity as the number of targets and sensors increases.

## B. Related Work

The majority of prior work addresses temporal data association for tracking, relying on the temporal continuity of target state to assist in associating observations across multiple frames. While the problem of spatial association studied in this paper has received relatively less attention, we provide a brief overview of conventional approaches in the literature that can be extended for the spatial problem.

The association problem between a pair of sensors can be optimally solved using the well-known Hungarian algorithm [4]. However, a naive approach to extend this to multiple sensor case by factorizing into pairwise (2D) association over consecutive sensors does not work well in the presence of detection anomalies such as miss, false alarm, clutter, and close-target interactions [5].

The multi-sensor association problem can be formulated as the Maximum A-Posteriori (MAP) estimation of most likely chain of observations across sensors. In order to solve this problem, a graphical model is defined, where a node represents sensor detection and an edge represents pairwise association likelihoods [6]. The association between the sensors is obtained by solving the Minimum Cost Maximum Flow (MCF) problem over this graph. A variety of methods such as Linear Programming [7], Dynamic Programming [8], [9], and push-relabel maximum flow [6] has been proposed to efficiently solve the MCF problem. Although those methods solve the optimization in polynomial time, they require specialized mechanisms such as expansion of observation set over successive iterations to resolve detection anomalies. Moreover, the complexity of the MCF problem grows quickly as  $O(N^3 \log N)$ , where N is the number of sensors [6]. In comparison with prior work, our approach reduces complexity by leveraging the high accuracy of sensor observations and their geometric properties.

Probabilistic approaches such as the gated Nearest Neighbor (NN) [10] method sequentially associate observations across the sensors. At each sensor, each observation is associated with its closest match to the state predicted by the chain of observations from the past sensors. However, using only single most likely observation to form association is prone to clutter and anomalies in noisy scenarios. Also, a single association error can cause significant contamination in final state estimate. This problem is well known in the literature on Simultaneous Localization and Mapping (SLAM), and various improvements such as Multiple Hypothesis Testing [11], Kbest assignment [12], and JPDAF [10] have been proposed. Bottom up approaches based on grid search over a set of candidate target states have been suggested in the literature [13]. In [14], an approach based on enumerating all possible candidates followed by pruning and merging shows promising results. Randomized adaptive search procedures such as random consensus sampling (RANSAC) [15], Interpretation Tree [16], Joint Compatibility Branch and Bound [17] have been shown to address the detection anomalies. These methods utilize a suitably defined metric to check the consistency of a set of associated observations and employ branch and bound type search strategies to reduce the search complexity. Our graphical approach also uses similar pruning techniques to perform the graph search, but with the additional use of geometric constraints and a geometric fitting error metric for guiding the search.

**Outline:** The rest of the paper is organized as follows. In Section II, we introduce the association problem in the single snapshot localization setting. In Section III, our graph association algorithm is presented. Then, the proposed algorithm is evaluated over different system parameters in Section IV and Section V concludes the paper.

**Notation:** a, a, A, A represent scalar, vector, matrix and set respectively. We use [.] to construct vector, matrix and  $\{.\}$  to construct set.  $\times, \cup, \cap$  denote the cartesian product, union and, intersection of two sets and  $\emptyset$  denotes a *NULL* value. n(A) represents the number of non-empty elements in set A.  $\circ$  denotes element-wise multiplication between vectors.  $A^T$  denotes transpose of matrix A and  $\wedge$  denotes logical "and" operator.

## **II. PROBLEM DESCRIPTION**

## A. System Model

Consider a linear array of  $N_S$  radar sensors in a twodimensional (2D) scene with  $N_T$  targets as in Figure 1. Without loss of generality, we assume that the sensor array is static and located along x-axis and centered at origin. The absolute kinematic state of the targets can be obtained by using the target location relative to this sensor array along with its own odometer information.

The kinematic state (i.e., instantaneous position and velocity information of all targets) of the scene is given by

$$\mathcal{Z} = \{oldsymbol{z}^k\}_{k=1}^{N_T}$$

where  $\mathbf{z}^k = (x^k, y^k, v_x^k, v_y^k)$  is the kinematic state of target k with an instantaneous velocity of  $(v_x^k, v_y^k)$  at position  $(x^k, y^k)$ .

The range-Doppler of target k observed at sensor i, can be expressed in terms of the desired kinematic state as follows,

$$r_i^k = \sqrt{(x^k - l_i)^2 + (y^k)^2}, \quad d_i^k = \frac{(x^k - l_i)v_x^k + y^k v_y^k}{r_i^k}.$$
(1)

where  $l_i$  is the x-coordinate of sensor *i*. We denote this nonlinear mapping as  $(r_i^k, d_i^k) = \mathcal{T}_i(\boldsymbol{z}^k)$ .

#### B. Single Snapshot Localization

In order to extract range and Doppler information of target k, each sensor i uses the signal (i.e.,  $m_i^{obs}(t)$ ) reflected back from the scene in monostatic mode. In this study, we focus on localization of the scene using a single snapshot. For that reason, the kinematic state of the scene is assumed to be constant for a certain time interval and the scene localization is performed based on the range and Doppler information gathered during that time interval. Based on those, the Maximum Likelihood Estimator (MLE) for the scene including all  $N_S$  sensors can be expressed as,

$$\hat{\mathcal{Z}}_{ML} = \arg \max_{\mathcal{Z}} \prod_{i=1}^{Ns} \mathcal{L}\left(m_i^{obs} | \mathcal{T}_i(\mathcal{Z})\right)$$
(2)

where  $m_i^{obs}$  corresponds to the observed signal in a single snapshot and  $\mathcal{L}\left(m_i^{obs} | \mathcal{T}_i(\mathcal{Z})\right)$  is the conditional log likelihood of the observed signal for scene  $\mathcal{Z}$ .

The optimization problem in (2) is difficult in general since the number of targets (i.e.,  $N_T$ ) is not known and a brute force search for  $\mathcal{Z}$  incurs exponential complexity in the number of targets; that is,  $n(\mathcal{D}(z))^{N_T}$  for a grid  $\mathcal{D}(z)$ . In addition, the observations contain a variety of anomalies such as clutter, missed detections, and false alarms, which further complicates the solution.

In order to facilitate the solution of the problem in (2), the problem is divided into two stages as follows:

1) Estimation: The Range-Doppler pairs of  $M_i \leq N_T$  nonoccluded targets are estimated from received signal  $m_i^{obs}$  at sensor *i* using efficient algorithms proposed in the literature [18]. The estimate at sensor *i* for  $k^{th}$  target can be modeled as follows,

$$(r_i^k) = (r_i^k)^{true} + w_i^R + \tilde{b}_i^k , \qquad (3a)$$

$$(d_i^k) = (d_i^k)^{true} + w_i^D + \bar{b}_i^k$$
 (3b)

where  $w_i^R \sim \mathcal{N}(0, \sigma_{r_i}^2)$  and  $w_i^D \sim \mathcal{N}(0, \sigma_{d_i}^2)$  denote independent Gaussian distributed noises with zero mean and  $\tilde{b}_i^k$  and  $\bar{b}_i^k$  denote the bias errors introduced due to proximity with any other  $M_i - 1$  targets in the scene. The noise variance depends on estimation accuracy at the given SNR which, in turn, depends on target radar cross section (RCS), path loss, and antenna directivity. For simplicity, we assume equal received signal power across all targets in the scene.

We denote the set of estimated range-Doppler pairs at sensor *i* by  $\Theta_i = \{\bigcup_{k=1}^{M_i} \theta_i^k\}$  where  $\theta_i^k = \left[(r_i^j), (d_i^j)\right]^T$ . It is important to note that the superscript of estimated range-Doppler pairs  $\theta_i^k$  is different from the true target index since we do not know the true target index that the observation at the sensor belongs to.

2) Association problem: The estimation of kinematic state  $\mathcal{Z}$  requires the association of those un-ordered range-Doppler pairs,  $\Theta_i$ , collected across all sensors. An association chain is defined as the ordered set of range-Doppler observations,  $\mathcal{A} : \{\{\boldsymbol{\theta}_i\}_{i=1}^{N_s} | \boldsymbol{\theta}_i \in \tilde{\Theta}_i\}$  which is constructed from the *NULL* augmented sets; that is,  $\tilde{\Theta}_i = \Theta_i \cup \emptyset$ .  $\boldsymbol{\theta}_i = \emptyset$  corresponds to the *NULL* state and represents the occurrence of missed detection at sensor *i*.

The spatial association problem can be formulated as the following maximum a posteriori (MAP) estimation problem,

$$\mathcal{A}^{*} = \underset{\mathcal{A} \subset \tilde{\Theta}_{1} \times \dots \times \tilde{\Theta}_{N_{S}}}{\operatorname{such that}} \log P(\mathcal{A}) P(\Theta|\mathcal{A})$$
(4)  
such that  $\mathcal{A}^{i} \cap \mathcal{A}^{j} = \emptyset \quad \forall i \neq j, \quad n(\mathcal{A}^{k}) \geq 2$ 

where  $\Theta = \bigcup_{i=1}^{N_S} \Theta_i$  denotes the set of all range-Doppler observations,  $\mathcal{A} = \{\mathcal{A}^1, \mathcal{A}^2, \cdots\}$  denotes a subset of association chains chosen from the set of all possible potential chains,  $\tilde{\Theta}_1 \times \tilde{\Theta}_2 \times \cdots \tilde{\Theta}_{N_S}$ . The optimal solution  $\mathcal{A}^*$  consists of the set of chains which jointly maximizes overall log likelihood while the constraints ensure that no two chains share a common observation and each chain contains at least two observations.

When the targets are well-separated, the bias terms in (3a) and (3b) vanish and the likelihood for the individual targets becomes independent across multiple targets. In this case, the log likelihood in (4) simplifies to

$$\log P(\mathcal{A})P(\Theta|\mathcal{A}) = \sum_{\mathcal{A}\in\mathcal{A}} \log P(\mathcal{A}) + \log P(\Theta|\mathcal{A})$$

where  $P(\Theta|A) = \prod_{i=1}^{N_S} P(\Theta_i|A)$  is the probability of detecting the range-Doppler pairs which can be modeled by a Bernoulli distribution,

$$P(\Theta_i | \mathcal{A}) = \begin{cases} \alpha & \text{, if target missed at sensor } i \,, \mathcal{A}_i = \varnothing \\ 1 - \alpha & \text{, else} \end{cases}$$

where  $\alpha$  denotes the probability of detection errors in (5) and is set to nominal value  $\alpha = 0.05$  [6]. This model accounts for the occurrence of both miss and false alarms across the sensors in the likelihood, which is given by

$$P(\Theta|\mathcal{A}) = \alpha^{N_S - n(\mathcal{A})} (1 - \alpha)^{n(\mathcal{A})}$$

Also,  $P(\mathcal{A})$  is the likelihood of chain modeled using the perceived range-Doppler pairs,  $(\hat{r}_i, \hat{d}_i) = \mathcal{T}_i(\hat{z})$  for a target state  $\hat{z}_k$  predicted by the chain (see Section III-B2). By ignoring the constant terms which preserve the MAP solution, we define the normalized negative log likelihood as follows,

$$\mathcal{L}(\mathcal{A}) = \sum_{\boldsymbol{\theta}_i \in \mathcal{A}} \left( \frac{(\hat{r}_i - r_i)^2}{\sigma_r^2} + \frac{(\hat{d}_i - d_i)^2}{\sigma_d^2} \right) + n(\mathcal{A}) \log \frac{\alpha}{1 - \alpha}$$
(5)

where  $\theta_i = [r_i, d_i]^T$  is the observation from  $i^{th}$  sensor in association chain  $\mathcal{A}$  and  $\sigma_r^2$  and  $\sigma_d^2$  are the nominal variance terms for range and Doppler, respectively (see Appendix A for details). The first term in (5) denotes the squared error between the estimated and observed range-Doppler pairs in the chain while the second term penalizes the selection of smaller chains which prevents formation of duplicate chains for the same target. Hence, the association problem is reduced to the following constrained minimization problem:

$$\mathcal{A}^{*} = \underset{\mathcal{A} \subset \tilde{\Theta}_{1} \times \dots \times \tilde{\Theta}_{N_{S}}}{\operatorname{argmin}} \sum_{\mathcal{A} \in \mathcal{A}} \mathcal{L}(\mathcal{A})$$
(6)  
such that  $\mathcal{A}^{i} \cap \mathcal{A}^{j} = \emptyset \ \forall i \neq j, \quad n(\mathcal{A}) \geq 2$ 

The joint minimization problem over all potential association chains in (6) is difficult in general. For that reason, we use an iterative approach where the most likely chains of observations are identified and removed from observation set  $\Theta$  sequentially,

$$\underset{\substack{\mathfrak{A}\in\tilde{\Theta}_{1}\times\cdots\times\tilde{\Theta}_{N_{S}}}{\operatorname{argmin}} \mathcal{L}(\mathcal{A}) \quad \text{such that} \quad n(\mathcal{A}) \geq 2.$$
(7)

Without any prior knowledge of association between the nodes, the number of potential chains  $\Theta_1 \times \Theta_2 \times \cdots \otimes_{N_S}$  still grows exponentially. However, the formulation in (7) enables the utilization of various network optimization methods to identify the most likely chain. Once the associated chains of range-Doppler observations are found across sensors, the kinematic state of the scene can be easily obtained by solving the inverse kinematic problem  $[\hat{x}, \hat{y}, \hat{v}_x, \hat{v}_y] = \mathcal{T}^{-1}(\mathcal{A})$  using Gauss-Newton algorithm [18].

## **III. GRAPHICAL ASSOCIATION**

In order to solve the association problem in (7), we formulate the spatial association problem using graphical models and present our low-complexity graphical search method to obtain association chains efficiently via geometric relations.

## A. Graph Generation

To begin with, we define a target-based graph to perform data association with following elements:

- Node θ<sup>k</sup><sub>i</sub> represents the k<sup>th</sup> range-Doppler pair at sensor i. Nodes for a given sensor are arranged along a single column of the graph as shown in Figure 2.
- Edge e<sup>kl</sup><sub>ij</sub> = [θ<sup>k</sup><sub>i</sub>, θ<sup>l</sup><sub>j</sub>] denotes the linkage between pairs of observation across sensor i and sensor j, which can correspond to a feasible target z<sup>kl</sup><sub>ij</sub> referred to as "candidate" location.
- Chain A<sup>j</sup> is represented by the sequence of two or more nodes spanning distinct sensors, which is associated to a single target, 2<sup>j</sup>.



Fig. 2. Target-based observation graph for a scene with 3 targets and 4 sensors. Sensors 1, 2 observe all 3 targets in different orders. Sensor 3 misses the observation of target state  $z^2$  while sensor 4 contains a false observation. Desired association chain, A is shown by the shaded set of nodes.

**Geometric Constraint:** A significant portion of the edges can be easily discarded in the graph generation phase by using the following geometric constraint on target's range (for noiseless case),

$$C_{\mathbf{G}}(e_{ij}): \quad (r_i - r_j < l_{ij}) \land (r_i + r_j > l_{ij})$$

$$\tag{8}$$

where  $l_{ij} = |l_i - l_j|$  represents the separation between sensor *i* and sensor *j*.

Graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  is initialized with vertices for all range-Doppler pairs  $\mathcal{V} = \{\Theta_i\}_{i=1}^{N_S}$  and edges  $\mathcal{E}$  between any two consecutive nodes that satisfy condition  $C_{\mathcal{G}}(e_{i-1,i}^{k,l}), \forall k \in [1, M_{i-1}], \forall l \in [1, M_i]$  for all  $i \in \{2, \ldots, N_S\}$  given in (8).

### B. Spatial Association using Geometric Features

In this subsection, we describe the solution of the association problem presented in (7) using graph  $\mathcal{G}$  by exploiting geometric relations between range, Doppler, and sensor geometry. For clarity of exposition, we focus on the association procedure of a single target  $\boldsymbol{z} = [x, y, v_x, v_y]$  and, therefore, drop the superscript k for the sake of simplicity.

1) Geometric Relations: The range of target observed at  $i^{th}$  sensor is given by

$$r_i = \sqrt{(x - l_i)^2 + (y)^2}$$
. (9)

The Doppler component is the rate of change of range and it is given by,

$$d_{i} = \dot{r}_{i} = \frac{(x - l_{i})v_{x} + yv_{y}}{r_{i}}$$
  
$$r_{i}d_{i} = (x - l_{i})(v_{x}) + yv_{y}.$$
 (10)

For a linear array of sensors, the range and Doppler measurements for a target satisfy the following relations based on (9) and (10):

$$r_i^2 = r_j^2 - 2x(l_i - l_j) + (l_i^2 - l_j^2)$$
 (11a)

$$r_i d_i = r_j d_j - (v_x)(l_i - l_j)$$
 (11b)

where  $r_i$   $(r_j)$  and  $d_i$   $(d_j)$  are the range and Doppler estimated at the  $i^{th}$   $(j^{th})$  sensor, respectively.  $l_i$   $(l_j)$  is the X-coordinate of  $i^{th}$   $(j^{th})$  sensor. (9) and (10) indicate that for the noiseless setting, the range-Doppler products and range squared are linear with respect to target's velocity and position at Xcoordinate, respectively. Therefore, the correct associations can be identified by fitting the observations to those geometric relations.

2) State Prediction and Fitting Error: The presence of noise in  $(r_i, d_i)$  causes high error in these geometric relations due to the quadratic dependence. An estimate of target state parameters  $\hat{x}, \hat{v}_x$  can be obtained by minimizing that error between observed and predicted range and Doppler values. Let  $q_1 = [r_i d_i | (r_i, d_i) \in \mathcal{A}]$  and  $l = [l_i | \theta_i \in \mathcal{A}]$  denote the vector of range-Doppler products using observations in chain  $\mathcal{A}$  and the vector of corresponding sensor X-coordinates, respectively. Predicted fit  $\hat{q}_1$  can be expressed using the geometric relation in (11b) as follows:

$$\hat{\boldsymbol{q}}_1 = -v_x \boldsymbol{l} + \kappa_1 \mathbb{1} = H \boldsymbol{s}_1$$

where H = [l, 1],  $s_1 = [-v_x \kappa_1]^T$ , and  $\kappa_1$  is a constant. Then, the least squares estimate for  $\hat{s}_1$  is obtained as

$$\hat{s}_{1} = \arg\min_{s_{1}} ||\boldsymbol{q}_{1} - H\boldsymbol{s}_{1}||^{2}$$
(12)  
=  $(H^{T}H)^{-1}H^{T}\boldsymbol{q}_{1}$ .

Therefore, the least square estimate is obtained as  $\hat{v}_x = u^T q_1$  where  $u = -H(H^T H)^{-1} e_1$  and  $e_1 = [1,0]^T$ .

Similarly, let  $q_2 = [r_i^2|(r_i, d_i) \in \mathcal{A}]$  denote the vector of range squared observations in chain  $\mathcal{A}$ , predicted fit  $\hat{q}_2$  can be expressed using the geometric relation in (11a) as follows:

$$\hat{\boldsymbol{q}}_2 - \boldsymbol{l} \circ \boldsymbol{l} = -2x\boldsymbol{l} + \kappa_2 \mathbb{1} = H\boldsymbol{s}_2$$

where  $s_2 = [-2x \ \kappa_2]^T$  and  $\kappa_2$  is a constant. The least squares estimate of  $\hat{x}$  is obtained as

$$\hat{\boldsymbol{s}}_2 = \arg\min_{\boldsymbol{s}_2} \quad \|\boldsymbol{q}_2 - \boldsymbol{l} \circ \boldsymbol{l} - H\boldsymbol{s}_2\|^2 \qquad (13)$$
$$= (H^T H)^{-1} H^T (\boldsymbol{q}_2 - \boldsymbol{l} \circ \boldsymbol{l}) \,.$$

Hence, we obtain  $\hat{x} = \boldsymbol{u}^T (\boldsymbol{q}_2 - \boldsymbol{l} \circ \boldsymbol{l})/2$ .

The remaining state parameters (i.e.,  $\hat{y}$  and  $\hat{v}_y$ ) are obtained using the geometric relations in (9) and (10) as

$$\hat{y} = \sqrt{\frac{1}{n\left(\mathcal{A}\right)}} \sum_{\boldsymbol{\theta}_i \in \mathcal{A}} \left(r_i^2 - (\hat{x} - l_i)^2\right)$$
$$\hat{v}_y = \frac{1}{n\left(\mathcal{A}\right)} \sum_{\boldsymbol{\theta}_i \in \mathcal{A}} \frac{r_i d_i - (\hat{x} - l_i) \hat{v}_x}{\hat{y}}.$$

The normalized geometric fitting error of a chain A can be computed using these estimates as follows:

$$F(\mathcal{A}) = \frac{\|\boldsymbol{q}_1 - \hat{\boldsymbol{q}}_1\|^2}{\eta_1} + \frac{\|\boldsymbol{q}_2 - \hat{\boldsymbol{q}}_2\|^2}{\eta_2}$$
(14)

$$= \frac{\left\| \left( I - H(H^T H)^{-1} H^T \right) \boldsymbol{q}_1 \right\|^2}{\eta_1}$$
(15)

$$+ \frac{\left\| \left(I - H^{T}(H^{T}H)^{-1}H\right) \left(\boldsymbol{q}_{2} - \boldsymbol{l} \circ \boldsymbol{l}\right) \right\|^{2}}{\eta_{2}}$$

where  $\eta_1$  and  $\eta_2$  are normalization constants that are set based on CRB (see Appendix B for details) and (15) is obtained by substituting the predicted fits into (14). It is important to note that the error in (15) is additive over the observations in chain  $\mathcal{A}$ . Therefore, the extension of the chain cannot reduce the fitting error. In other words,  $F(\mathcal{A})$  is monotonically non-decreasing over the length of chain  $\mathcal{A}$ . For that reason, the fitting error provides a simple measure of the geometric consistency of a chain, which can be used to traverse the graph and extract the chains efficiently.

3) Geometric Association: We now present a graph search procedure which obtains the associated chains by minimizing geometric fitting error F(A) in (15) and negative log likelihood  $\mathcal{L}(A)$  in (5). We apply the geometric relations by adding constraints on the desired chain, A to the optimization problem in (7) as follows,

$$\begin{array}{l} \min_{\mathcal{A}\in\tilde{\Theta}_{1}\times\cdots\times\tilde{\Theta}_{N_{S}}} \mathcal{L}(\mathcal{A}) \\ \text{such that} \quad n(\mathcal{A}) \geq \gamma, \end{array} \tag{16a}$$

$$F(\mathcal{A}) < \tau_f^{n(\mathcal{A})} \tag{16b}$$

The constraint in (16a) restricts the number of missed observations to be less than  $N_S - \gamma$  and the constraint in (16b) only allows chains with good geometric fit to be selected. In order to provide a solution for the optimization problem in (16), we perform Depth First Search (DFS) over the graph generated in Section III-A to extract the chains, where those additional constraints help in reducing the search complexity. Our complete Spatial Association using Geometry Algorithm (SAGA) is outlined in Algorithm 1. Here is a brief description:

- 1) We start the graph search by setting  $\gamma = N_S$  so that only chains that include observations from all sensors are extracted. For that reason, we consider a graph having edges between consecutive sensors only. This helps to reduce the chains encountered during initial DFS procedure (see Appendix C for details).
- 2) The DFS is guided by geometric fitting error  $F(\mathcal{A})$ . After each node is visited, the fitting error of candidate chain is calculated and the chain is ignored if it has a fitting error higher than predefined threshold  $\tau_f^{N_S}$ . Since the fitting error is non-decreasing over the length of the chain, most of the candidate chains are eliminated before reaching at the end of the graph, which reduces the complexity further. Details of DFS are shown in Appendix D. At the termination of the DFS, the corresponding chain of nodes is added to solution  $\mathcal{A}^{\dagger}$  if it satisfies all the constraints in (16) and the negative log-likelihood of the association chain is below a predefined threshold (i.e.,  $\mathcal{L}(\mathcal{A}) < \tau_l^{n(\mathcal{A})}$ ). The nodes belonging to the selected chains are removed from the graph together with their corresponding edges to keep subsequent chains disjoint.
- 3) In order to deal with missed detection cases at sensors, the minimum chain length constraint (i.e.,  $\gamma$ ) is relaxed in steps up to robustness level  $\rho$ . Due to that relaxation, the graph includes not only the edges between consecutive sensors but also the edges among the nodes that skip over h consecutive sensors. Those edges are called Skiph edges where  $h = N_S - \gamma$ . Then, the DFS procedure is repeated for different minimum chain length constraints. Consequently, in this procedure, NULL states are taken into account and the generated chain does not include any observation from a sensor that misses the corresponding target by skipping over the observations of that sensor via Skip-h edges. In addition, the DFS procedure implicitly accounts for NULL state in the beginning and end of a chain by starting searching from different nodes in consideration of minimum chain length constraint.
- 4) The thresholds (i.e., τ<sub>f</sub> and τ<sub>l</sub>) for the geometric fitting error and the likelihood depend on length of the chain n(A) and their initial value is set based on CFAR criteria (see Appendix B for details). Using a tight initial threshold τ<sub>f</sub> for F(A) restricts the number of branches to be explored at each node to a smaller set. This reduces the initial complexity of DFS while allowing only a subset of association chains A<sup>†</sup> ⊂ A<sup>\*</sup> to be found. The thresholds are later relaxed by a factor of β > 1 to allow the observations contaminated with noise to be selected. The relaxation is stopped when no further chains with

length  $n(\mathcal{A}) \geq N_S - \rho$  exist in the graph.

Al	gorithm	1	Spatial	Association	using	Geometric	Assistance
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**Input:** Graph  $\mathcal{G}$ , Robustness level  $\rho$ 1: INITIALIZE Chains  $\mathcal{A}^{\dagger} = \emptyset, \boldsymbol{\tau} = [\tau_f, \tau_l]_{init}$ 2: repeat REMOVE all Skip edges 3: 4: for h = 0 to  $\rho$  do Set minimum chain length:  $\gamma = N_S - h$ 5: ADD SKIP-h EDGES TO GRAPH G6: for  $v \in \mathcal{V}$  do 7: DFS from node v:  $\mathcal{A} \leftarrow GA\text{-}DFS(v, \gamma, \tau)$ 8: if Valid Chain,  $\mathcal{A}$  is found then 9: 10:  $\mathcal{A}^{\mathsf{T}} \leftarrow \mathcal{A}$ Remove chain from graph  $\mathcal{V} = \mathcal{V} - \{\mathcal{A}\}$ 11: 12: end if end for 13: 14: end for 15: Relax thresholds:  $\boldsymbol{\tau} \leftarrow \beta \boldsymbol{\tau}$ 16: **until** Chains with length  $n(\mathcal{A}) \geq N_S - \rho$  exists in  $\mathcal{G}$ **Output:** Selected chains  $\mathcal{A}^{\dagger}$ 

**Robustness:** During chain length relaxation, a *Skip* edge is added between the observations across sensor i and sensor q if

- 1) Observations  $\theta_i$  and  $\theta_q$  satisfy the geometric constraint  $C_{\rm G}(e_{iq})$  in (8), and,
- 2) The target state predicted by  $\theta_i$  and  $\theta_q$  differs by a predefined threshold  $\tau_z$  from the ones predicted by using all observations on the paths that connect  $\theta_i$  and  $\theta_q$ .

$$C_{\mathbf{S}}(e_{iq}): \quad \left\| \hat{\mathbf{z}}_{\mathcal{A}_{p}} - \hat{\mathbf{z}}_{iq} \right\| > \tau_{z}, \forall \mathcal{A}_{p}: \{ \boldsymbol{\theta}_{i}, \boldsymbol{\theta}_{q} \} \in \mathcal{A}_{p}$$
(17)

where  $\mathcal{A}_p$  is in the form of  $\mathcal{A}_p = \{\boldsymbol{\theta}_i, \boldsymbol{\theta}_j, \cdots, \boldsymbol{\theta}_q\}$  with  $\boldsymbol{\theta}_i$  and  $\boldsymbol{\theta}_q$  at the edges of the path,  $\tau_z$  is set based on CRB (see Appendix E),  $\hat{\boldsymbol{z}}_{iq}$  indicates the predicted target state based on  $\boldsymbol{\theta}_i$  and  $\boldsymbol{\theta}_q$ , and  $\hat{\boldsymbol{z}}_{\mathcal{A}_p}$  shows the predicted target target state using the observations in  $\mathcal{A}_p$ .

Enforcing the condition in (17) avoids the formation of multiple chains corresponding to the same target and avoids unnecessary increase in the number of edges. The number of skip connections introduced in the graph is controlled by the robustness level; that is,  $0 \le \rho \le (N_S - 2)$ , which sets the maximum number of missed detections that can be tolerated across the sensor array. In this way, addition of such edges provides a flexible mechanism to provide robustness against missed detection in the sensors while keeping search space in control.

**Complexity:** The non-decreasing property of F(A) is used to discard unlikely chains in the early stages of DFS. This allows for rapid extraction of associations without requiring search over all possible chains in the graph. The minimum track length threshold,  $\gamma$ , is reset to its maximum value after each relaxation. Therefore, the skip edges in the graph can be removed at the end of the inner loop to reduce search complexity further. Therefore, our approach exploits the geometric structure of observations across multiple sensors to reduce search complexity.

# C. Spatial Association using Edge-based State Likelihoods

Before evaluating the performance of our main algorithm, we describe an iterative search method, which relies on the fact that an approximate kinematic state estimate can be derived by using two connected observations in a graph. In other words, a state estimate can be obtained for each edge in a graph, which is a part of the association chain A. Therefore, the search space for the association problem in (7) can be reduced to the set of edges.

The likelihood of a candidate  $z_e$  corresponding to an edge  $e \in \mathcal{E}$  can be computed as,

$$\mathcal{L}(\boldsymbol{z}_e) = \sum_{i=1}^{N_S} \left[ \min_{\boldsymbol{\theta} \in \Theta_i} \left( \frac{(r'_i - r_i)^2}{\sigma_r^2} + \frac{(d'_i - d_i)^2}{\sigma_d^2} \right) \right]$$
(18)

where  $[r'_i, d'_i] = \mathcal{T}_i(\boldsymbol{z}_e)$  is the perceived range and Doppler at sensor *i* for target state  $\boldsymbol{z}_e$ . Then, the most likely candidate can be selected by evaluating (18) over all edges and choosing the one that achieves the minimum negative log likelihood; that is,  $\boldsymbol{z}^* = \boldsymbol{z}_{e^*}$  for  $e^* = \arg \min_{e \in \mathcal{E}} \mathcal{L}(\boldsymbol{z}_e)$ . Then, the observations associated with  $\boldsymbol{z}^*$  can be identified via the following neighborhood constraint:

$$\mathcal{N}(\boldsymbol{z}^*) = \bigcup_{i=1}^{N_S} \{ (r_i, d_i) | (r_i - r_i^*) \le \delta_r \land (d_i - d_i^*) \le \delta_d \}$$

where  $[r_i^*, d_i^*] = \mathcal{T}_i(\boldsymbol{z}^*)$  are the perceived range-Doppler at sensor *i* and  $\delta_r$  and  $\delta_d$  are the range and Doppler resolution parameters defined in Appendix A. The algorithm carrying out this Spatial Association using Edge-based State Likelihoods (*SAESL*) procedure is presented in Algorithm 2.

Algorithm 2 SAESL Algorithm

- 1: INITIALIZE GRAPH WITH OBSERVATIONS  $\Theta$ :  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
- 2: AUGMENT GRAPH with skip edges
- 3: for h = 0 to  $\rho$  do
- 4: ADD SKIP-h EDGES TO GRAPH G

```
5: end for
```

```
6: INITIALIZE \mathcal{Z} = \emptyset
```

- 7: while  $\mathcal{E} \neq \emptyset$  do
- 8: FIND MOST LIKELY CANDIDATE,  $\mathcal{Z} \leftarrow \boldsymbol{z}^*$ from edge  $\boldsymbol{z}^* = \arg \min_{e \in \mathcal{E}} \mathcal{L}(\boldsymbol{z}_e)$
- 9: Remove all vertices explained by  $\boldsymbol{z}^*$ ,  $\boldsymbol{\mathcal{V}} \leftarrow \boldsymbol{\mathcal{V}} - \boldsymbol{\mathcal{N}}(\boldsymbol{z}^*)$

11: end while

12: RETURN Selected candidates  $\mathcal{Z}$ 

Since all edges in the graph are checked while selecting the candidates, this approach exhibits higher complexity than our proposed algorithm. Moreover, evaluation of state likelihood  $\mathcal{L}(\boldsymbol{z}_e)$  in (18) is more expensive than evaluation of chain likelihood  $\mathcal{L}(\mathcal{A})$  in (5) as it involves a minimization over all other observations. In Section IV, we use this algorithm as a benchmark against our proposed algorithm.

## **IV. SIMULATION RESULTS**

In this section, we evaluate the performance of the proposed spatial association algorithm, SAGA against the SAESL algorithm through various performance metrics. We consider a linear array of  $N_S$  FMCW radar sensors each of which collects range and Doppler observations from the scene. The FMCW radar system parameters are set based on the ones that are used in typical low cost automotive systems at mm-Wave frequencies [19] where range and Doppler resolutions are  $\delta_r = 0.3 \text{ m}$  and  $\delta_d = 0.5 \text{ m/s}$  respectively. In the simulations, a single snapshot of the scene is considered with multiple targets having equal received SNR at all sensors. The kinematic states of targets are randomly selected based on uniform distributions  $x \sim \mathcal{U}(-8m, 8m), y \sim \mathcal{U}(2m, 12m), v_x \sim \mathcal{U}(-10m/s, 10m/s), v_y \sim \mathcal{U}(-10m/s, 10m/s).$ 

It is important to note that when range and Doppler separation between two targets gets small, the estimation algorithm either provides a merged estimate or results in detection anomalies such as miss and false alarm. In order to differentiate the scenes with such estimation errors due to range-Doppler proximity, we consider two different scenarios with two different scenes. The *well-separated* scene is generated by enforcing a minimum separation between the range and Doppler of the targets at all sensors. The *adverse* scene does not have such constraints and contains additional missed detection anomalies by randomly removing measurements from the sensors with probability  $P_{miss}$ . Unless stated otherwise, the nominal values of system parameters are presented in Table I.

TABLE I Simulation Parameters

Number of targets	$N_T = 20$
Number of radar sensors	$N_S = 6$
SNR	-10 dB
Sensor Array Width	$L_W = 4 \text{ m}$
Max range, Doppler	19.2 m, ±16 m/s
Simulated misses	$P_{miss} = 0.05$
Robustness Level	$\rho = 4$

#### A. Localization Accuracy

In this subsection, we analyze the localization accuracy of kinematic state estimates obtained using associated sensor observations. This depends on the accuracy of underlying range-Doppler estimates. The position and velocity estimation errors for state estimates  $\hat{Z}$  are computed as follows:

$$D_p(\hat{\boldsymbol{\mathcal{Z}}}) = \frac{1}{n(\hat{\boldsymbol{\mathcal{Z}}})} \sum_{\hat{\boldsymbol{z}} \in \hat{\boldsymbol{\mathcal{Z}}}} \min_{\boldsymbol{z} \in \boldsymbol{\mathcal{Z}}^{\text{true}}} d_p(\boldsymbol{z}, \hat{\boldsymbol{z}})^2$$
$$D_v(\hat{\boldsymbol{\mathcal{Z}}}) = \frac{1}{n(\hat{\boldsymbol{\mathcal{Z}}})} \sum_{\hat{\boldsymbol{z}} \in \hat{\boldsymbol{\mathcal{Z}}}} \min_{\boldsymbol{z} \in \boldsymbol{\mathcal{Z}}^{\text{true}}} d_v(\boldsymbol{z}, \hat{\boldsymbol{z}})^2$$

where  $d_p(z, z') = \sqrt{(x - x')^2 + (y - y')^2}$  and  $d_v(z, z') = \sqrt{(v_x - v'_x)^2 + (v_y - v'_y)^2}$  are the errors in position and velocity, respectively. The CRBs for Range-Doppler and Position-Velocity estimates are evaluated in Appendix A and Appendix E, respectively. Figure 3 shows the Root Mean Square Error (RMSE) in range-Doppler estimated at sensor level for different number of targets in a *well-separated* case. We observe that range-Doppler RMSE at individual sensors achieves CRB at a SNR= -15 dB threshold. The RMSE for position-velocity estimates obtained from sensor observations



Fig. 3. Range-Dopper estimation accuracy and Position-Velocity estimation accuracy versus SNR. The position-velocity RMSE converges to the CRB bound as SNR increases and the SNR at which this convergence occurs is called as SNR threshold. The SNR threshold provides an indicator for the localization performance when multiple targets,  $N_T > 1$  are present.

also achieve their CRB at the same SNR threshold. This shows that association using *SAGA* does not introduce any additional errors to the localization process when SNR is above this threshold. However, the RMSE increases sharply below the SNR threshold due to the difficulty in associating noisy range-Doppler pairs. Therefore, we use nominal SNR = -10 dB in our simulations to perform further analysis.

**Cardinality Error and OSPA**: In the case of multiple targets, the number of *valid* targets identified by the system is also an important performance metric. An estimated target  $\hat{z}$  is classified to be *valid* only if it lies within a region "close" to the true targets,  $\min_{z \in \mathbb{Z}^{\text{true}}} ||\hat{z} - z|| < \bar{d}$  where  $\bar{d}$  sets the maximum error threshold. The cardinality error is defined as the difference between actual number of targets and the number of estimated target; that is,  $N_T - N_e = |\mathbb{Z}^{\text{true}}| - |\hat{\mathbb{Z}}|$ . That error is caused due to the detection anomalies in the estimation algorithm at sensor level as well as during the association stage. In such cases, the localization accuracy by itself does not capture the true performance of the system. Therefore, we use the OSPA metric [20], which combines the localization and cardinality error into a single performance metric and is given by,

$$OSPA(\hat{\boldsymbol{\mathcal{Z}}}) = \sqrt{\frac{1}{n(\hat{\boldsymbol{\mathcal{Z}}})} \left(\sum_{i=1}^{m} \min\left(d_c(\hat{\boldsymbol{z}}_i), \bar{d}\right)^2 + |N_e - N_T|\bar{d}^2\right)}$$

where *m* is the number of *valid* targets,  $N_e - N_T$  is the cardinality error and,  $d_c(\hat{z}_i)$  is the localization error computed relative to *closest* true target given as

$$d_c(\hat{oldsymbol{z}}_i) = \min_{oldsymbol{z} \in oldsymbol{\mathcal{Z}}^{ ext{true}}} d_p(oldsymbol{z}, \hat{oldsymbol{z}}_i)^2 + d_v(oldsymbol{z}, \hat{oldsymbol{z}}_i)^2$$
 .

Figure 4 shows the OSPA error along with the localization and cardinality errors with increasing scene density in the *well*separated case. Both localization error and cardinality error start to increase as the scene gets denser until a breaking point where the cardinality error increases significantly. At SNR=-15 dB, this breaking point occurs near  $N_T = 17$  for SAGA whereas  $N_T = 21$  for SAESL algorithm. Notice that the localization error is misleading beyond this point since it only considers the errors in the reduced set of *valid* targets. Hence, the OSPA metric effectively combines both quantities so that it represents localization error only when scene is sparse and cardinality errors when the scene is dense. We observe that SAGA has slightly worse overall performance compared to the SAESL as the number of targets increases. However, the performance difference reduces as we increase SNR. Moreover, SAGA obtains the association with significantly lower complexity than SAESL as we show in the next section.



Fig. 4. Overall localization accuracy versus number of targets at SNR = -15, -10 dB. The solid and dotted lines represent the performances of SAGA and SAESL association algorithms, respectively.

#### B. Complexity Reduction

In this section, we analyze the computational savings achieved by the proposed *SAGA* algorithm and provide comparison against traditional approaches. In order to effectively compare the performance, we now consider adverse scenes in which the sensor observations contain detection anomalies. Figure 5 shows the graph truncation over the iterations of the graph search with different miss probabilities for *SAGA* and *SAESL*. When miss probability is low, *SAGA* rapidly extracts all chains. As the missed detections increase, the robust scheme automatically increases the number of iterations by allowing the relaxation of constraints in DFS graph search. On the other hand, *SAESL* always requires large number of iterations.

SAGA provides robustness to the missed detections by selectively adding skip edges to the graph. This mechanism reduces the OSPA error in adverse scenarios at the expense of more computational complexity. The level of robustness can be tuned using parameter  $\rho$  based on the adversity of the scene. Figure 5 also shows the estimation performance for different robustness levels with increasing scene adversity (i.e., increasing miss detections). OSPA error reduces with higher robustness levels. However, low robustness level (e.g.,  $\rho = 1$ ) is sufficient to obtain good performance at typical miss detection probability  $P_{miss} < 0.05$ . Similarly, the higher robustness level helps to reduce the cardinality errors when the scene contains higher number of targets. The highest robustness level is  $\rho = 4$ , which corresponds to the minimum chain length constraint in (16a) with  $n(\mathcal{A}) \ge 2$ .



Fig. 5. (Top) Graph size at end of each iteration of association algorithm for different  $P_{miss}$ . SAGA is denoted by solid line while SAESL is denoted by dotted line. (Bottom) OSPA versus  $P_{miss}$  with different robustness levels  $\rho$ 

**Runtime Comparison:** We now compare the computational complexity of our approach against *SAESL*. Computing the number of operations that occur during the association process is difficult since the number of chains visited depends on a variety of factors such as the fitting error thresholds and minimum chain length. However, given the same sensor estimates for the simulated scenes, we compare the relative complexities of *SAGA* against other methods in Figure 6 in terms of total number of operations of Floating Point

operations (FLOPS) conducted during association and the total runtime. We observe that *SAGA* exhibits an order of magnitude lower complexity reduction compared to the *SAESL* algorithm. Moreover, this improvement increases as the number of targets increases, which highlights the advantage of our approach. In addition, the increase in the robustness level of the proposed algorithm (e.g., from  $\rho = 0$  to  $\rho = 4$ ) causes a slight increase in complexity compared to the *SAESL* approach.

We also compare the complexity against traditional methods such as gated Nearest neighbor filter (NN) and Minimum cost flow (MCF). The NN association scheme [10] builds the association chain by starting with a local kinematic state estimate from a pair of sensor observations and sequentially adding the nearest measurement from other sensors to update this state. The MCF association scheme [6] identifies the most likely set of chain by solving the minimum cost maximum flows over the graph. The cost of each edge is set based on its relative likelihood similar to our *SAESL* method. We use an optimized implementation [21] of *MCF* for the comparison purposes.

In order to compare the complexity of those algorithms, we count the number of times that the primary objective function (i.e., the likelihood cost in (7)) is computed during the graph search procedure. Figure 7 provides that comparison with increasing number of targets. Since the *NN* method is not able to predict the correct chain due to the greedy criteria and requires repeated search over the graph, the total complexity of it approaches to the complexity of the *SAESL* algorithm as the scene becomes dense. On the other hand, *MCF* algorithm predicts the chains relatively well and its complexity lies between our approach and *NN*.

Figure 8 shows the overall runtime of algorithms for an increasingly denser scene. We observe that *SAGA* is faster than the other methods by an order of magnitude. Since, the FLOPS count is not available from those implementations, we only compare the overall runtime which follows similar trends as FLOPS count and provides a reasonable estimate of algorithmic complexity.



Fig. 7. Number of evaluations of Likelihood  $\mathcal{L}(\mathcal{A})$  (solid) and Geometric fitting error  $F(\mathcal{A})$ (dotted) with increasing number of targets. *SAGA* has the lowest complexity across all scene densities while *NN* and *MCF* lie between *SAGA* and *SAESL*.

#### C. Benefit of Super-Resolution

Our algorithm extracts the geometric relationships between range-Doppler measurements based on the sensor array geometry and builds the association chains by adding likely observations at new sensors to the existing chains. In this section, we investigate the role of enhanced accuracy of range and Doppler estimates obtained using NOMP [3] super-resolution algorithm in spatial association by providing comparison against coarse estimates obtained using DFT. Figure 9 shows the localization and cardinality error for both cases. From the figure, it is obtained that the localization accuracy using NOMP estimates achieves the CRB when number of targets are moderate, whereas DFT has higher RMSE as expected. However, the RMSE of NOMP deviates away from CRB as the number of targets increases and approaches to the DFT accuracy for dense scenes. It is important to note that our association algorithm works even with the coarse estimates even though NOMP provides accuracy boost for our algorithm which identifies more targets resulting in lower cardinality errors compared to DFT in the presence of multiple targets.

Since NOMP estimates are more accurate, their geometric fitting errors are better than DFT. This allows the reduction in association time at the expanse of some computation



Fig. 6. Association complexity versus the number of targets averaged over 100 trials using nominal parameters with robustness levels  $\rho = 0$  and  $\rho = 4$ . Total number of FLOPS is denoted by blue line while the runtime is in red.



Fig. 8. Runtime comparison with traditional algorithms.



Fig. 9. Estimation accuracy (thin) and cardinality error (thick) versus number of targets at SNR = -15 dB.

overhead over DFT during estimation. Figure 10 compares the runtime of the estimation and association stages with different number of sensors for  $N_T = 20$  targets. We observe that the association time with NOMP estimates is 10 times lower than the one with DFT estimates while the estimation overhead is about 2 - 3 times higher. This complexity reduction is due to the lower geometric fitting error of association chains formed using higher accuracy NOMP estimates. Figure 10 also shows that the complexity of association stage becomes more significant than the one of estimation stage for the overall complexity as the number of targets and the number of sensors increases. Therefore, the overall complexity reduction achieved via enhanced accuracy estimates becomes more pronounced with a denser scene and larger number of sensors.



Fig. 10. Runtime comparison of association (solid) & estimation (dotted) stages versus number of sensors.

# D. Array Geometry

In this subsection, we analyze the localization performance of linear sensor arrays from the perspective of data association. We consider the adverse scene with  $P_{miss} = 0.2$  to emphasize our findings. The array width and the number of sensors affect both localization accuracy and association complexity.

Increasing the array width generates more spatial diversity in range-Doppler measurements across sensors. This helps to reduce the OSPA error for a given number of sensors. On the other hand, larger distance among the sensors weakens the pruning criteria for the graph edges used in (8) resulting in a denser graph with a higher number of potential associations between sensors. Therefore, the overall localization performance improves with wider arrays at the expense of slightly more association complexity. The available sensor width is an important design constraint in practical applications (e.g., length of side profile of a vehicle). For that reason, we analyze the effect of number of sensors in the presence of fixed array width  $L_W = 4$  m.

We find that increasing the number of sensors also improves association performance as well as association complexity. Figure 11 shows OSPA versus number of sensors for *SAESL* and *SAGA*. While the OSPA for *SAESL* association reduces with more sensors, we observe that the OSPA for *SAGA* with robustness level  $\rho$  achieves minimum OSPA with  $N_S = \rho + 3$ sensors, and increases for  $N_S > \rho + 3$ . This is caused due to the missed observations, which prevent formation of chains with minimum length constraint  $N_S - \rho$ . For an array with  $N_S$  sensors and a robustness level of  $\rho$ , the expected number of missed targets can be expressed as

$$\mathbb{E}[\text{miss}] = \sum_{k=1}^{\min(N_S-2,\rho+1)} \binom{N_S}{k} P_{miss}^k (1 - P_{miss})^{N_S-k}.$$

Figure 12 shows that missed targets observed using our approach closely match this expected value for various values of  $\rho$  and  $N_S$ .

As a result, we obtain that the robustness level needs to be increased to avoid higher cardinality errors even though localization error reduces with more sensors. That causes increase in the complexity of our algorithm; however, it still achieves lower complexity compared to *SAESL* algorithm. We leave as an open issue the design of more sophisticated methods for selection of a subset of sensors during the association stage to reduce the complexity further.

# V. CONCLUSION

We have shown that simple constraints relating range-Doppler observations to sensor geometry can be exploited



Fig. 11. Association versus number of sensors for SAESL (thick dotted) and SAGA (thin solid) with different robustness levels.



Fig. 12. Comparison of simulated (solid) and theoretical (dotted) cardinality error.

to significantly reduce the complexity of spatial association. Our system-level simulations demonstrate that our framework for spatial association based on these geometric constraints is robust to noisy observations and detection anomalies, and that it scales well with the number of sensors and targets. Our approach is compatible with standard FFT-based range-Doppler processing, but enhanced accuracy estimation at each sensor (i.e., super-resolution of range and Doppler) significantly improves both localization accuracy and association complexity. Important topics for future investigation include extending these concepts to more complex target models (e.g., for extended targets, and targets causing both specular and diffuse reflection), and combining them with complementary strategies utilizing platform and/or target motion across multiple snapshots.

# APPENDIX A CRB FOR RANGE AND DOPPLER

The Cramér Rao Bound provides an estimation theoretic lower bound on the sample covariance of range-Doppler estimates; that is,  $Cov(\theta_i) \ge I(\theta_i)^{-1}$  where  $I(\theta_i)$  is Fisher Information Matrix (FIM) given by,

$$I(\boldsymbol{\theta}_i) = \mathbb{E}\left[\left(\nabla_{\boldsymbol{\theta}_i} \mathcal{L}(m_i^{obs} | \boldsymbol{\theta}_i)\right) \left(\nabla_{\boldsymbol{\theta}_i} \mathcal{L}(m_i^{obs} | \boldsymbol{\theta}_i)\right)^H\right]$$

where  $\mathcal{L}(m_i^{obs}|\boldsymbol{\theta}_i)$  is the log likelihood of the observed signal for a given target range-Doppler  $\boldsymbol{\theta}_i$ . For an FMCW radar, this expression simplifies to [18],

$$I(\boldsymbol{\theta}_i) = \kappa SNR \begin{bmatrix} 1/\delta_r^2 & 0\\ 0 & 1/\delta_d^2 \end{bmatrix}$$
(19)

where  $\kappa$  is a constant,  $\delta_r$  and  $\delta_d$  are the Rayleigh range and Doppler resolutions, respectively. We set the nominal variance of range-Doppler estimates based on the value of CRB at nominal SNR = -20 dB; that is,  $\sigma_{r_i}^2 = \delta_r^2/(\kappa SNR)$  and  $\sigma_{d_i}^2 = \delta_d^2/(\kappa SNR)$ .

#### APPENDIX B

# ASSOCIATION CONSTRAINT RELAXATION

The choice of initial stopping thresholds  $\tau_f^n$  and  $\tau_l^n$  and scaling factor  $\beta$  for subsequent relaxations in *SAGA* algorithm governs the total complexity of association algorithm. In order

to initialize the association algorithm, we set tight thresholds for  $\mathcal{L}(\mathcal{A})$  and  $\mathcal{F}(\mathcal{A})$ . Assuming the range-Doppler observations have small error (i.e.,  $w_i^R \ll r_i$ ,  $w_i^D \ll d_i$  in (3)), the expected negative log likelihood in (5) can be approximated as

$$\mathcal{L}(\mathcal{A}) \approx \sum_{\boldsymbol{\theta}_i \in \mathcal{A}} \left( \frac{(w_i^R)^2}{\sigma_r^2} + \frac{(w_i^D)^2}{\sigma_d^2} \right) \,.$$

Since  $w_i^R \sim \mathcal{N}(0, \sigma_r^2)$  and  $w_i^D \sim \mathcal{N}(0, \sigma_d^2)$  are standard Normal distributed random variables,  $\mathcal{L}(\mathcal{A}^k)$  has chi-squared distribution,  $\chi^2_{2n(\mathcal{A})}$  with  $2n(\mathcal{A})$  degrees of freedom. Then, the expected fitting error in (14) can be approximated as

$$F(\mathcal{A}) = \sum_{\theta_i \in \mathcal{A}} \frac{\left( (\hat{r}_i \hat{d}_i) - (r_i d_i) \right)^2}{\eta_1} + \frac{\left( (\hat{r}_i)^2 - (r_i)^2 \right)^2}{\eta_2}$$
$$\approx \sum_{\theta_i \in \mathcal{A}} \frac{\left( r_i w_i^D + d_i w_i^R \right)^2}{\eta_1} + \frac{\left( 2r_i w_i^R \right)^2}{\eta_2} \tag{20}$$

where  $(\hat{r}_i \text{ and } \hat{d}_i)$  denote the perceived range-Doppler pair at sensor *i* for predicted state *z* and  $\theta_i = (r_i, d_i)$  denotes the observed range-Doppler pair at sensor *i*. The normalization factors  $\eta_1, \eta_2$  are set to the variance of numerator terms which is,

$$\begin{split} &(\eta_1)_i = \operatorname{Var}[r_i w_i^D + d_i w_i^R] \approx \sigma_{r_i}^2 d_i^2 + r_i^2 \sigma_{d_i}^2 + \sigma_{r_i}^2 \sigma_{d_i}^2 \\ &(\eta_2)_i = \operatorname{Var}[2r_i w_i^R] \approx 4r_i^2 \sigma_{r_i}^2 \;. \end{split}$$

Using those values to normalize (20) results in  $\mathcal{F}(\mathcal{A}^k) \sim \chi^2_{2n(\mathcal{A})}$  being chi-squared distributed with  $2n(\mathcal{A})$  degrees of freedom. Hence, the thresholds for the association algorithm are determined as follows,

$$\tau_f^{n(\mathcal{A})} : \Pr(F(\mathcal{A}) > \tau_f^{n(\mathcal{A})}) = P_{\text{FA}}$$
  
$$\tau_l^{n(\mathcal{A})} : \Pr(\mathcal{L}(\mathcal{A}) > \tau_l^{n(\mathcal{A})}) = P_{\text{FA}}$$

where  $P_{\rm FA}$  is the nominal false alarm rate set to  $P_{\rm FA} = 0.01$ .

Note that while the normalization factors  $\eta_1, \eta_2$  depend on  $r_i, d_i$ , we set this based on the maximum range, Doppler values to get a conservative initial value. This does not cause a problem since the successive relaxation procedure loosens that threshold so that chains with high fitting error can be extracted.

The relaxation factor,  $\beta$  should be set appropriately. Choosing a high value causes faster convergence but might lead to false chains being identified. On the other hand, a low value delays the extraction of *loose* chains. In the simulations, we find that  $\beta = 2$  performs well.

# APPENDIX C MINIMUM AMBIGUITY ASSOCIATION

**Lemma 1.** In the ideal detection scenario (i.e., no miss or false alarms), the number of candidate locations generated between a pair of sensors is minimum for consecutive sensors.

*Proof.* Recall that candidate locations are generated when range perceived at a pair of sensors satisfy conditions in (8). For a candidate,  $z_{ij}^{pq}$  generated by incorrectly associated observations,  $\theta_i^p, \theta_j^q$ , across consecutive sensors i, j, the following relations hold,

$$r_i^p - r_j^q < l_{ij}, \quad r_i^p + r_j^q > l_{ij}.$$
 (21)

Now consider  $\bar{q}^{th}$  observation at sensor k adjacent to sensor j which corresponds to same target as  $\theta_j^q$ , the following relations hold,

$$\begin{aligned} r_j^q - r_k^{\bar{q}} &< l_{jk} \qquad (\text{using (8)}) \\ r_j^q + l_{jk} &> r_k^{\bar{q}} \qquad (l_{jk} \geq 0, \text{ Triangle inequality}) \end{aligned}$$

Using these along with (21) we obtain,

$$r_i^p - r_k^q < l_{ik}, \quad r_i^p + r_k^q > l_{ik}$$

Hence any candidate produced between consecutive sensors i, j also generates a candidate between sensors i, k by skipping over intermediate sensor j. Hence,

$$\sum_{p=1}^{n(\Theta_i)} \sum_{q=1}^{n(\Theta_{i+1})} n(\boldsymbol{z}_{i,i+1}^{pq}) \le \sum_{p=1}^{n(\Theta_i)} \sum_{q=1}^{n(\Theta_k)} n(\boldsymbol{z}_{i,k}^{pq})$$

Therefore, the number of candidates generated between a pair of sensors is minimum for consecutive sensors.  $\Box$ 

Association complexity is due to the presence of unwanted candidate targets which need to be discarded based on their likelihood. When a target is observed at all sensors, it is sufficient to associate observations along consecutive sensors. Lemma 1 states that the association of observations along consecutive sensors generates the lowest number of phantoms during graph search. Hence, the number of potential ambiguities is minimized when the graph search procedure is conducted across consecutive sensors first.

# APPENDIX D Depth First Search

A depth first search algorithm is outlined in Algorithm 3. At each node, the DFS procedure traverses through all branches which have geometric fitting error below the maximum error threshold  $\tau_f^{N_S}$ . On reaching the end of the graph, we select the chain if it satisfies the likelihood, fitting error, and minimum chain length constraints. In addition, we check for possible chain termination at each node after going through all its branches. This step implicitly accounts for the NULL state at the end of a chain.

# APPENDIX E CRB FOR POSITION AND VELOCITY

Using the range-Doppler model in Section 3, we evaluate the single target CRB for kinematic parameters  $\bar{z}$  using the log likelihood of range-Doppler observations  $\mathcal{A} = \{\boldsymbol{\theta}_i\}_{i=1}^{N_S}$  given kinematic state  $\bar{z}$ , which is

$$\mathcal{L}\left(\{\theta_i | \bar{z}\}_{i=1}^{N_S}\right) = \sum_{i=1}^{N_S} \left(\frac{(\bar{r}_i - r_i)^2}{\sigma_{r_i}^2} + \frac{(\bar{d}_i - d_i)^2}{\sigma_{d_i}^2}\right)$$

where  $\theta_i = (r_i, d_i)$  is the observed range-Doppler pair for sensor i,  $(\bar{r}_i, \bar{d}_i) = \mathcal{T}_i(\bar{z})$  is true range-Doppler pair for given target state  $\bar{z}$  and  $\sigma_{r_i}^2$  and  $\sigma_{d_i}^2$  are, respectively, the range and Doppler CRBs obtained in (19). The FIM for  $\bar{z}$  can be evaluated as

$$I(ar{m{z}}) = \mathbb{E}\left[ 
abla_{m{z}} \mathcal{L}\left( \{m{ heta}_i | ar{m{z}}\}_{i=1}^{N_S} 
ight) 
ight] \,.$$

Algorithm 3 Geometry Assisted Depth First Search						
1:	<b>procedure</b> GA-DFS $(v, A, \gamma, \tau)$					
2:	Get list of children of $v$ that geometrically fit,					
	$B(v) = \left\{ v_j : \mathcal{F}([\mathcal{A}, v_j]) < \tau_f^{N_S}  ight\}$					
3:	if $B(v) \neq \emptyset$ then					
4:	Sort $B(v)$ using geometric fitting error, $F([\mathcal{A}, v_j])$					
5:	for $v_j \in B(v)$ do					
6:	BRANCH out a new chain $\mathcal{A}^j : \mathcal{A} \leftarrow v_j$					
7:	$\mathcal{A}^o \leftarrow \text{GA-DFS}(v_j, \mathcal{A}^j, \gamma, \boldsymbol{\tau})$					
8:	Exit loop if valid chain $\mathcal{A}^o$ is found.					
9:	end for					
10:	end if					
11:	CHECK IF CHAIN CAN BE TERMINATED AT $v$					
12:	if $n(\mathcal{A}) \geq \gamma, \mathcal{L}(\mathcal{A}) < \tau_l^{n(\mathcal{A})}, F(\mathcal{A}) < \tau_f^{n(\mathcal{A})}$ then					
13:	Select $\mathcal{A}^o \leftarrow \mathcal{A}$ ,					
14:	end if					
15:	Output: $\mathcal{A}^o$					
16:	end procedure					

The CRB obtained from inverse FIM is used to find position and velocity CRB as follows,

$$CRB_p = I(\bar{z})_{(1,1)}^{-1} + I(\bar{z})_{(2,2)}^{-1}$$
$$CRB_v = I(\bar{z})_{(3,3)}^{-1} + I(\bar{z})_{(4,4)}^{-1}$$

The CRB of velocity is a function of both range and Doppler variances whereas the CRB of position only depends on the variance of range. We use the nominal range and Doppler CRB values to set the minimum separation distance threshold,  $\tau_z = 10\sqrt{CRB_p + CRB_v}$  between targets. This threshold is also used to check similarity between chains in the association algorithm.

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