ECE 146B: Introduction to mmWave Radar Sensing

Reading: Posted tutorial material from Texas Instruments; lecture notes; for background on the need for using tapered windows for FFT, see Lecture Notes on Signals and Systems (Madhow), Section 9.6 and Problem 9.10.

Lab Objectives: The goal of this lab is to provide an introduction to the fundamentals of radar sensing, along with hands-on signal processing of data acquired using a mmWave MIMO radar.

1 Background

The basic concept of radar is as follows. The transmitter sends out a signal $s(t)$. Assuming that the receiver is co-located with the transmitter, when the signal bounces back from a target, the "return" signal is received with a delay equal to the round-trip time between the transceiver and the target. The radar signal is designed so as to enable accurate estimation of this delay, which in turn provides an estimate of the distance between the transmitter and the target, termed the *range* R. Figure 1 shows a nominal geometry for a monostatic radar in which the transmitter and receiver are co-located.

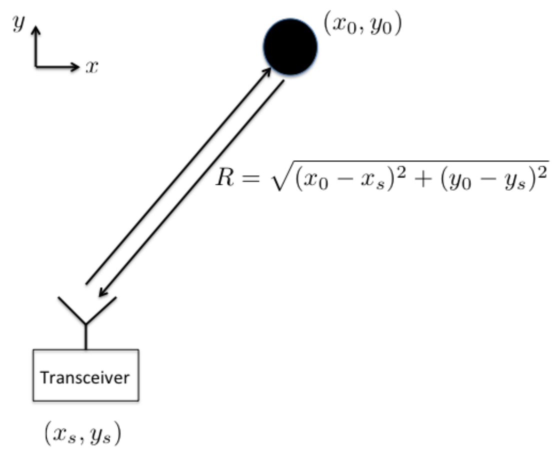


Figure 1: Basic radar geometry.

In the simplest possible scenario, the radar transmitter emits a complex baseband signal $s(t)$, and obtains a received signal

$$y(t) = \alpha s(t - \tau)e^{-j2\pi f_c \tau} + n(t) \tag{1}$$

where $n(t)$ is complex AWGN, α is a complex amplitude that accounts for attenuation and phase change during the round trip to the target and back, and $\tau = \frac{2R}{c}$ is the round-trip delay (c denotes the speed of wave propagation, taken to be speed of light in free space in our setting). For a moving target, τ can be a function of time. We often assume that the variations in τ can be ignored for the baseband waveform $s(t-\tau)$, but they will affect the phase term in (11). Specifically, making the time dependence of τ explicit in the phase term

$$\theta(t) = -2\pi f_c \tau(t) \tag{2}$$

we see that target motion causes a Doppler frequency shift:

$$f_D = \frac{1}{2\pi}\frac{d\theta(t)}{dt} = -f_c\frac{d\tau}{dt} = -f_c\frac{2dR/dt}{c} = -2f_c\frac{v_R}{c} \tag{3}$$

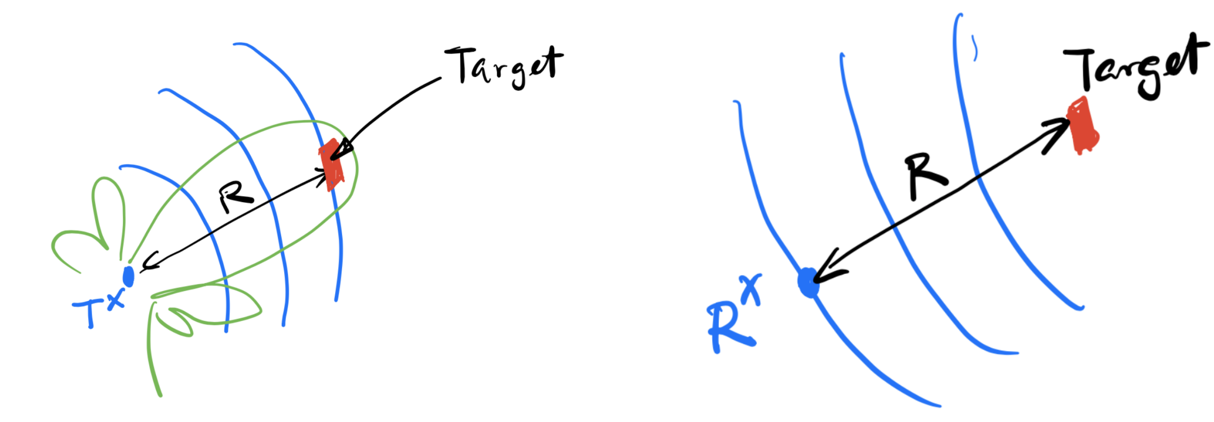
where v_R is the velocity of the target projected in the radial direction (i.e., along the vector connecting the radar to the target).

Thus, range estimation maps to delay estimation, while estimation of the radial velocity maps to estimation of the Doppler frequency offset from the carrier. Roughly speaking, the resolution of the delay estimate depends on the inverse of the signal bandwidth B, and the resolution of the frequency estimate depends on the inverse of the signal duration, or observation interval, T_o. It is possible to "super-resolve" beyond these resolutions at high enough SNR and accurate enough modeling, but this coarse notion of resolution suffices to come up with high-level design guidelines for radar waveforms.

In Section 1.1, we use the model (11) to obtain a basic link budget calculation that predicts SNR as a function of range. If the radar receiver performs matched filtering against all possible delays, then the maximum SNR that it can obtain, integrating over the signal duration, is $\frac{|\alpha|^2||s||^2}{N_0}$.

In Section 1.2, we specialize the model (11) to the linear chirp waveform, which is a radar waveform that is designed to simplify the signal processing involved in range/Doppler estimation.

1.1 Radar range equation



(a) Radar TX to target

(b) Target to radar RX

Figure 2: Two-way propagation from radar to target and back.

For round trip propagation to a target and back as depicted in Figure 2, the amount of power "caught and reflected" by a target at distance R is given by

$$P_{target} = \frac{P_{TX} G_{TX}}{4\pi R^2} A_{RCS} \tag{4}$$

where $\frac{P_{TX} G_{TX}}{4\pi R^2}$ is the power per unit area at distance R over the cone shaped by the transmit beam, and A_{RCS} is the effective area of the target, which is termed the radar cross section (RCS). While the RCS is a convenient concept, it is a coarse and highly variable characterization of the impact of the target, and depends heavily on many factors, especially the target pose relative to the radar (e.g., the RCS would be expected to be larger for the front view of a human compared to a side view). It also sweeps under the rug the microstructure of the target (e.g., the reflection from an "extended target" like a car, and indeed, even a human, may correspond to multiple scattering points).

The power emitted back by the target is now "caught" by the radar receiver to obtain a received power

$$P_{RX} = \frac{P_{target}}{4\pi R^2} A_{RX} = \frac{P_{target}}{4\pi R^2} \frac{\lambda^2 G_{RX}}{4\pi} \tag{5}$$

where A_{RX} is the effective aperture of the receiver, and where we have used the standard relationship between aperture and antenna gain,

$$G_{RX} = \frac{4\pi A_{RX}}{\lambda^2}$$

to express the received power in terms of the receive antenna gain.

Putting (4) and (5) together, we obtain the so-called "radar range equation" relating received power to transmitted power:

$$P_{RX} = \frac{P_{TX} G_{TX} G_{RX} \lambda^2 A_{RCS}}{(4\pi)^3 R^4} \tag{6}$$

Thus, in free space, radar designers must contend with the substantially worse $1/R^4$ decay due to two-way propagation, compared to the $1/R^2$ decay faced by designers of one-hop communication links.

If we are integrating over an observation interval of length T_o, the integrated SNR is given by

$$SNR = \frac{P_{RX} T_o}{N_0} \tag{7}$$

Given the transmitter and receiver specifications, therefore, we can estimate the SNR as a function of range and RCS, which provides us with rough guidelines on the combination of range and target characteristics that the radar system can be expected to handle.

1.2 The chirp waveform

With recent advances in radio frequency integrated circuits (RFICs) at millimeter wave (mmWave) frequencies, reasonably priced mmWave radar boards are now available in the 60 GHz unlicensed band (57-64 GHz) and in the 77 GHz automotive radar band (76-81 GHz). These are all based on (variants of) a particularly popular type of radar waveform called the linear chirp, or frequency modulated continuous wave (FMCW), where the instantaneous frequency varies linearly with time: $f(t) = f_c + St$, $0 \leq t \leq T_d$, where S is termed the *slew rate* or chirp slope, and T_d is the chirp duration. The phase of the waveform is the integral of the frequency, and is given by

$$\theta(t) = 2\pi \int_0^t f(u)du = 2\pi f_c t + \pi S t^2 \tag{8}$$

The passband transmitted chirp is therefore described as

$$s_p(t) = \cos(\theta(t)) = \cos(2\pi f_c t + \pi S t^2)\ ,\ 0 \leq t \leq T_d \tag{9}$$

Taking f_c as the reference, the corresponding complex baseband waveform is given by

$$s(t) = e^{j\pi S t^2}\ ,\ 0 \leq t \leq T_d \tag{10}$$

A particularly interesting feature of the chirp signal is that we can vastly simplify the signal processing required for matched filtering against various delay/Doppler combinations via bandwidth compression. Consider a passband return signal of the form

$$y_p(t) = \sum_k A_k \cos\left(\theta(t - \tau_k) + \phi_k\right) + n_p(t) = \sum_k A_k \cos\left(2\pi f_c (t - \tau_k) + \pi S (t - \tau_k)^2\right) + n_p(t) \tag{11}$$

where A_k, ϕ_k and τ_k are parameters associated with scatterer k in the environment. Instead of standard downconversion, we mix this with a replica of the passband transmitted chirp waveform to obtain the mixer output:

$$z_{mixer}(t) = y_p(t) s_p(t) \tag{12}$$

Consider now the term corresponding to a generic scatterer k:

$$A_k \cos\left(\theta(t - \tau_k) + \phi_k\right) \cos(\theta(t)) = \frac{A_k}{2} \cos\left(\theta(t) + \theta(t - \tau_k) + \phi_k\right) + \frac{A_k}{2} \cos\left(\theta(t) - \theta(t - \tau_k) - \phi_k\right)$$

The first term on the extreme right hand side is a double frequency term that can be rejected by baseband or IF filtering. Let us simplify the difference of phases in the second term:

$$\beta_k(t) = \theta(t) - \theta(t - \tau_k) - \phi_k = 2\pi f_c t + \pi S t^2 - 2\pi f_c (t - \tau_k) - \pi S (t - \tau_k)^2 = 2\pi f_c \tau_k + 2\pi (S\tau_k) t - \pi S \tau_k^2 - \phi_k \tag{13}$$

Differentiating this time-varying phase to obtain the associated frequency offset gives us:

$$\Delta f_k(t) = \frac{1}{2\pi} \frac{d}{dt} \beta_k(t) = f_c \frac{d\tau_k}{dt} + S\tau_k + St \frac{d\tau_k}{dt} - S\tau_k \frac{d\tau_k}{dt} \tag{14}$$

The term $\frac{d\tau_k}{dt}$ is the Doppler frequency; the term $S\tau_k$ is a mapping of the delay τ_k into a frequency shift (which is typically significantly larger than the Doppler frequency); the term $St\frac{d\tau_k}{dt}$ can be neglected relative to the Doppler, since St is much smaller than f_c. The last term $S\tau_k \frac{d\tau_k}{dt}$ couples the range and Doppler together, but again, $S\tau_k$ is much smaller than f_c, hence we usually neglect it.

Thus, assuming that the double frequency terms in the mixer output (12) are filtered out, each scatterer gives rise to frequency terms encoding the range and the Doppler. The largest frequency in (14) corresponds to the largest delay/range we wish to accommodate: $\tau_{max} = \frac{2R_{max}}{c}$. For example, for a maximum range of 30 m, the round-trip delay is 200 ns, or 0.2 μs, which is much smaller than the typical chirp durations T_d of 10s of μs that we consider. Thus, we obtain a substantial bandwidth compression (down to $S\tau_{max}$) at the mixer output, compared to the original chirp bandwidth ST_d. Thus, while digitizing a high-bandwidth radar signal after conventional downconversion would be challenging due to the cost and power consumption of high-speed analog-to-digital converters (ADCs), the bandwidth compressed intermediate frequency (IF) signal at the mixer output can be efficiently digitized with high precision. The IF signal can either be digitized directly, or we can downconvert and digitize the complex baseband waveform. The sampling rate required depends on the maximum bandwidth $S\tau_{max}$ that we wish to accommodate.

We typically estimate the range-dependent frequency shifts $S\tau_k$ using an FFT carried out on the samples obtained for each chirp. The Doppler shifts $f_c \frac{d\tau_k}{dt}$ are too small to be estimated over the short duration of a single chirp, and we employ multiple chirps in a "chirp frame" in order to estimate it, typically by performing an FFT across chirps.

In addition to the use of time-frequency degrees of freedom (the product of chirp bandwidth and the duration of the chirp frame) for estimating range and Doppler, existing mmWave radar boards also use spatial degrees of freedom, or MIMO. MIMO radar is discussed in class, and will not be reviewed in this document.

The preceding is a bare bones description of the setting of interest. You are referred to the posted tutorial material from TI (*Introduction to mmwave Sensing: FMCW Radars*, by Sandeep Rao) for an exposition with plenty of figures. See also the associated videos.

Running Example with mmWave chirp parameters: For example, consider a mmWave radar sensor using almost the entire bandwidth (76–81 GHz: 4.98 GHz) using a chirp with ramp time = 60 μs and idle time = 7 μs, corresponding to a slew rate S = 83 MHz/μs. The ADC sampling rate is $F_s = 10$ MHz, which would correspond to 600 samples if taken over the entire ramp duration. However, sampling doesn't take place from the beginning of the ramp (ADC start time = 6 μs) in order to avoid edge effects; hence we consider $N = 512$ samples per chirp (so we can take a 512-point FFT to go to the delay/range domain). We consider 60 chirps per frame, and employ a 64-point Doppler FFT across chirps (for each range bin).

Because the radar only samples a portion of the chirp, it is not able to utilize the full transmitted bandwidth of 4.98 GHz for range processing. Instead, the usable bandwidth is determined by the frequency sweep occurring during the sampled time interval. Therefore, the effective bandwidth is

$$BW_{\text{eff}} = S \cdot \frac{N}{F_s}.$$

With $S = 83$ MHz/μs, $N = 512$, and $F_s = 10$ MHz,

$$BW_{\text{eff}} = 83 \times \frac{512}{10} \approx 4.25 \text{ GHz}.$$

The data provided to you corresponds to 600 chirp frames (150 ms frame time, recorded for 90 s), obtained from a MIMO radar with parameters as in the running example, and with 3 transmitters and 4 receivers, corresponding to 12 virtual antennas. The data is shared as a 5D radar cube format (Frames $\times$ Chirps per Frame $\times$ Rx $\times$ Tx $\times$ ADC Samples).

2 Lab Assignment

1) What is the range resolution (in meters) for the running example? When you take a 512-point range FFT, what is the size of each range bin (in meters)?

2) What is the maximum unambiguous range (in meters) for the running example?

3) What is the maximum unambiguous magnitude of the radial velocity that we can measure? What is the Doppler resolution (in meters/sec)?

4) Suppose that each transmit element emits a power of 13 dBm, that the receiver noise figure is 13 dB, and that the transmit and receive antennas each have directivity 10 dBi. what is the integrated SNR obtained at each virtual element at a range of 10 meters for a target with RCS 0.1 m^2 (i.e., -10 dBsqm)?

5) For the data provided to you, perform a 2D-range Doppler FFT for each virtual element in each frame, using a suitable window taper (e.g., a Hann window or Hamming window). Average the *magnitudes* of the range-Doppler plots across virtual elements, and display the result as a video. How many moving objects do you "see"?

6) For each virtual element, estimate the contribution due to background clutter by averaging the complex values of the range-Doppler FFT across frames (i.e., compute the average for each range-Doppler bin). To visualize the estimate of the background that you get, average the *magnitudes* of the background estimates across virtual elements and display it.
Comment on whether you obtain areasonable estimate of the clutter that you see in 5).

7) Now, subtract the estimated background for each virtual element from its range-Doppler map: for a range-Doppler bin with complex value z and mean background z_b, replace z by $z - z_b$.
After background subtraction, visualize the range-Doppler maps as a video as in Step 5. Does the view of moving targets look "cleaner"?

8) Use Constant False Alarm Rate (CFAR) processing on the results from Step 7 (e.g., see the tutorial in Matlab `https://www.mathworks.com/help/phased/ug/constant-false-alarm-rate-cfar-detection.html`) to better isolate target features. Use your judgment in setting parameters. Again, visualize the range-Doppler maps as a video as in Step 5 and comment on whether the view is "cleaner."

9) Based on the results of Step 8, pick a range-Doppler bin in a given frame that you think might contain a target, and estimate the azimuth angle using MIMO radar processing across virtual elements (no need to bother with the elevation angle). From the result, estimate the 2D location of the target relative to the radar.

10) The preceding steps are meant to give you a preliminary feel of radar signal processing. You are encouraged to explore further: for example, you could perform Step 9 for each of the range-Doppler bins that you deem salient in each frame, compute the (x, y) location based on the range and angle estimates, and see how the resulting raw estimates of target locations evolve across frames. In practice, these range/Doppler/angle estimates might be fed into a multi-target tracking algorithm (these typically use some variant of the Kalman filter, along with an association algorithm (which associates new measurements with existing tracks, or creates new tracks).

3 Lab Report

- Discuss the results you obtain, answer any specific questions that are asked, and print out the most useful plots to support your answers.
- Append your programs to the report. Make sure you comment them in enough detail so they are easy to understand.
- Write a paragraph about any questions or confusions that you may have experienced with this lab.