Millimeter wave wireless networking and sensing

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The Plan

- Introduction: why mm wave
- The millimeter wave channel
- Mm wave networking standards
- Compressive tracking
- Protocol-level approaches to blockage & mobility
- Mm wave sensing
- LoS MIMO
- Open issues

Introduction

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2 decades of wireless growth

- Digital cellular is now global
  - 6B mobile phone subscribers today!
  - Connects the most remote locations to the global economy
- WiFi is pervasive and growing
  - Huge growth in carrier and enterprise markets
  - Huge potential in residential markets in developing nations
- Technology ~~is~~ was converging
  - MIMO, OFDM part of all modern standards

mmWave represents a fundamental disruption

mm-wave: what's different?

- System goals: multiGbps wireless
- Bandwidth no longer a constraint
- Channel characteristics
  - Sparse rather than rich scattering
- The nature of MIMO
  - Beamforming, diversity, multiplexing all different at tiny wavelengths
- Signal processing at multiGbps speeds
  - ADC is a bottleneck, OFDM may not be the best choice
- Networking with highly directional links

So really, everything is different!

Interdisciplinary approach is essential

Example: NSF GigaNets project

Circuits

Rodwell (UCSB)

Buckwalter (UCSB)

Arbabian (Stanford)

+

Systems

Madhow (UCSB)

Zheng (U. Chicago)

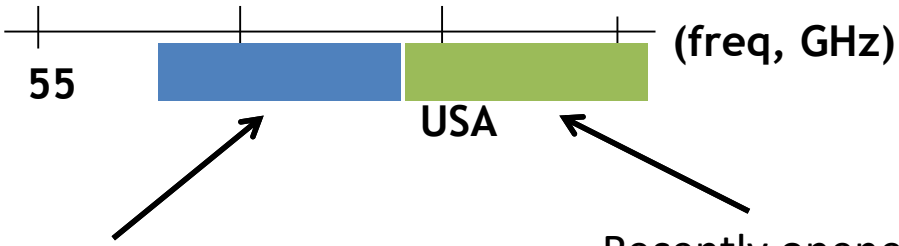
Zhang (UCSD)

The Opportunity

Technology and Applications

The end of spectral hunger?

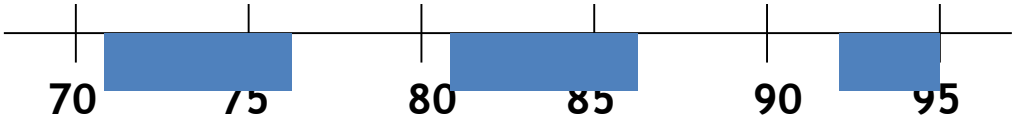
60 GHz: 14 GHz of unlicensed spectrum!



Oxygen absorption
(ideal for short-range, multihop)

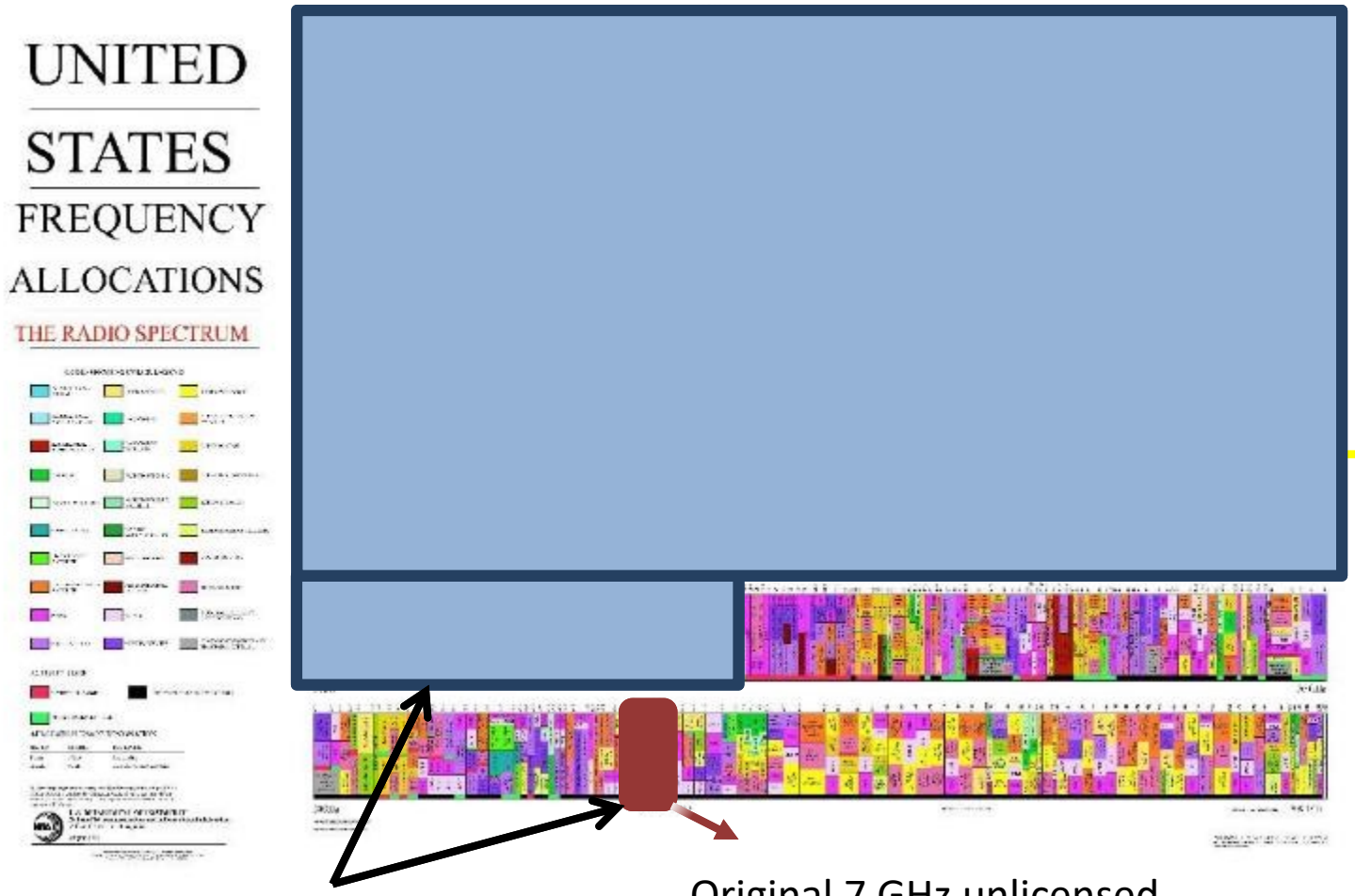
Recently opened up
No oxygen absorption
Longer range links, esp with no rain

E/W bands: 13 GHz of semi-unlicensed spectrum

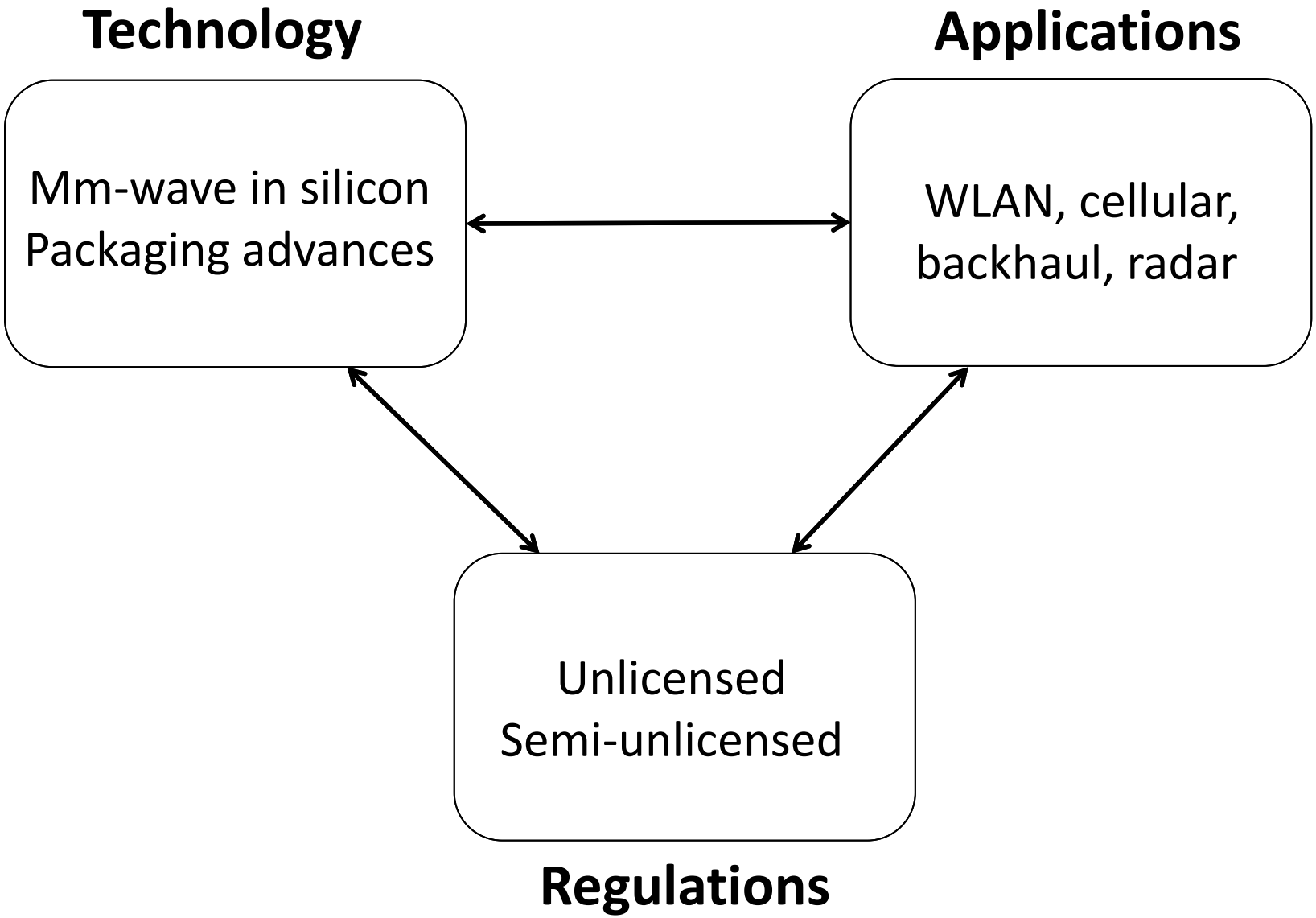


Bands beyond 100 GHz becoming accessible as RFIC and packaging technology advances

To put it in perspective



We now have a virtuous cycle



Initial industry focus: indoor 60 GHz networks

- WiGig spec/IEEE 802.11ad standard: up to 7 Gbps
- Support for moderately directional links
- 32 element antennas that can steer around obstacles

www.technologyreview.com

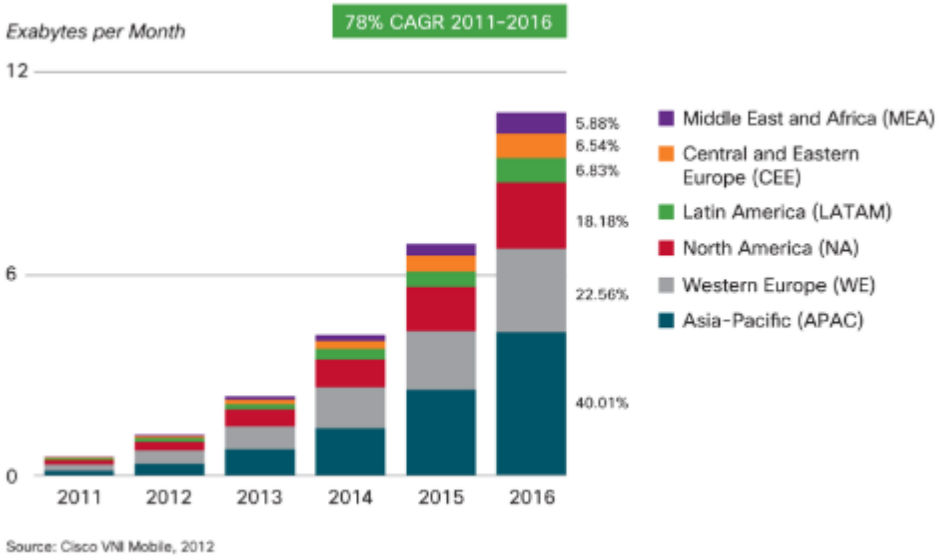
Progress due to push for WiGig

- 60 GHz CMOS RFICs ✓
  - WiFi-like economies of scale if and when market takes off
- Antenna array in package (32 elements) ✓
  - Good enough for indoor consumer electronics applications
- MAC protocol supporting directional links ✓
  - Good enough for quasi-static environments
  - Does not provide interference suppression
  - Does not scale to very large number of elements
- Gigabit PHY ✓
  - Standard OFDM and singlecarrier approaches
  - Does not scale to 10 Gbps at reasonable power consumption (ADC bottleneck)

Now: mmWave for 5G

Driven by exponential growth in cellular data demand

Figure 2. Global Mobile Data Traffic Forecast by Region



Source: Cisco VNI Mobile, 2012

NEED EXPONENTIAL INCREASE IN CELLULAR NETWORK CAPACITY (without breaking the bank)

Industry consensus on the need for Cellular 1000X

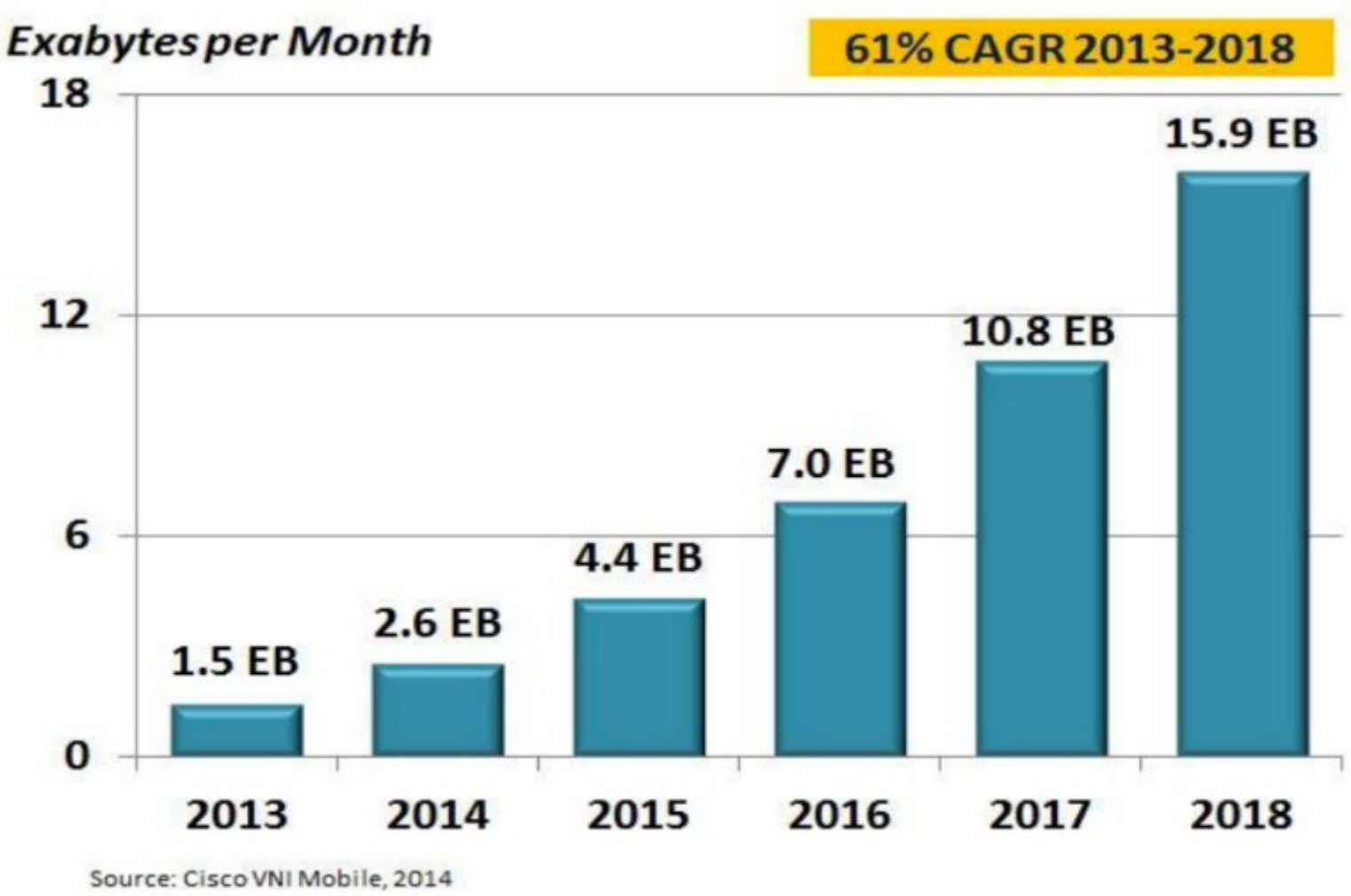


Figure courtesy: Nokia

Figure courtesy: Qualcomm

Figure courtesy: Cisco

Figure 2.1. Global Mobile Data Traffic growth 2013 to 2018 (Cisco VNI).

Mm-wave enables aggressive spatial reuse

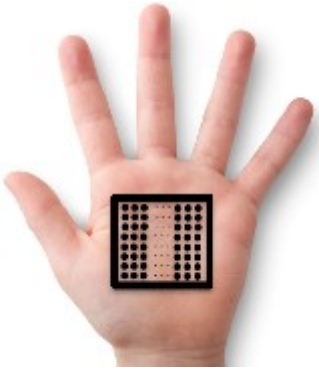
Large arrays in small form factors

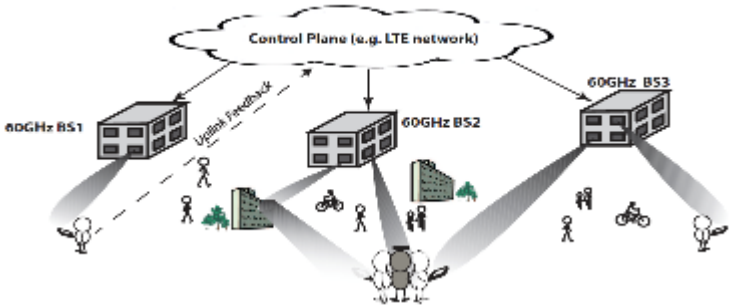
Directive links

Limited interference

Dense cells / much higher spatial reuse

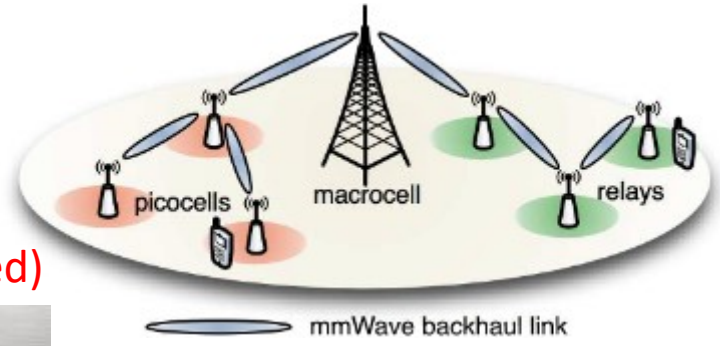
32 x 32
8 x 8 cm^2





Access

Terragraph
(WiGig repurposed)



Backhaul

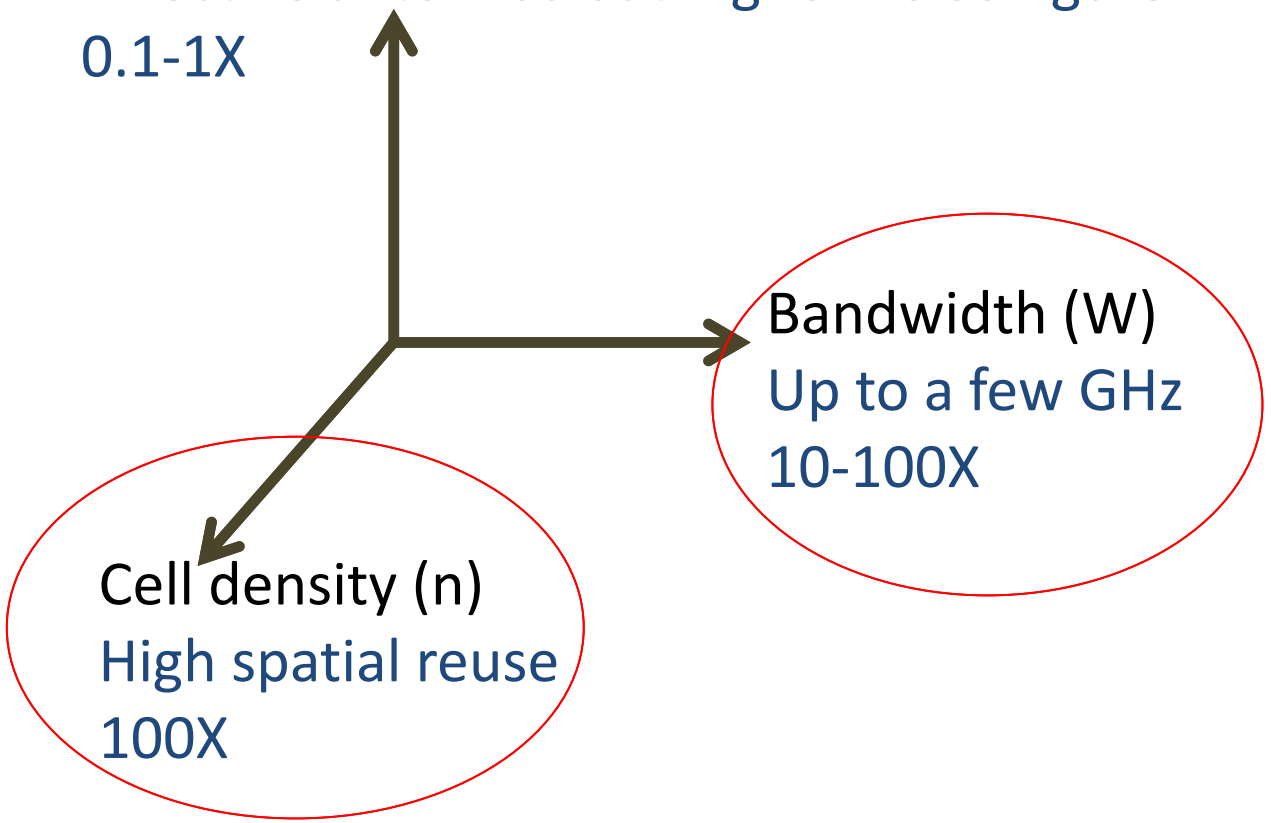
1000X via mm-wave

$$C = nWlog(1 + SNR)$$

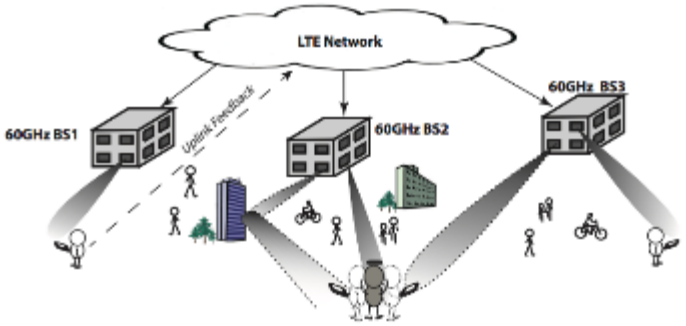
Spectral efficiency (SNR)

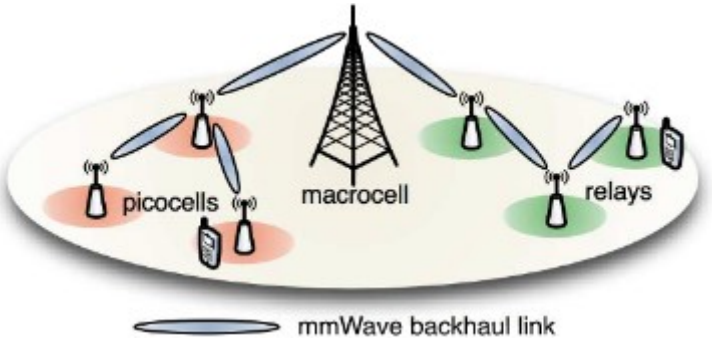
Directive antennas but higher noise figure

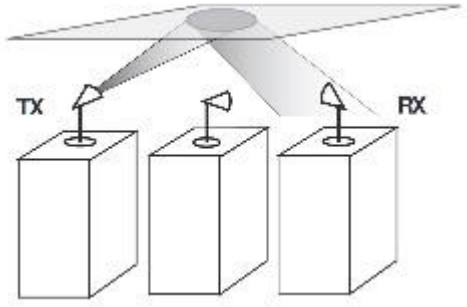
0.1-1X



mm-wave comm: a snapshot

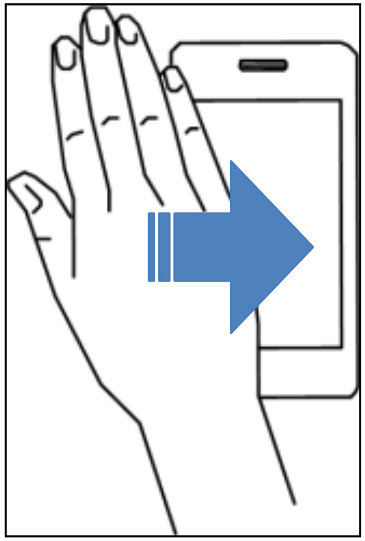






In addition…mmWave commodity radar

Vehicular situational awareness



Gesture recognition

Designs constrained by cost, complexity and geometry
Very different from classical long-range military radar

Challenges

(aka Research Opportunities)

Must revisit all key concepts in wireless design

- Revisiting channel models for tiny wavelengths
  - Sparse, easily blocked
  - Critical role of directionality
  - Geometric rather than statistical view of MIMO
- Revisiting signal processing architectures
  - The ADC bottleneck
- Revisiting networking
  - Highly directional links change MAC design considerations
  - Multi-band operation (e.g., 1-5 GHz and 60 GHz)
- Revisiting radar
  - Short-range geometry and hardware constraints
- Inherently cross-layer even at the level of comm and estimation theory
  - Node form factor, hardware constraints, propagation geometry

Channel Modeling

Slides mostly due to: Maryam Eslami Rasekh

Step 0: can we close the link?

Take-away from link budget
Low-cost silicon works for indoor links
Low-cost silicon also works for short outdoor links
(~100 meters)

Is propagation on our side?

- Can we attain the kind of system specs we want with technology compatible with the mass market?
  - Link budget for indoor links
  - Link budget for outdoor links (oxygen absorption)
- CMOS power amps: sweet spot 0-10 dBm
- SiGe power amps can go higher
- Using antenna arrays, can we go far enough so it is interesting?

Free space propagation

The simplest model for how transmit power translates to received power

Isotropic transmission ➔ at range *R,* the power is distributed over the surface of a sphere of radius *R*

Receiver antenna provides an aperture with an effective area for catching a fraction of this power

$$P_{RX} = \frac{P_{TX}}{4\pi R^2} \cdot$$ Receive antenna aperture

If the transmitter uses a directional antenna:

$$P_{RX} = \frac{P_{TX}}{4\pi R^2} \cdot A_{RX}$$

Transmit antenna gain

Relating gain to aperture

Antenna gain = ratio of aperture to that of an isotropic antenna

$$G = \frac{A}{\text{Aperture for an "isotropic" antenna}} = \frac{4\pi A}{\lambda^2}$$

Remarks

--For given aperture, gain decreases with wavelength

--Aperture roughly related to area ➔ at lower carrier frequencies (larger wavelengths) we need larger form factors to achieve a given antenna gain

Friis' formula for free space propagation

Given the antenna gains:

$$P_{RX} = P_{TX}\, G_{TX}\, G_{RX}\, \frac{\lambda^2}{16\pi^2 R^2}$$

For fixed antenna gains, the larger the wavelength the better

Given the antenna apertures:

$$P_{RX} = P_{TX}\, \frac{A_{TX} A_{RX}}{\lambda^2 R^2}$$

For fixed antenna apertures (roughly equivalent to fixed form factors), the smaller the wavelength the better, provided we can point the transmitter and receiver at each other

Applying Friis' formula

Going to the dB domain:

$$P_{RX,\mathrm{dBm}} = P_{TX,\mathrm{dBm}} + G_{TX,\mathrm{dBi}} + G_{RX,\mathrm{dBi}} + 10\log_{10}\frac{\lambda^2}{16\pi^2 R^2}$$

More generally:

$$P_{RX,\mathrm{dBm}} = P_{TX,\mathrm{dBm}} + G_{TX,\mathrm{dBi}} + G_{RX,\mathrm{dBi}} -$$

Plug in your favorite model for path loss

Free space path loss model gives us back the first formula:

$$L_{pathloss,\mathrm{dB}}(R) = 10\log_{10}\frac{16\pi^2 R^2}{\lambda^2}$$

Link budget

Given a desired receiver sensitivity (i.e., received power),
what is the required transmit power to attain a desired range?
OR
what is the attainable range for a given transmit power?
Must account for transmit and receive directivities, path loss, and add on a link margin (for unmodeled, unforseen contingencies)

$$P_{TX,\mathrm{dBm}} = P_{RX,\mathrm{dBm}}(\mathrm{min}) - G_{TX,\mathrm{dBi}} - G_{RX,\mathrm{dBi}} + L_{pathloss,\mathrm{dB}}(R) + L_{margin,\mathrm{dB}}$$

Link budget analysis

Basic comm theory maps modulation & coding scheme to Eb/N0 requirement; we then need to map to received power needed

Receiver sensitivity: minimum received power required to attain a desired error probability
(depends on the modulation scheme, bit rate, channel model, receiver noise figure)

We can now design the physical link parameters: transmit and receive antennas, transmit power, link range

Link budget: Once we know the receiver sensitivity, we can work backward and figure out the physical link parameters required to deliver the required received power (plus a margin of safety)

Example 60 GHz indoor link budget

2.5 Gbps link using QPSK and rate 13/16 code operating 2 dB from Shannon limit

$$(E_b/N_0)_{reqd} \approx 2.5dB$$

Noise figure 6 dB

Receiver sensitivity = -71.5 dBm

4x4 antenna array at each end, 2 dBi gain per element
➔ 14 dBi gain at each end

10 m range ➔ free-space path loss is about 88 dB

Transmit power with 10 dB link margin is only about -1.5 dBm!
(➔ can use less directive antennas)

Example 100 m outdoor 60 GHz link (backhaul, base-to-mobile)

Using 10 m indoor link budget as reference

Free space propagation loss increases by 20 dB

Oxygen absorption (16 dB/km) leads to 1.6 dB additional loss

Rain margin (25 dB/km for 2 inches/hr): 2.5 dB

Required transmit power goes up to **22.6 dBm**
For 4x4 array, TX power per element is **10.6 dBm**
(doable with CMOS, easy with SiGe)
EIRP = 22.6 dBm + 14 dBi = **36.6 dBm** < FCC EIRP limit of 40 dBm

What the link budgets tell us

- 60 GHz is well matched to indoor networking and to picocellular networks
  - Oxygen absorption has limited impact at moderate ranges
  - Heavy rain can be accommodated in link budget
  - Moderate directivity suffices
  - Electronically steerable links give flexibility in networking
  - Low-cost silicon implementations are possible
- For truly long range, need to avoid oxygen absorption
  - 64-71 (unlicensed), 71-76, 81-86 GHz (semi-unlicensed)
  - Bands above 100 GHz
  - Need very high directivity (can we steer effectively?)

Step 1: Channel Characterization

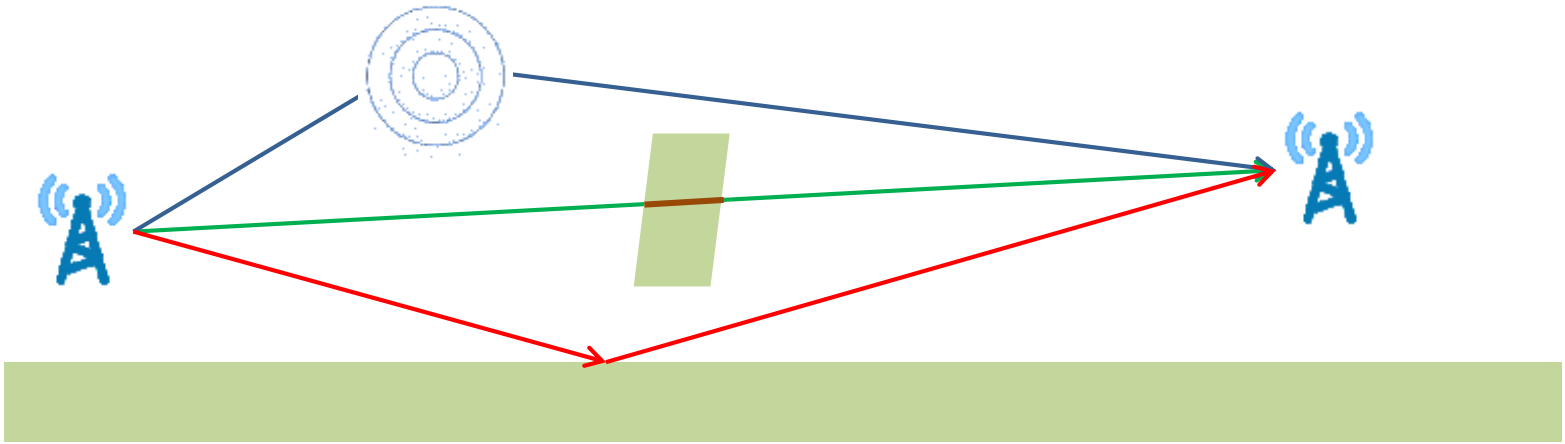
Take-away

Sparse, geometrically predictable channels

Very different from statistical models used at lower frequencies

Basics of channel modeling

- Sum of propagation paths
  - Free space propagation (LOS)
  - Specular reflection
  - Propagation through dielectric obstacles
  - Diffraction and scattering



All these components are strong in conventional lower frequency bands (<6GHz)

but in mmwave..?

REFLECTION

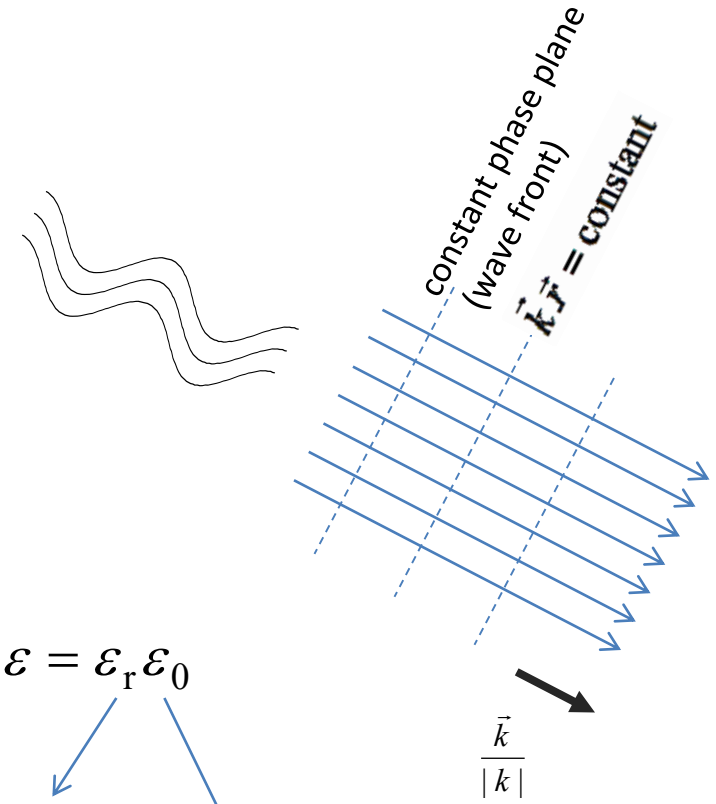
Reflection

- Plane wave traveling in homogenous environment

$$\vec{E}(\vec{r},t) = |\vec{E}_0| \cos(2\pi f t - \vec{k}.\vec{r} + \angle\vec{E}_0)$$

(phasor) $\vec{E}(\vec{r}) = \vec{E}_0 e^{-j\vec{k}.\vec{r}}$

$$|k| = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = 2\pi f\sqrt{\mu\varepsilon}$$



Magnetic permeability $\mu = \mu_r \mu_0$

Relative permeability

Magnetic permeability of vacuum

Electric permittivity $\varepsilon = \varepsilon_r \varepsilon_0$

Relative permittivity

Electric permittivity of vacuum

Reflection

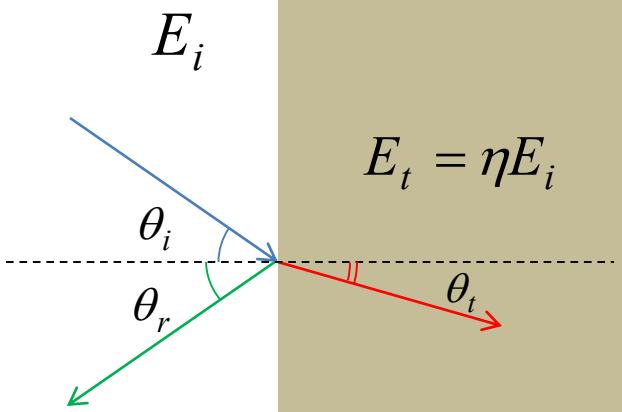
- Plane wave reflection and transition: Snell's law

$$\theta_r = \theta_i$$

$$\sqrt{\mu_1 \varepsilon_1}\sin\theta_i = \sqrt{\mu_2 \varepsilon_2}\sin\theta_t$$

$$v = \frac{c}{n} = \frac{1}{\sqrt{\mu\varepsilon}}$$

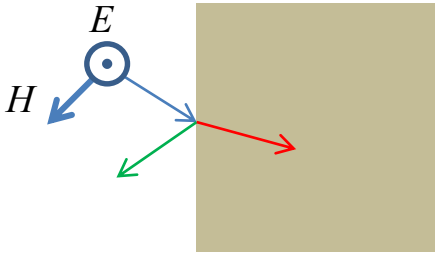
Speed of light in substance



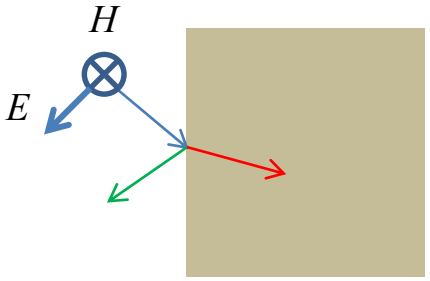
Reflection coeffs

- Fresnel formula derived from Maxwell's equations
  - Depend on magnetic permeability and electric permittivity

$$\rho_{\perp} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_t}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_i + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_t}$$

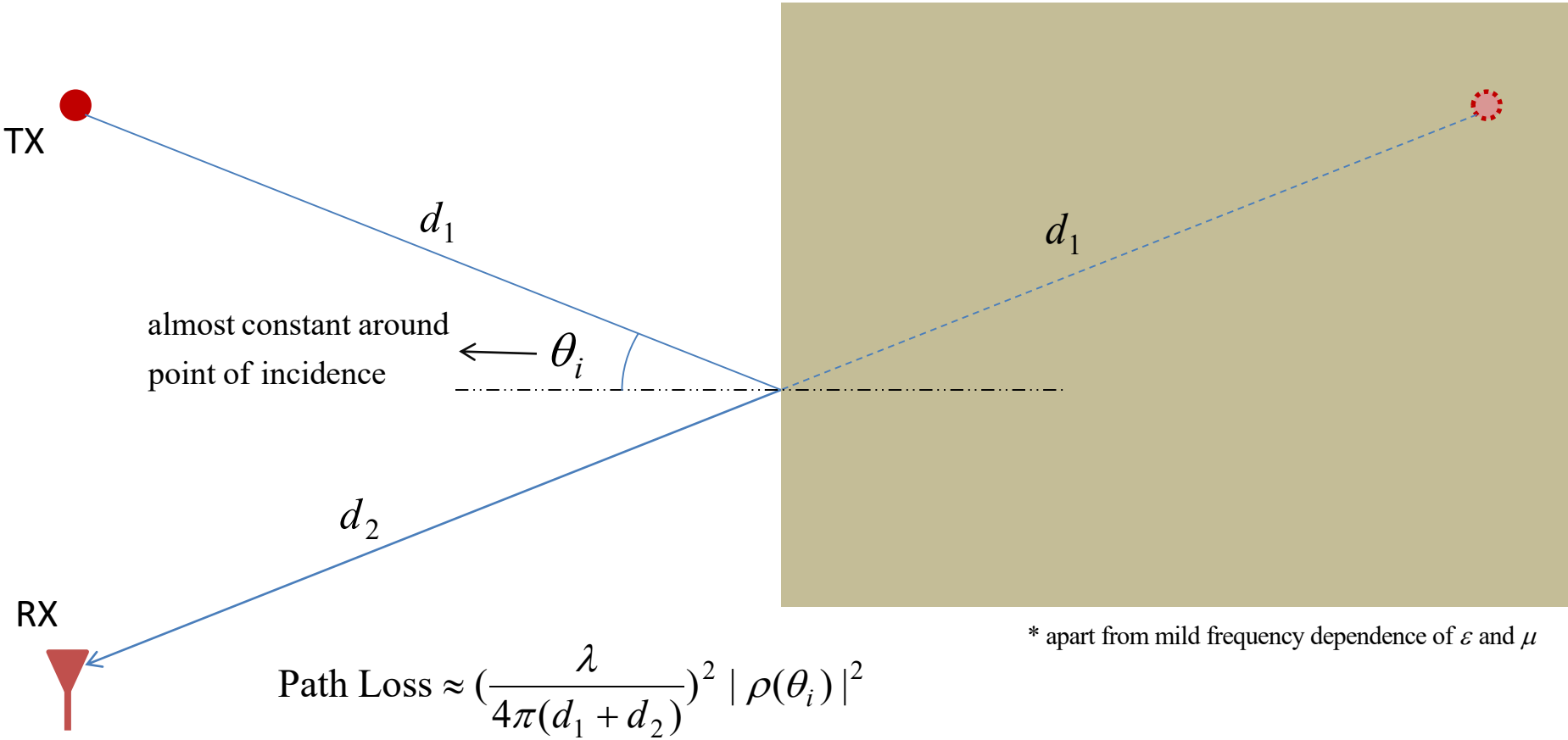


$$\rho_{\parallel} = \frac{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t - \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i}{\sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t + \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_i}$$



Reflection coeffs have mild freq dependence

- Quasi-plane wave:



* apart from mild frequency dependence of ε and μ

$$\text{Path Loss} \approx \left(\frac{\lambda}{4\pi(d_1+d_2)}\right)^2 |\rho(\theta_i)|^2$$

$$\text{Excess Loss} \approx |\rho(\theta_i)|^2 \Rightarrow \text{independent of frequency}^*$$

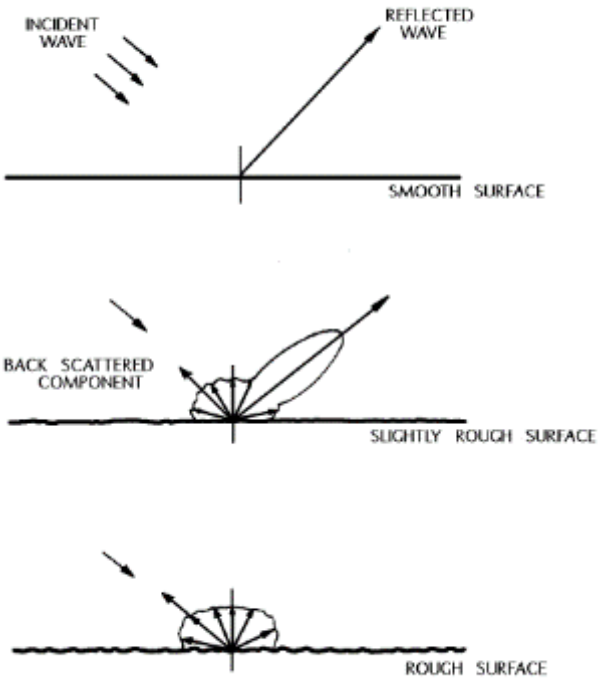
Main freq dependence is from rough scattering

- Reflection from **rough** surfaces:
  part of wave energy is **scattered**

$$\text{Path Loss} \approx \left(\frac{\lambda}{4\pi(d_1+d_2)}\right)^2 |\rho(\theta_i)|^2 \exp\left(-\frac{1}{2}\left(\frac{4\pi h_s \cos\theta_i}{\lambda}\right)^2\right)$$

h_s = std deviation of surface height *

$$\text{Excess Loss} \approx |\rho(\theta_i)|^2 \exp\left(-\frac{1}{2}\left(\frac{4\pi h_s \cos\theta_i}{\lambda}\right)^2\right)$$



$\Rightarrow$ Higher loss at higher frequencies (exponential)

(surfaces are *rougher* at shorter wavelengths)

* assuming Gaussian distribution of surface heights without sharp edge and shadowing effects

Roughness: 5 GHz vs 60 GHz

- Surface roughness std deviation varies from 0 (e.g. glass) to a few mm
- At low frequencies (f < 6 GHz, λ > 5 cm) most surfaces are smooth

$$h_s < 2\,\text{mm},\quad \lambda > 5\,\text{cm} \Rightarrow \exp(-8(\frac{\pi h_s \cos\theta_i}{\lambda})^2) \geq 0.88$$

$$\text{roughness loss} \leq 0.55\,\text{dB}$$

- At 60 GHz a surface with 0.6 mm roughness causes 5 dB of excess loss

$$h_s = 0.6\,\text{mm},\quad \lambda = 5\,\text{mm} \Rightarrow \exp(-8(\frac{\pi h_s \cos\theta_i}{\lambda})^2) = 0.32$$

$$\text{roughness loss} = 4.95\,\text{dB}$$

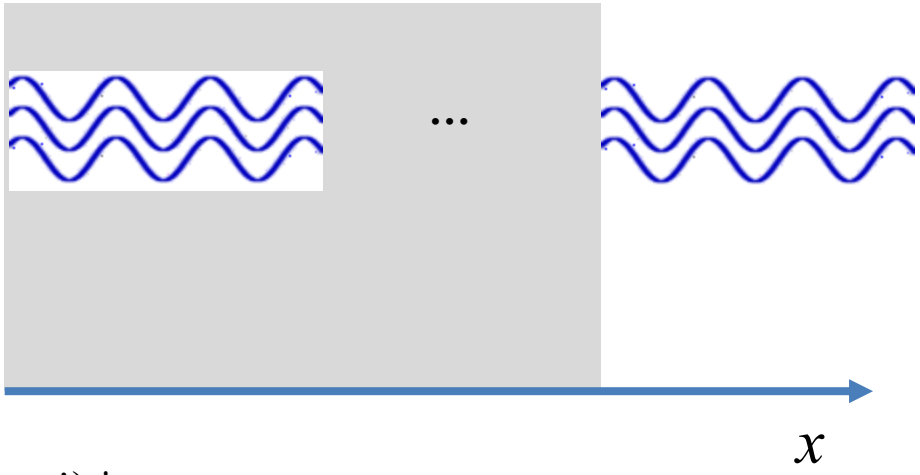
Take-away on reflections

One bounce path is usable
(typically 5-10 dB weaker than LoS)

Multi-bounce paths usually too weak to be useful
(bonus: less worry about interference)

BLOCKAGE

Propagation through dielectrics



$$E(x) = E_0 e^{-jkx}$$

$$k = 2\pi f\sqrt{\mu\varepsilon} \qquad \varepsilon = \varepsilon' - j\varepsilon'' \quad (\varepsilon'' << \varepsilon')^*$$

$$k \approx 2\pi f\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{2\varepsilon'}\right)$$

Phase change

Attenuation

$$P(x)/P_0 = |E(x)/E_0|^2 \approx \exp\left(-2\pi f\sqrt{\mu\varepsilon'}\frac{\varepsilon''}{\varepsilon'}x\right)$$

$\Rightarrow$ **Penetration loss increases exponentially with depth and frequency**

* for substances with conductivity σ, effectively $\varepsilon = \varepsilon' - j\varepsilon'' - j\frac{\sigma}{2\pi f}$

Propagation through dielectrics

Example: relative permittivity of concrete

@ 5 GHz: $\varepsilon_r = 4.8 - j0.6$

@ 60 GHz: $\varepsilon_r = 3.3 - j0.38$

$$P(x)/P_0 = \exp(-2\pi f\sqrt{\mu\varepsilon'}\frac{\varepsilon''}{\varepsilon'}x)$$

$$\sqrt{\mu_0\varepsilon_0} = \frac{1}{c} = 0.33\times10^{-8} \quad (\text{s/m})$$

$$\text{Loss} = \exp(-2\pi f\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r'}\frac{\varepsilon_r''}{\varepsilon_r'}x)$$

$\Rightarrow$ Loss of a 3 cm thick slab of concrete

@ 5 GHz: $\text{Loss} = \exp(-2\pi\times5\times10^9\times0.33\times10^{-8}\times\sqrt{4.8}\times\frac{0.6}{4.8}\times0.03) = 3.7\ \text{dB}$

@ 60 GHz: $\text{Loss} = \exp(-2\pi\times60\times10^9\times0.33\times10^{-8}\times\sqrt{3.3}\times\frac{0.38}{3.3}\times0.03) = 34\ \text{dB}$

How thick can a slab of concrete be for <10dB attenuation?

@ 5 GHz: 8.04 cm

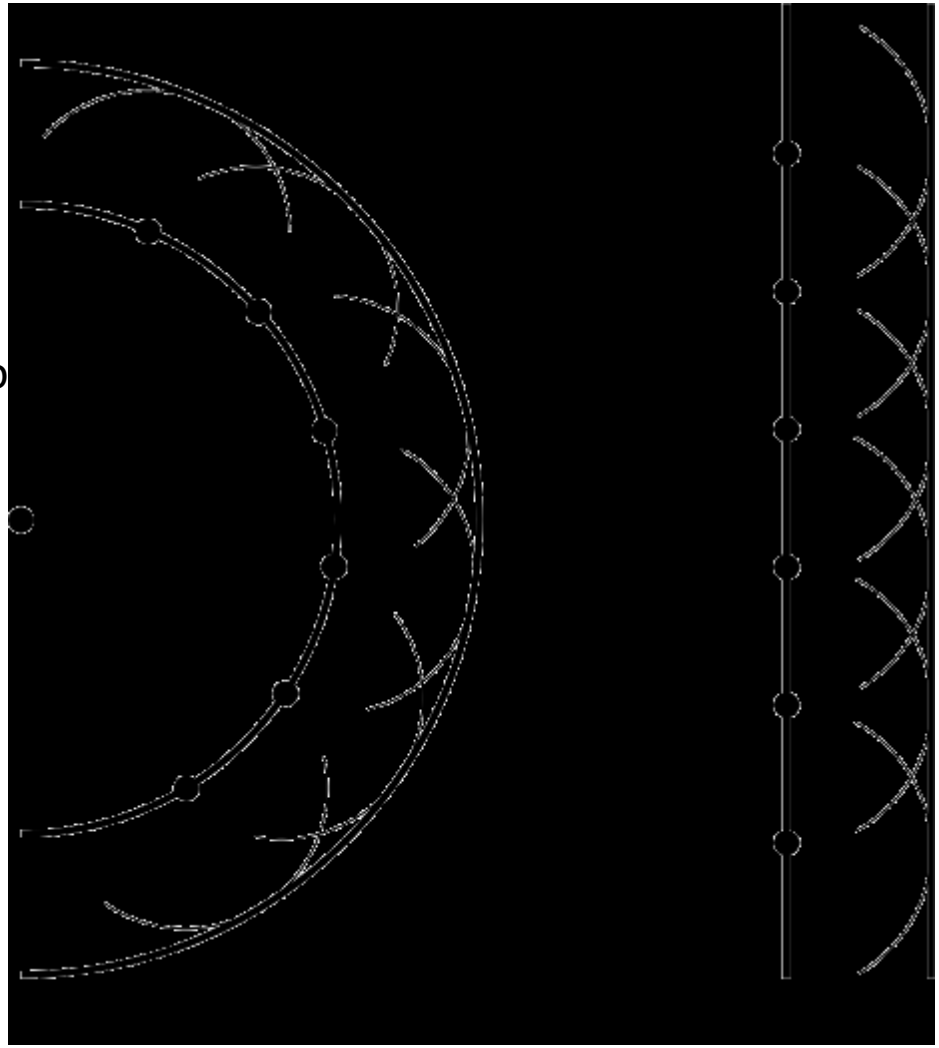
@ 60 GHz: 8.8 mm

Mm-wave cannot propagate through obstacles

Can we diffract *around* obstacles?

Need Huygen's principle to understand this

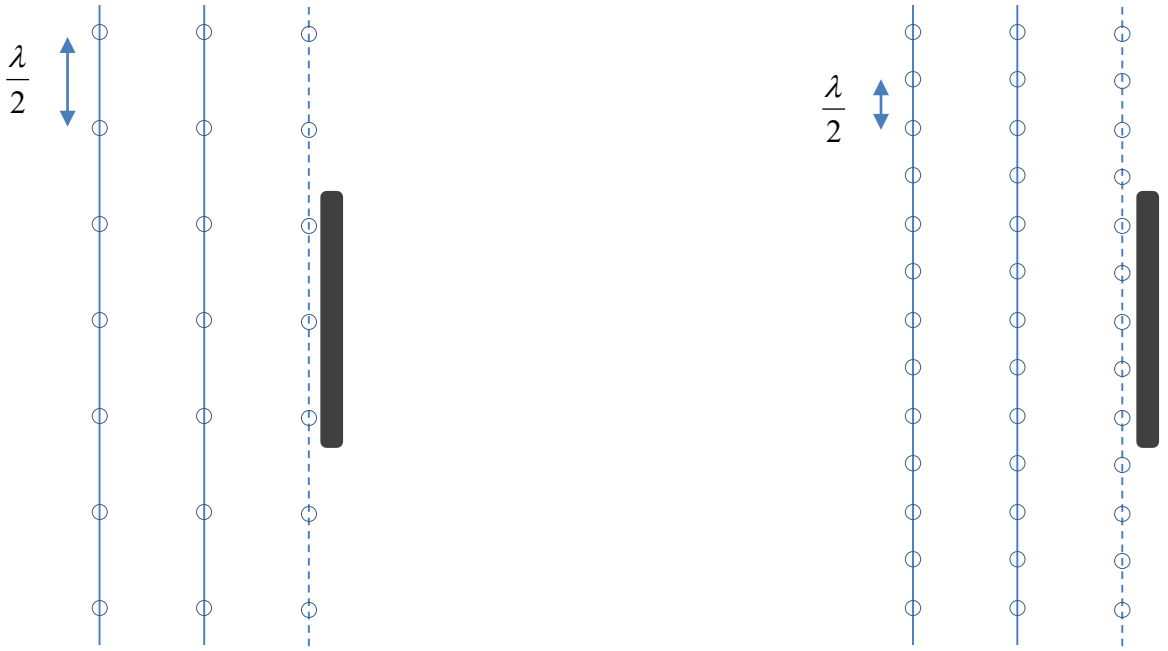
wavefront decompo
for point source



wavefront decomposition
for plane wave

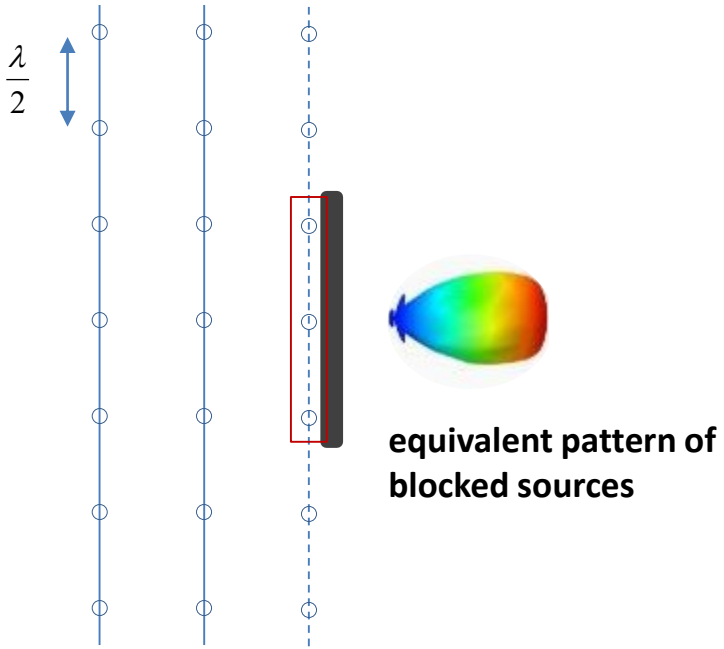
Huygens' principle

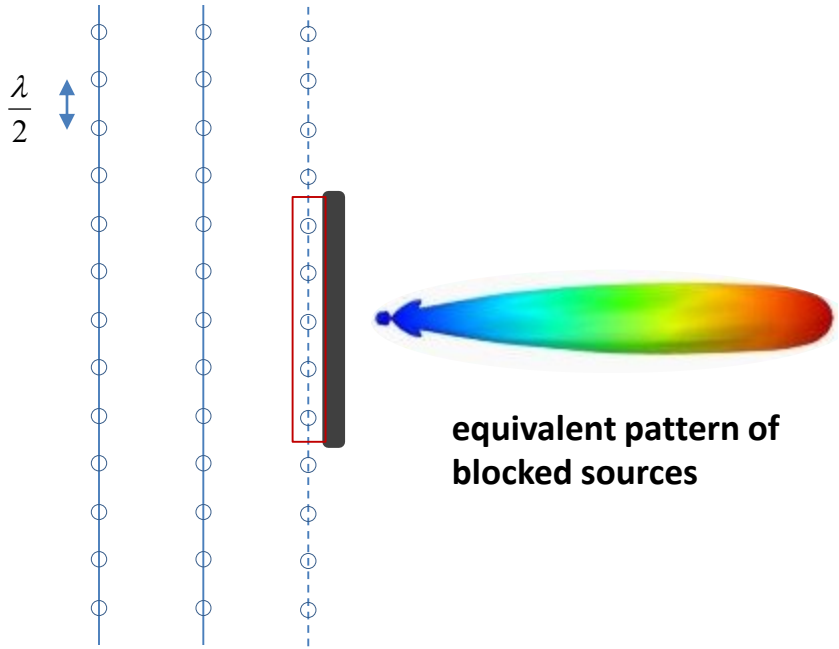
- If part of wavefront is blocked, contribution of that portion is lost



Huygens' principle

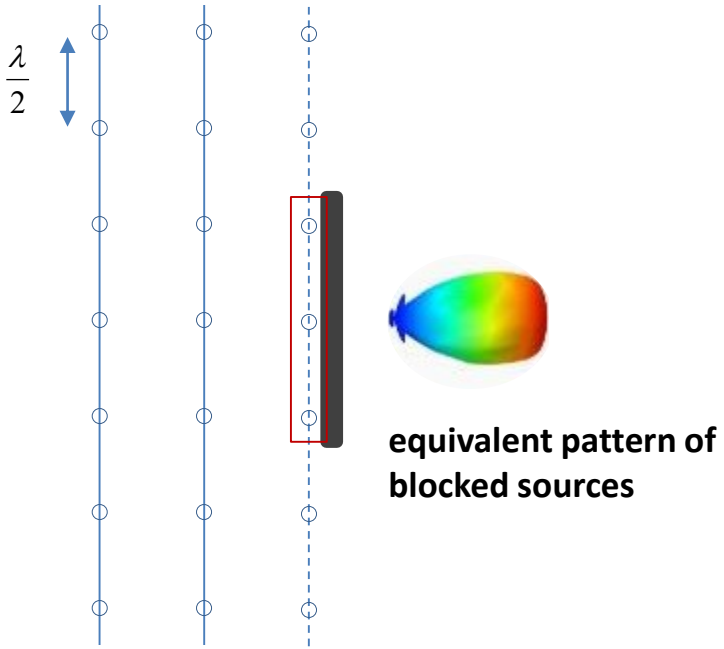
- If part of wavefront is blocked contribution of that portion is lost



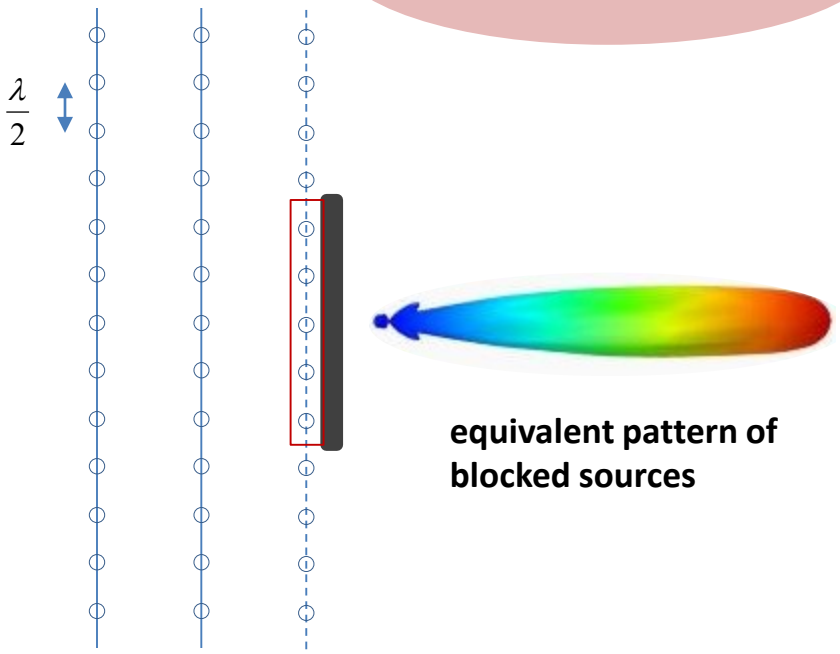


Huygens' principle

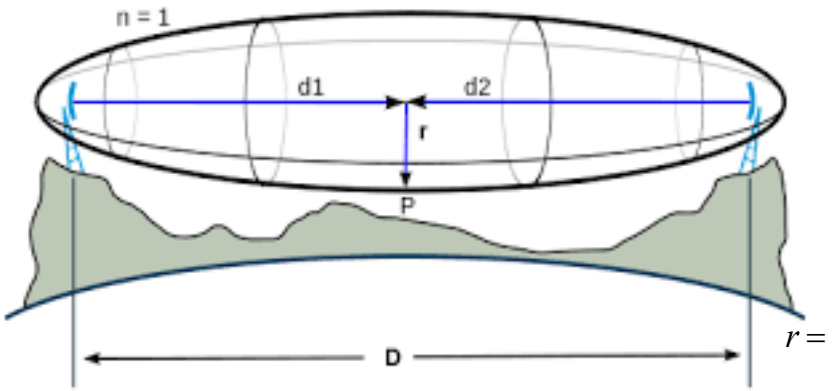
- If part of wavefront is blocked contribution of that portion is lost



more of power reaching front of obstacle is lost



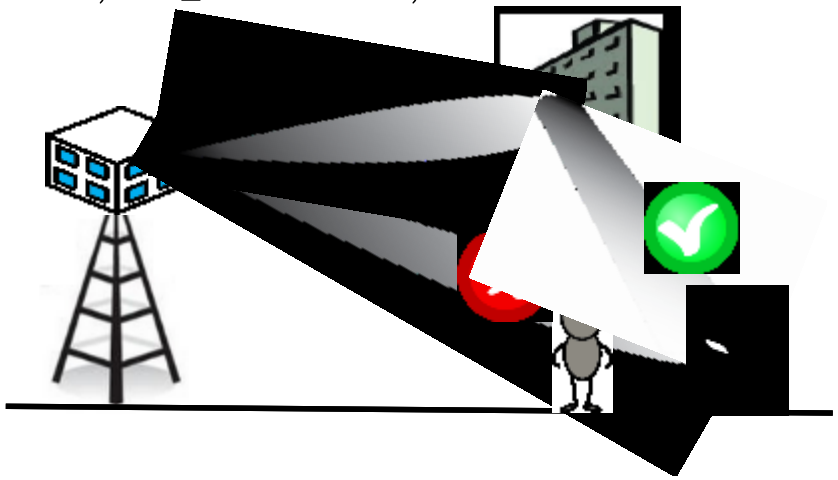
Can mostly write off diffraction



Fresnel zone heuristic

$$r = \sqrt{\frac{\lambda\, d_1 d_2}{d_1 + d_2}}$$

$$d_1 = 20\ m,\ d_2 = 2\ \mathrm{m},\ \lambda = 5\ \mathrm{mm} \rightarrow r \approx 10\ \mathrm{cm}$$



Can’t diffract around human obstacles in picocells, for example

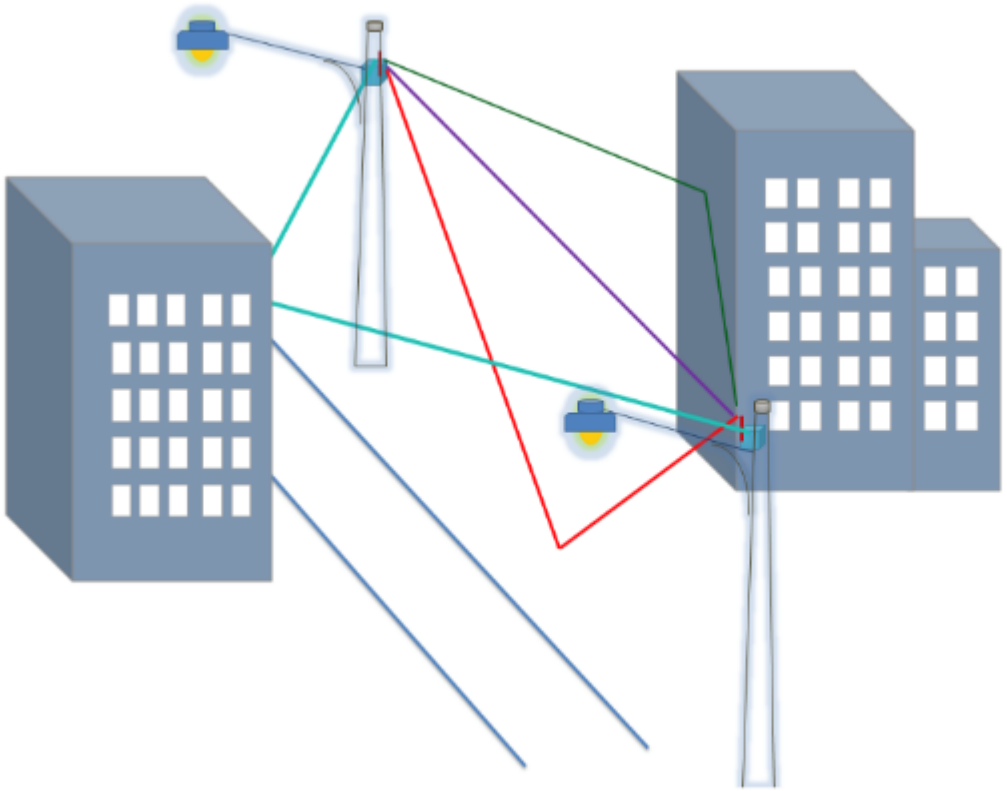
Take-away on blockage

When a path is blocked, it's blocked
(can't burn through it, can't diffract around it)

Must steer around obstacles

OVERALL CHANNEL MODEL

Quasi-deterministic modeling

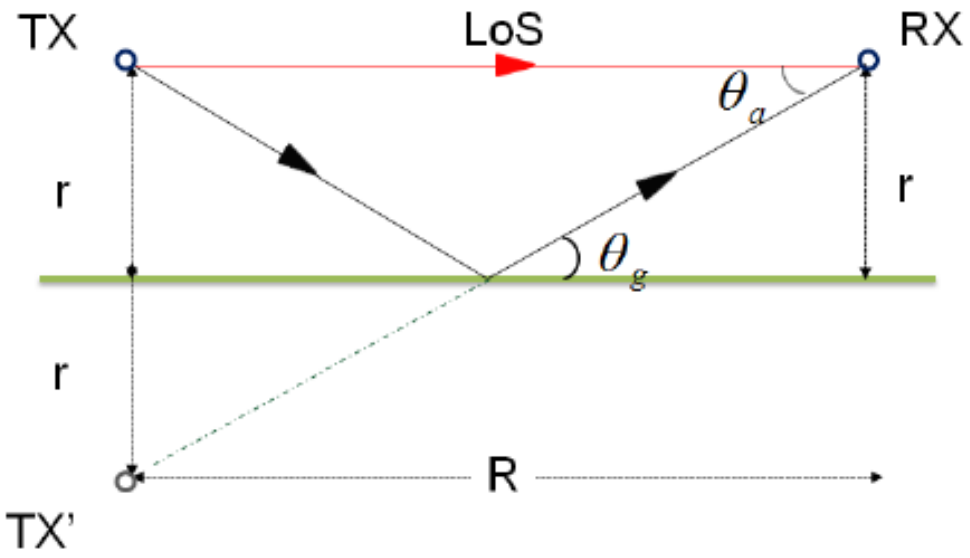


Example lamppost-to-lamppost link

LoS + single bounce reflections from side walls and road

Can vary lamppost heights and street width to get multiple realizations

Basic ray tracing

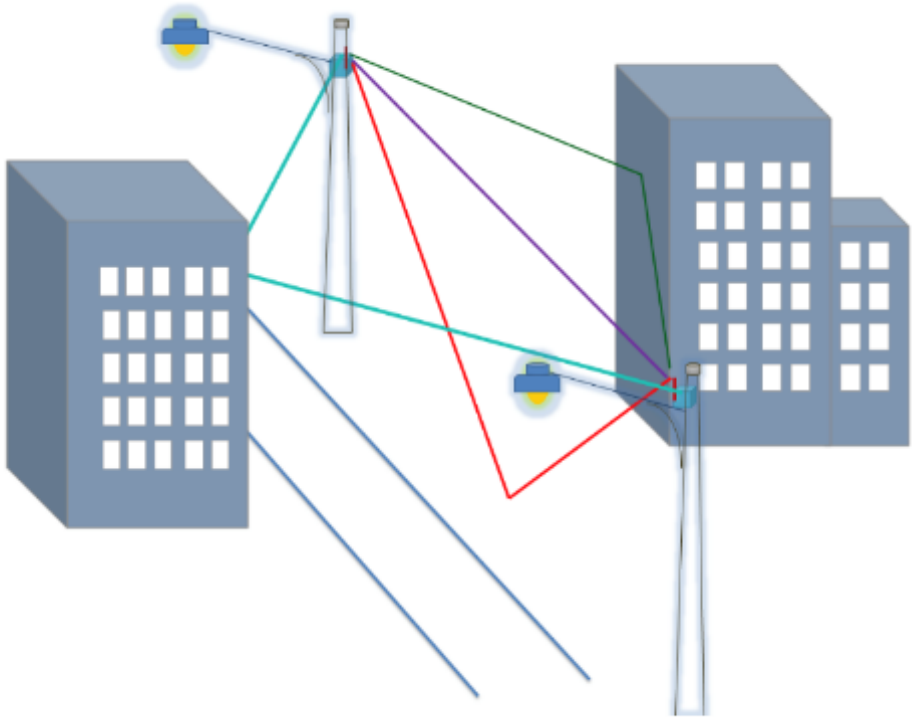


Relative delays: compute using geometry

Path strength: reflection model, propagation distance, beam patterns

Relative phase: uniform (small path length differences cause large phase differences)

Sparse channel impulse response

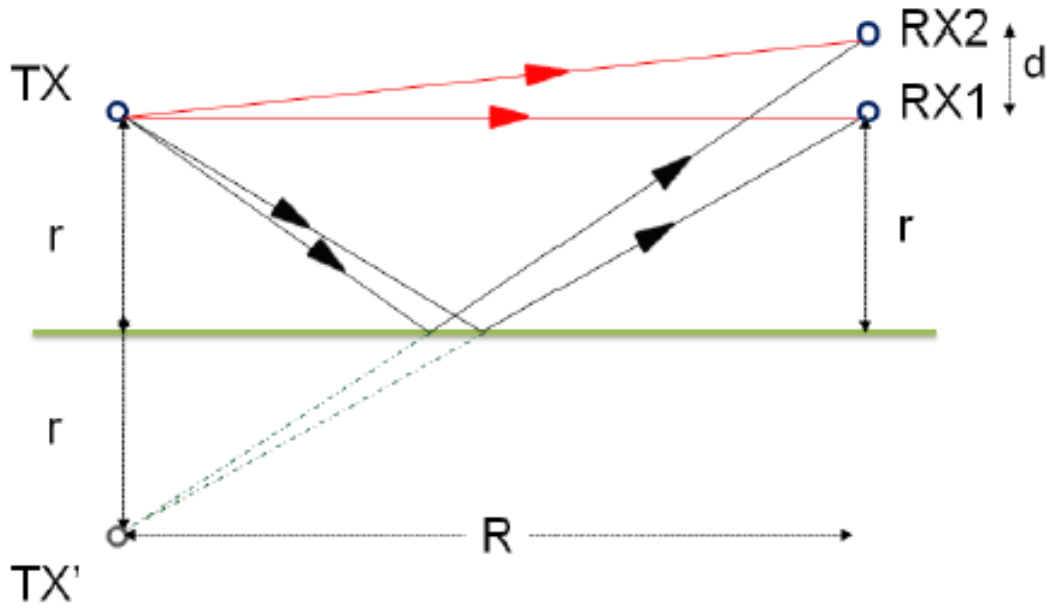


$$h(t) = \delta(t) + \sum_{i=1}^{N_r - 1} \alpha_i e^{-j\phi_i} \delta(t - \tau_i)$$

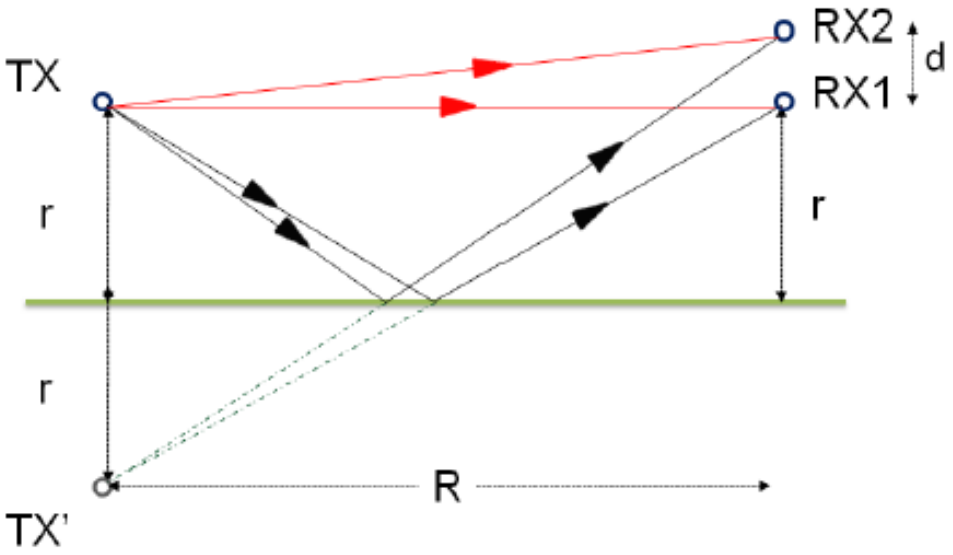
Gains α_i account for TX and RX directivity

Phases ϕ_i modeled as uniform over $[0, 2\pi]$

Easily extends to multiple antennas



Need to be careful with relative phases



Example SIMO system: relative channel gains are

$$h_1 = 1 - \alpha e^{-j\phi}\ , \quad h_2 = 1 - \alpha e^{-j(\phi+\gamma)}$$

Phase difference:

$$\gamma = \frac{2\pi}{\lambda}\Delta L_{reflected} \approx \frac{4\pi r d}{R\lambda}$$

$$\Delta L_{reflected} = \sqrt{R^2 + (2r+d)^2} - \sqrt{R^2 + (2r)^2} \approx \frac{2rd}{R}$$

Take-away on millimeter wave channel

Sparse and geometrically predictable

Take-way on millimeter wave channel

Sparse and geometrically predictable

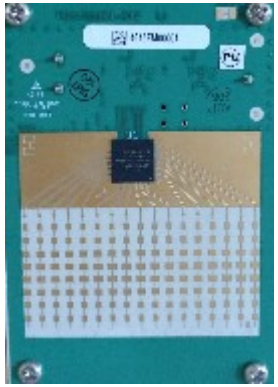
Do measurements back this up?

Measurements on UCSB campus

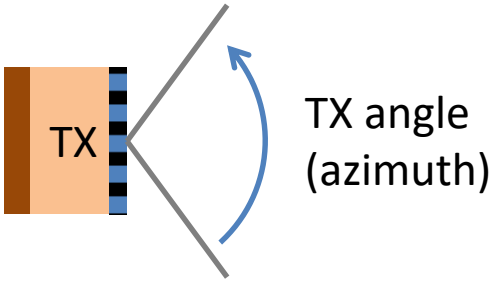
Measurements at different locations on campus

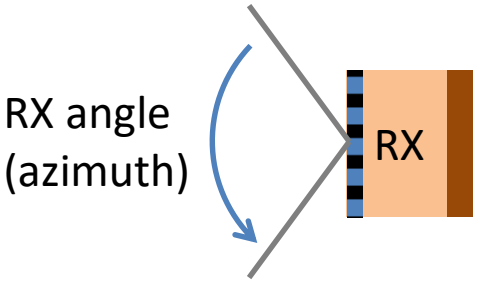
16-element phased array @ 60 GHz

Steers beam in horizontal plane (azimuth)



Facebook Terragraph node

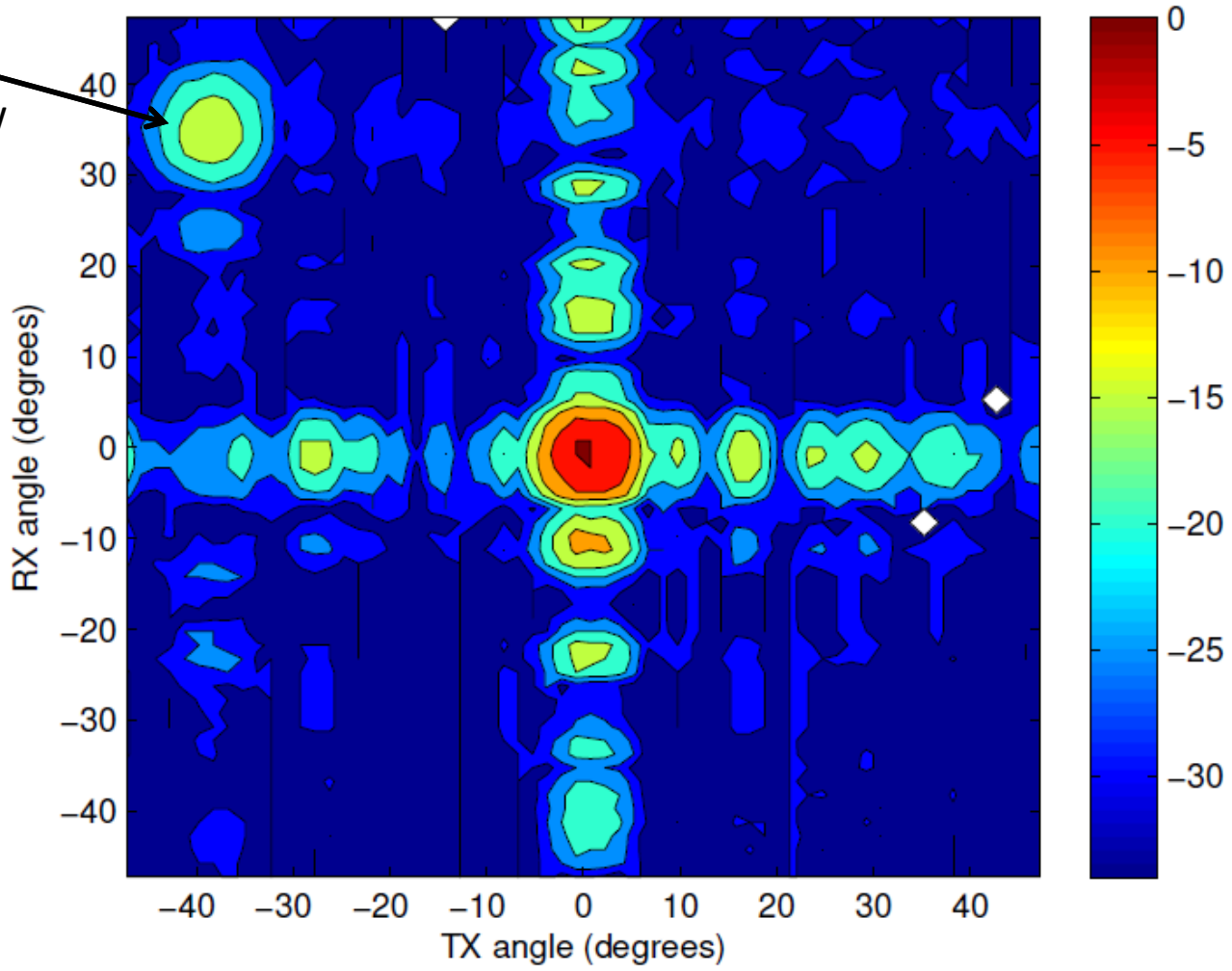




(Narrow vertical beam ➔ ground reflection not present on typical measurements)

Received power: angular profile

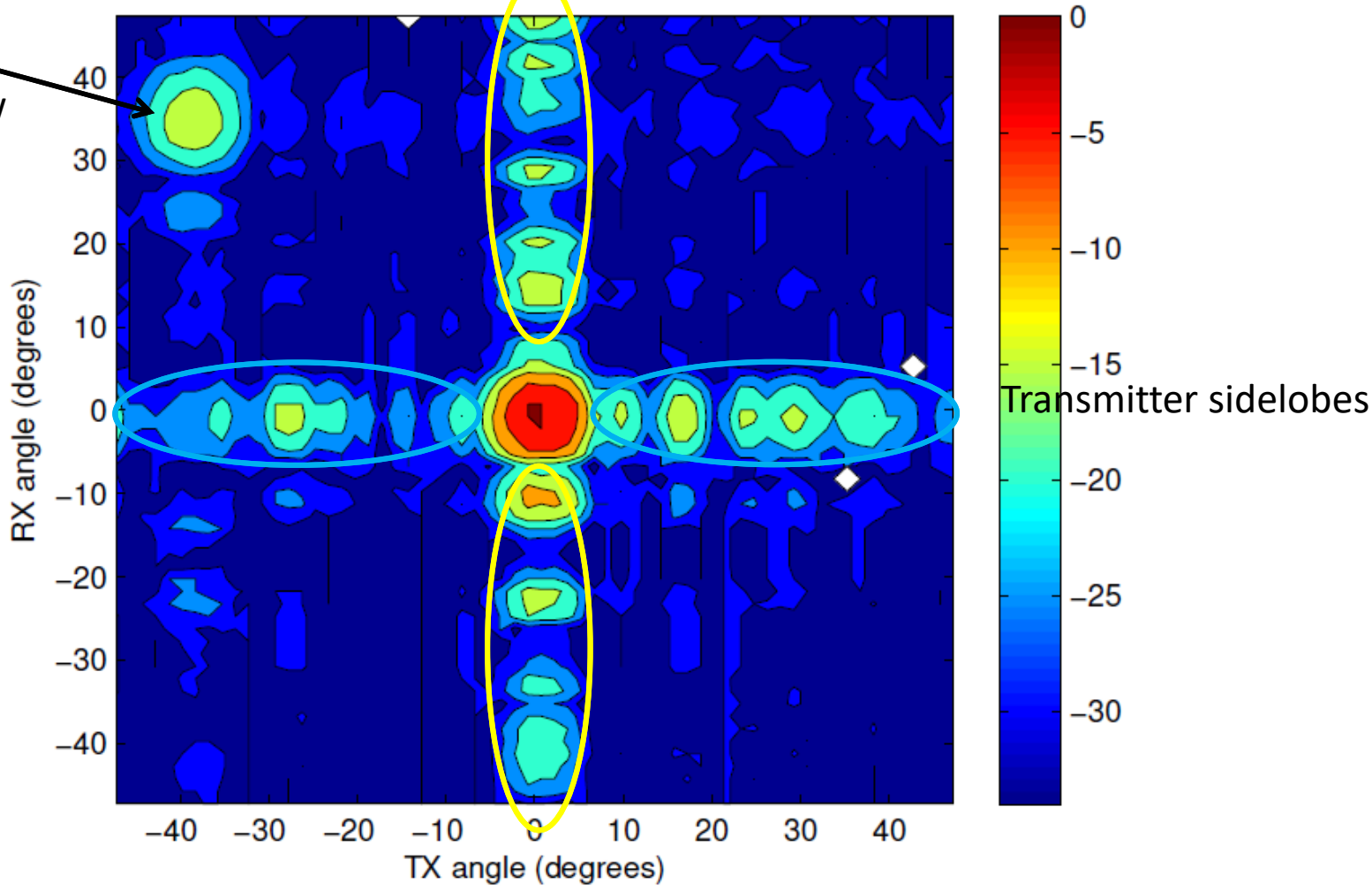
Reflection from window on right wall



Consistent with 2 rays

Receiver sidelobes

Reflection from window on right wall

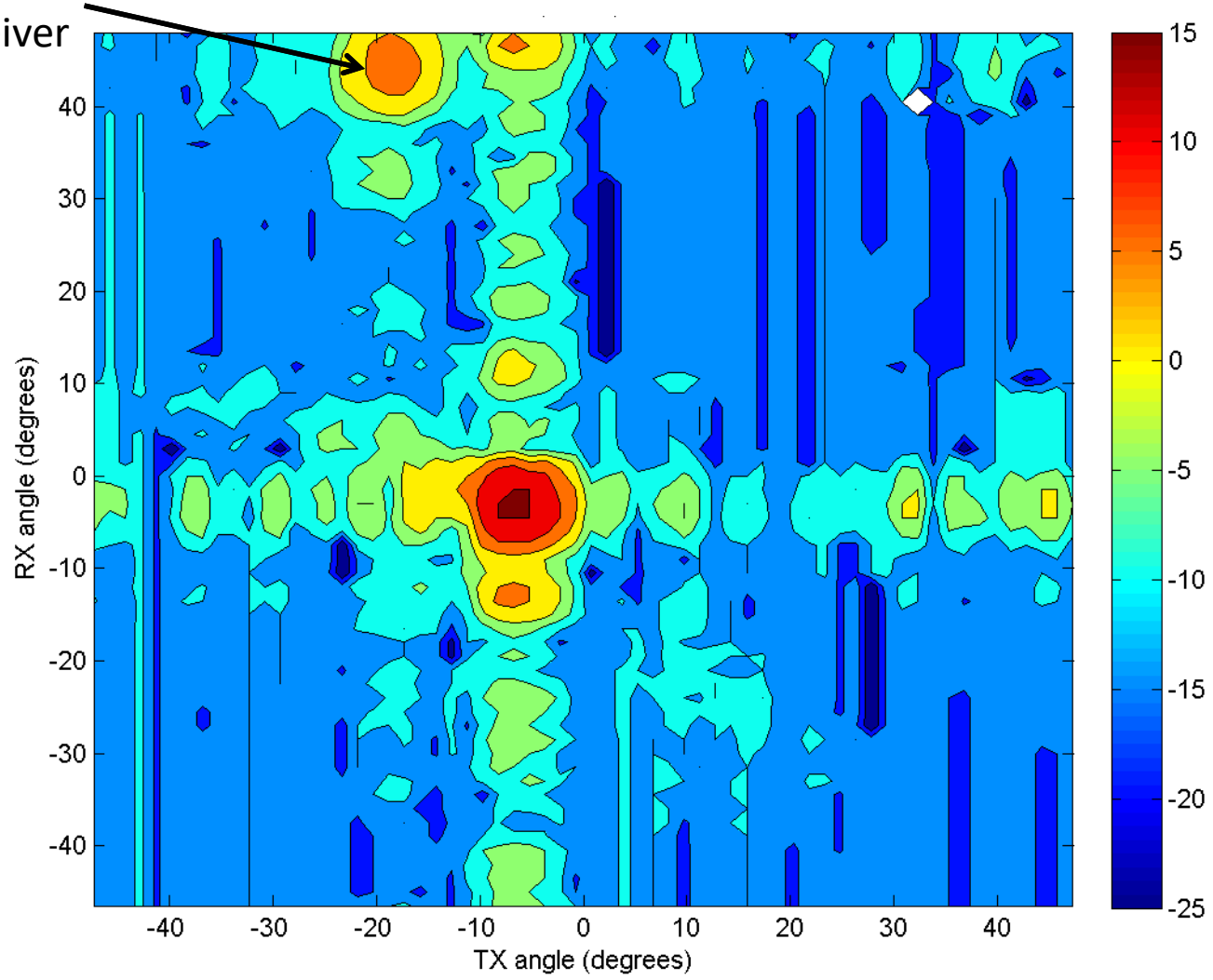


Transmitter sidelobes


RX

Another 2-ray channel

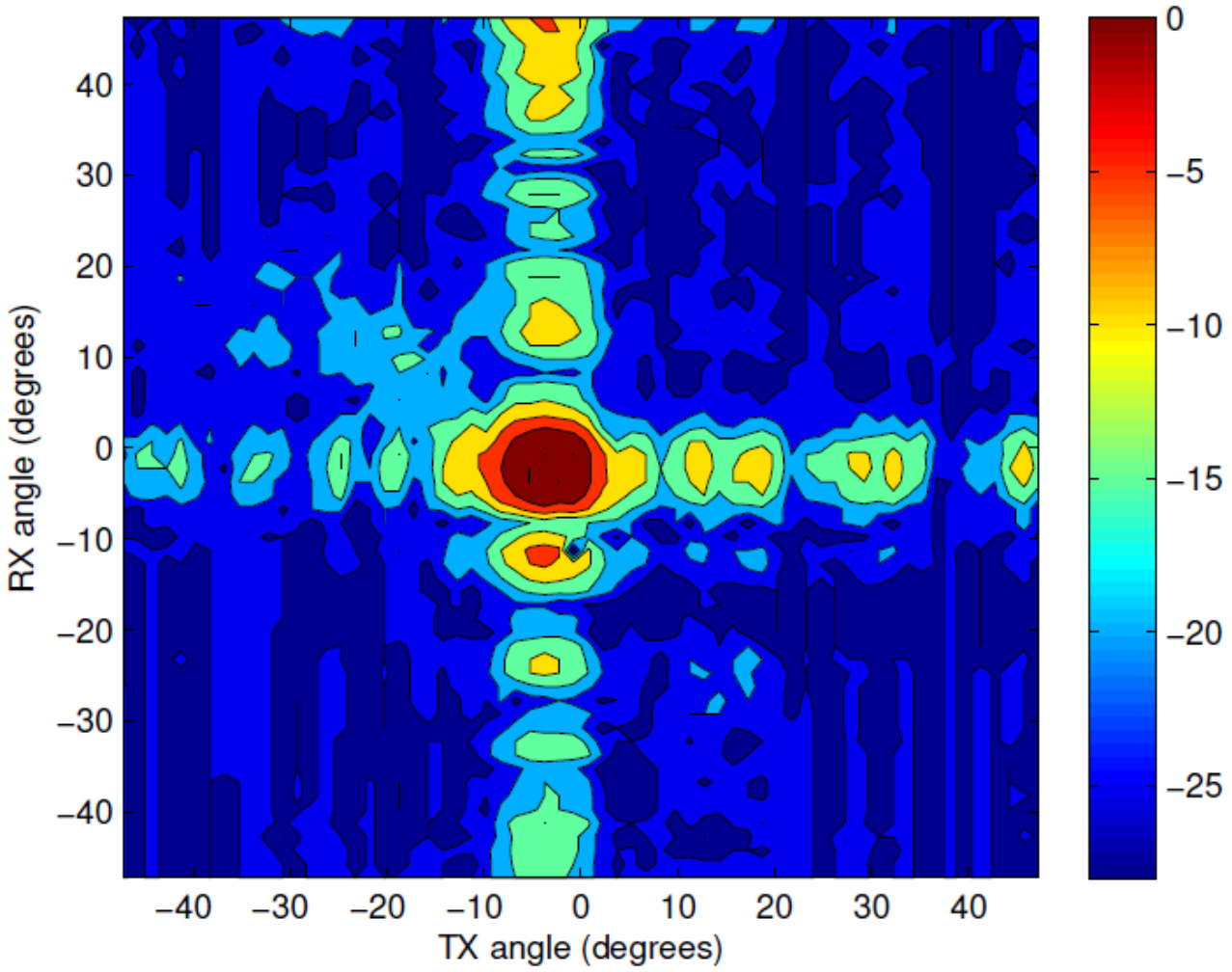
Reflection from
wall near receiver



Reflected path for 2nd ray

We expect only the LoS path here

1-ray model


RX angle (degrees)
40
30
20
10
0
−10
−20
−30
−40
−40
−30
−20
−10
0
10
20
30
40
TX angle (degrees)
0
−5
−10
−15
−20
−25

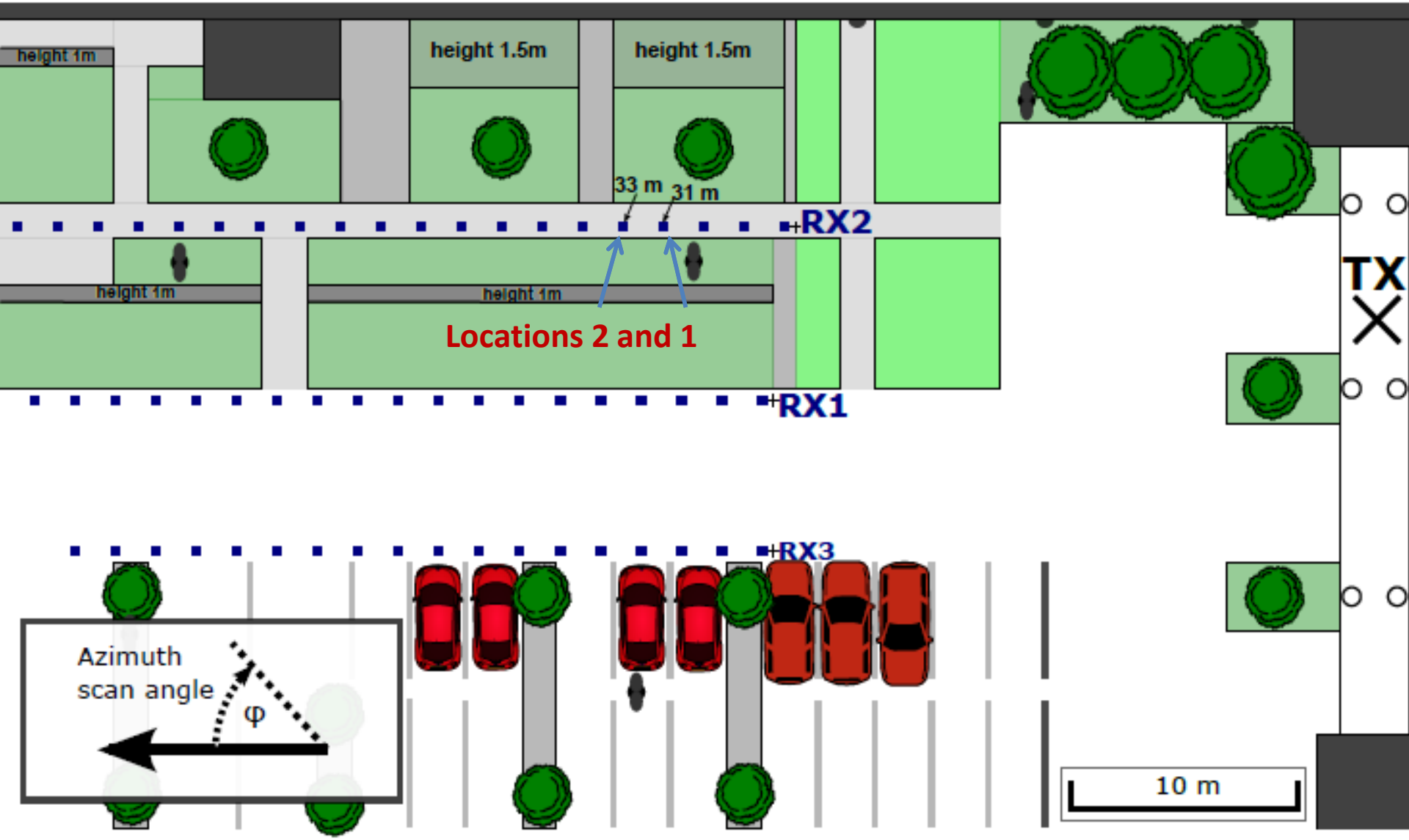
Other measurement campaigns show similar results

Previous measurement results

Measurement campaign by Weiler et al:
Transmitter performs beamsteering in 2D (azimuth and elevation)
Receiver is omnidirectional

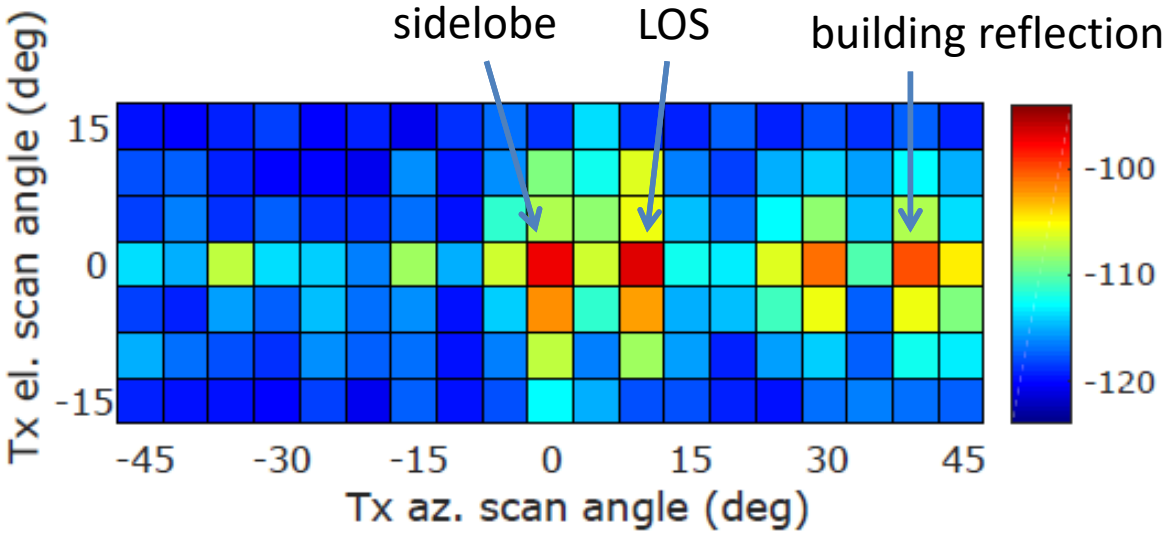
Weiler, R.J., Keusgen, W., Maltsev, A., Kühne, T., Pudeyev, A., Xian, L., Kim, J. and Peter, M., 2016, April. Millimeter-wave outdoor access shadowing mitigation using beamforming arrays. In *Antennas and Propagation (EuCAP), 2016 10th European Conference on* (pp. 1-5). IEEE.

Previous measurement results

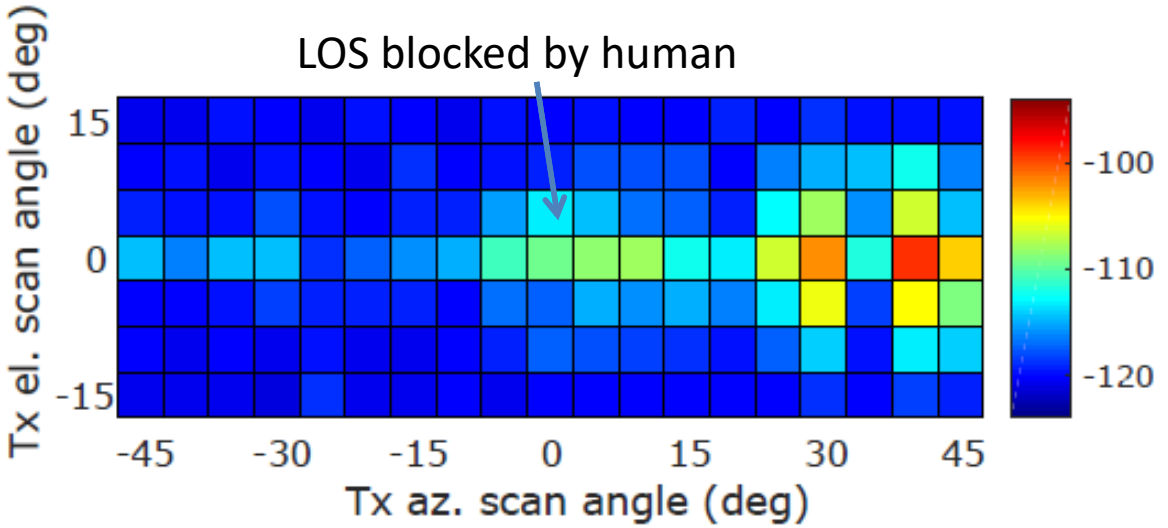


First location

Line of sight free:



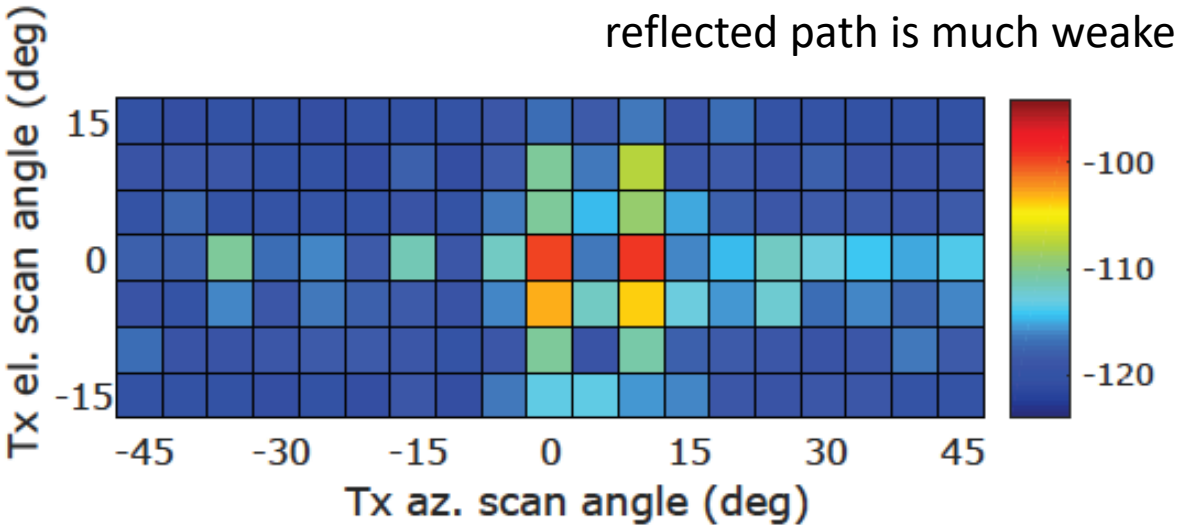
Line of sight blocked:



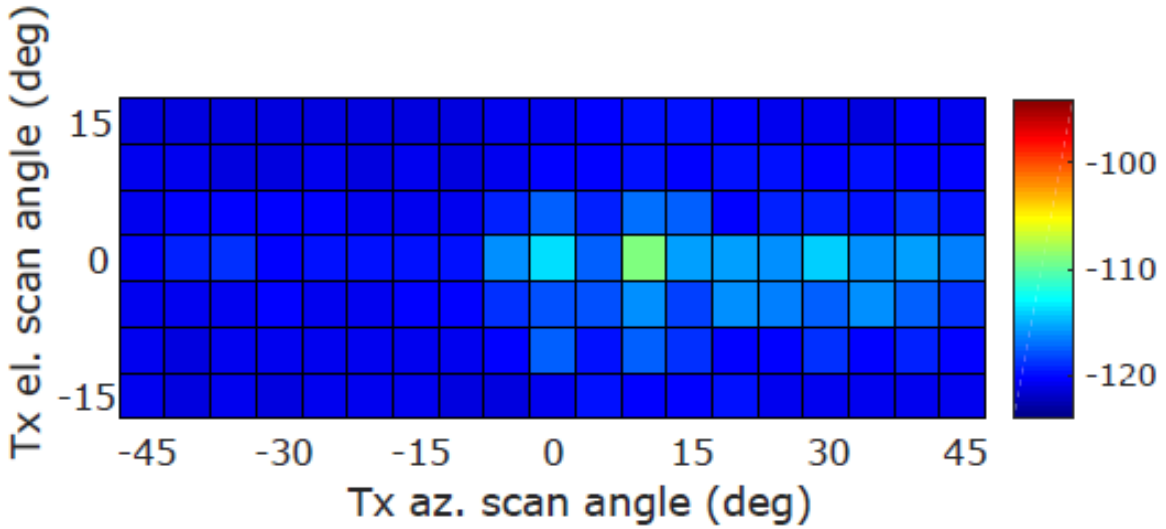
Second location

reflected path is much weaker

Line of sight free:



Line of sight blocked:



WORTH REPEATING

Millimeter wave channel is
sparse and geometrically predictable

BEAMFORMING

Beamforming is critical for utilizing sparse mm-wave channels

Link budgets require directionality

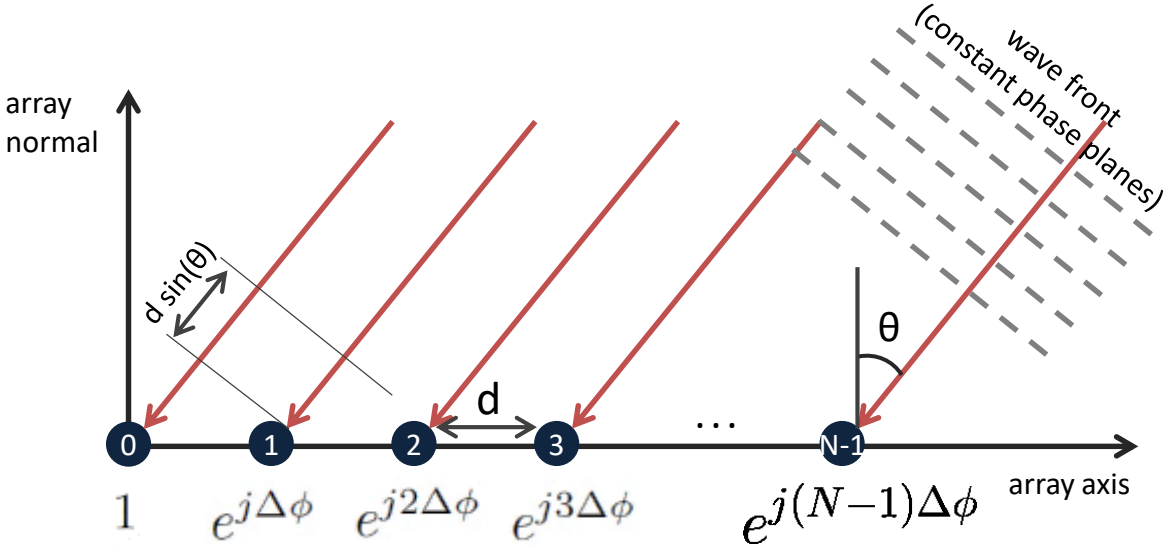
Tiny wavelength ➔ compact steerable antenna arrays

BEAMFORMING BASICS

Beamforming

Wave front incident from angle θ reaches elements with different phases

The signal reaching neighboring elements will have phase lag of k d sin(θ)

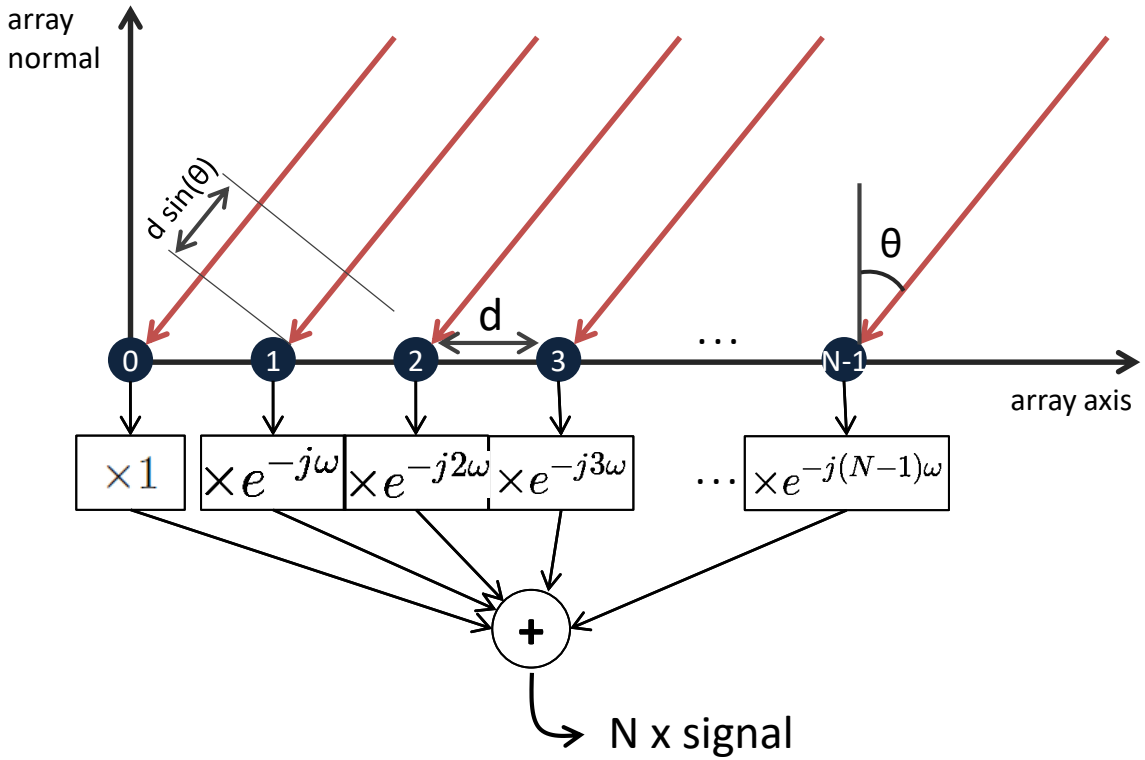


$$\Delta\phi = \frac{2\pi d}{\lambda}\sin(\theta) = \pi\sin(\theta) \qquad d = \frac{\lambda}{2}$$

$$\text{spatial frequency: } \omega = kd\sin\theta$$

Beamforming

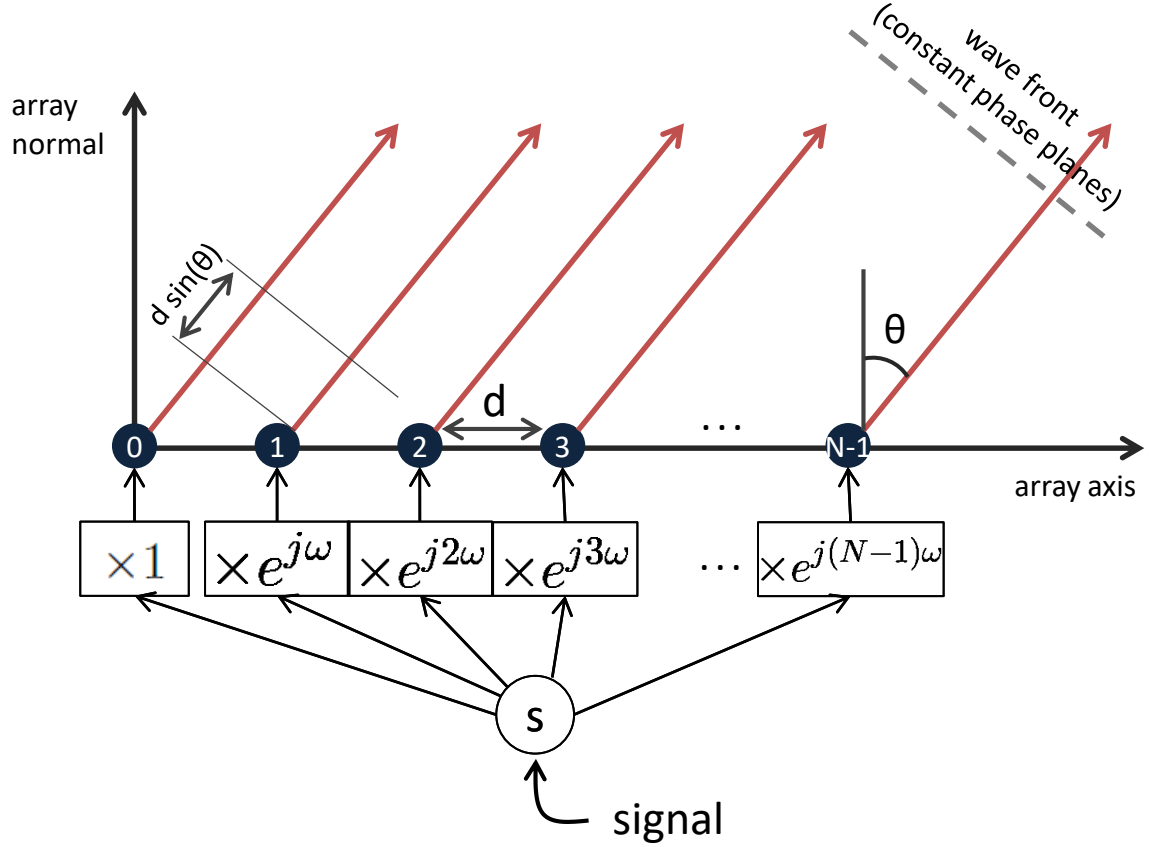
Receive beamforming: multiply by conjugate of phase offsets for constructive reception of signal coming from angle θ



$$\omega = \pi \sin\theta$$

Beamforming

Transmit beamforming: excite elements with phase offset to *generate* wave in direction θ



$$\omega = \pi \sin\theta$$

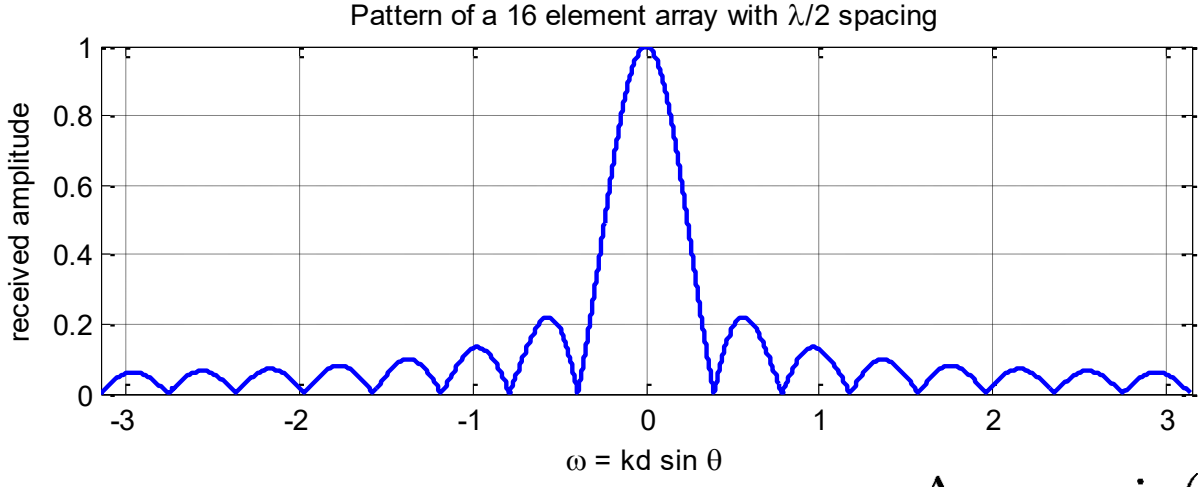
Array pattern (as a function of spatial frequency)

When array is beamformed toward angle θ, what is the signal received from angle θ+Δθ?

$$\text{array response: } \mathbf{a} = [1, e^{-j(\omega+\Delta\omega)}, e^{-j2(\omega+\Delta\omega)}, ..., e^{-j(N-1)(\omega+\Delta\omega)}]$$

$$\text{normalized weights: } \mathbf{w} = \frac{1}{N}[1, e^{j\omega}, e^{j2\omega}, ..., e^{j(N-1)\omega}]$$

$$\text{response} = |\langle \mathbf{w}, \mathbf{a} \rangle| = \frac{1}{N}\left|\sum_{n=0}^{N-1} e^{jn\Delta\omega}\right| = \frac{\sin(N\Delta\omega/2)}{N\sin(\Delta\omega/2)}$$



$$\omega = \pi \sin\theta$$

$$\Delta\omega = \pi \sin(\theta + \Delta\theta) - \pi \sin(\theta)$$

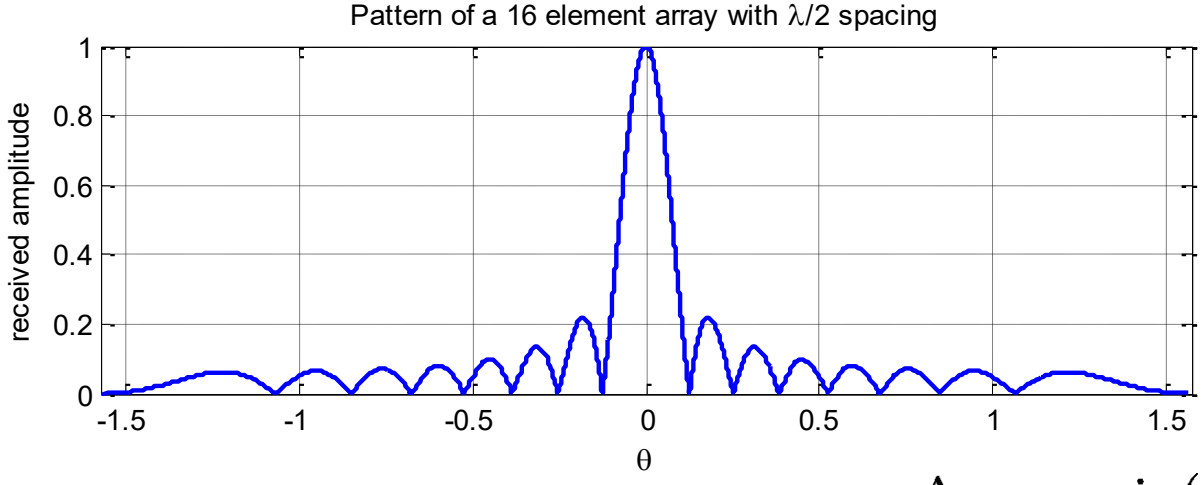
Array pattern (as a function of physical angle)

When array is beamformed toward angle θ, what is the signal received from angle θ+Δθ?

$$\text{array response: } \mathbf{a} = [1, e^{-j(\omega+\Delta\omega)}, e^{-j2(\omega+\Delta\omega)}, ..., e^{-j(N-1)(\omega+\Delta\omega)}]$$

$$\text{normalized weights: } \mathbf{w} = \frac{1}{N}[1, e^{j\omega}, e^{j2\omega}, ..., e^{j(N-1)\omega}]$$

$$\text{response} = |\langle \mathbf{w}, \mathbf{a} \rangle| = \frac{1}{N}\left|\sum_{n=0}^{N-1} e^{jn\Delta\omega}\right| = \frac{\sin(N\Delta\omega/2)}{N\sin(\Delta\omega/2)}$$

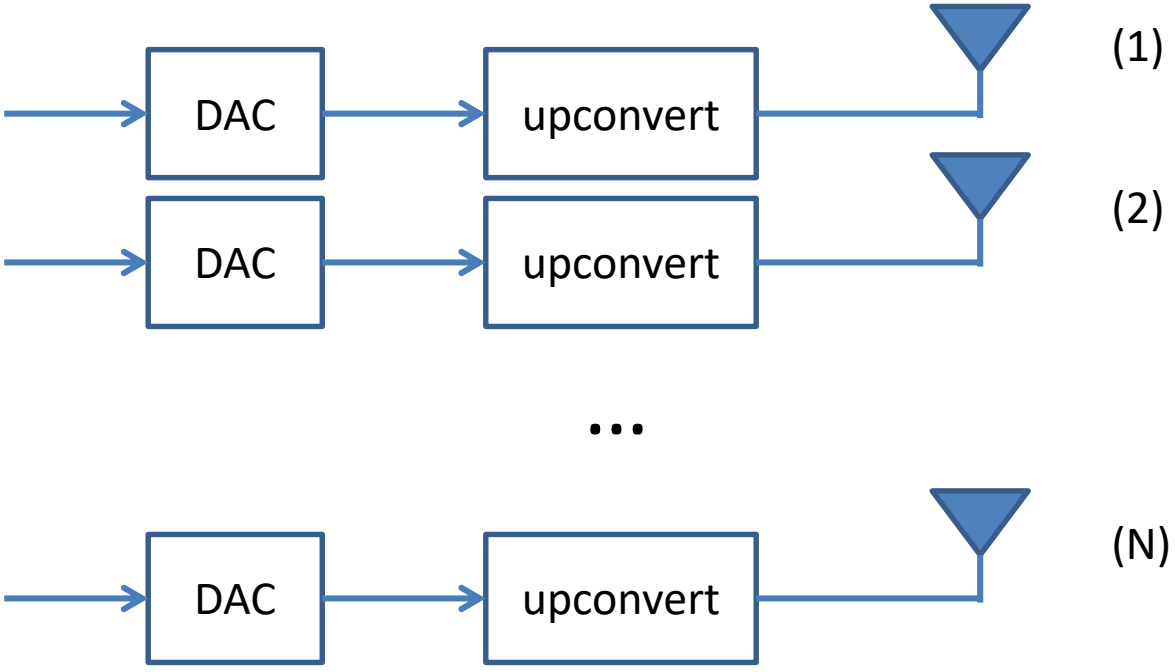


$$\omega = \pi \sin\theta$$

$$\Delta\omega = \pi \sin(\theta + \Delta\theta) - \pi \sin(\theta)$$

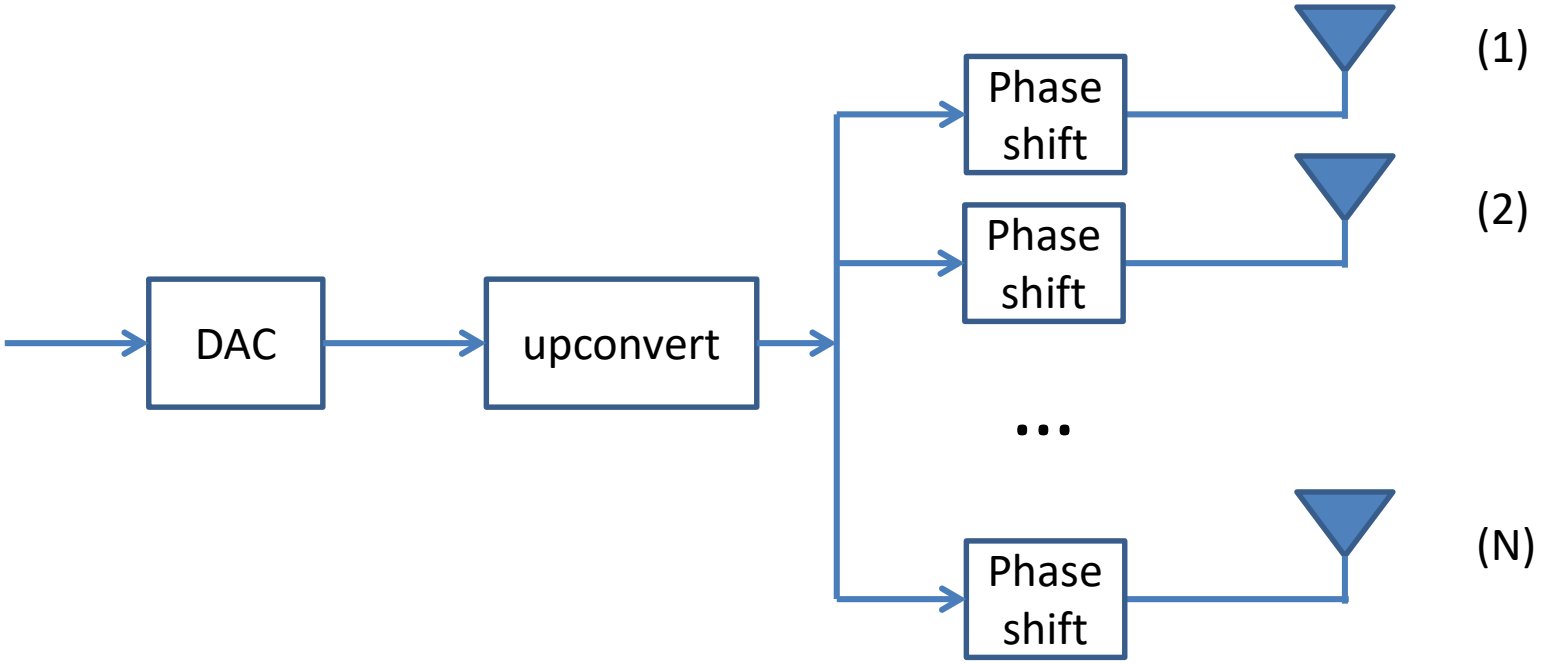
What we do today
Digital Beamforming

Today's systems have a small #antennas
#RF chains can't keep up with mm-wave array scaling



In order to scale to large arrays RF beamforming

Single RF chain
Less flexible, more scalable



We will assume RF beamforming

But hybrid models are worth exploring