Mathematical Modeling and Performance Analysis for a Two-Stage Acquisition Scheme for Direct-Sequence Spread-Spectrum CDMA

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***Abstract*—The problem of acquisition of synchronism is considered for direct-sequence spread-spectrum (DS/SS) code-division multiple-access (CDMA) systems. For large systems with large timing uncertainties, it has been shown recently that acquisition in the presence of multiple-access interference may impose a significant limitation on capacity. This leads us to consider a system in which timing uncertainties are relatively small and to propose an acquisition scheme which exploits this to reduce complexity and acquisition overhead. Our proposal may be appropriate for a microcellular environment for personal communications in which CDMA packet transmission is employed for both voice and data. Packetized transmission would imply that the overhead available for acquisition is small, and the large number of microcells would restrict the cost (and therefore the complexity) of the acquisition scheme used in the receiver in each microcell. The acquisition time required for a simple serial search scheme may therefore be unacceptably large. On the other hand, while acquisition using a passive matched filter is fast, the filter length required for reliable acquisition is liable to be excessive in terms of cost and complexity. Motivated by these considerations, we propose a two-stage acquisition scheme which employs a short programmable matched filter for initial detection, followed by a correlator for verification. Numerical results based on an approximate analysis of acquisition performance in the presence of multiple-access interference are employed to compare our scheme with conventional acquisition schemes. In particular, it is shown that, for a given probability of successful acquisition, our scheme acquires much more rapidly than a serial search scheme and is much less complex than a passive matched filter scheme.**

I. Introduction

Acquisition of timing for a direct-sequence spread-spectrum (DS/SS) signal in the presence of multiple-access interference is considered. A DS/SS signal is generated by spreading the data signal using a spreading, or *signature*, sequence. The signature sequence consists of a sequence of pulses, or *chips*, of a duration much smaller than the duration of a data symbol. The choice of signature sequences with good crosscorrelation properties enables the simultaneous transmission of several DS/SS signals over the same channel, resulting in a code-division multiple-access (CDMA) system. However, in order for the receiver to despread and demodulate the transmission of interest (henceforth called the *desired transmission*), the receiver must first estimate the delay of the desired transmission. *Acquisition* refers to the task of obtaining a coarse estimate of the delay before initiating the tracking and demodulation procedures. For this purpose, it is convenient to express the delay as a multiple of the chip duration referred to as the *phase* of the signature sequence.

It has been shown recently [5] that for CDMA systems with many simultaneous transmissions and large timing uncertainties, the problem of acquisition in the presence of multiple-access interference significantly limits the system capacity. However, the results of [5] also imply that acquisition performance can be improved significantly if the timing uncertainties are small. In this paper, we propose a method of exploiting small timing uncertainties to reduce both the complexity of the acquisition scheme and the time required to acquire.

While this work is not limited to a specific application, a strong motivation for it is the possibility that packetized CDMA may be used to transmit both voice and data in emerging wireless mobile networks for personal communications. In the latter application, the division of the geographical area of interest into microcells ensures that each receiver only hears transmissions within a small radius, so that both the number of transmissions and the timing uncertainty are relatively small. On the other hand, the large number of microcells that are likely to be used for personal communications implies that the complexity of the hardware used for acquisition in each microcell should be low, and packetizing transmission implies that the overhead available for acquisition may be small. Conventional methods for acquisition such as serial search schemes (low complexity but large acquisition time) and passive matched filter schemes (small acquisition time but high complexity) do not match these constraints very well. Preliminary results presented here show that the two-stage acquisition scheme proposed here, which consists of a short matched filter followed by a correlator, may be more successful in balancing the tradeoffs between complexity and acquisition overhead.

Initial detection in our scheme is achieved using a short programmable matched filter, and a correlator is used for verification. The acquisition scheme is designed so that the

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timing uncertainty never increases beyond its initial value, which is assumed to be small. We will see that this consideration makes it necessary for the matched filter to be programmable. Prior to [5], the effect of multiple-access interference on acquisition, and hence the issue of maintaining small timing uncertainties, had not been considered in the literature. Thus, our mathematical model for the acquisition process differs from those for two-stage acquisition schemes considered previously [7], [3, p. 419], [9], [12].

We note that additional mechanisms, such as constraining transmissions to start at one of a set of prespecified times, may be required to achieve the small initial timing uncertainties assumed here. The specific mechanism used is not our primary concern since it may depend on other system constraints. Slotted transmission, for instance, would automatically ensure a small timing uncertainty, since the start times are constrained to be at the beginning of time slots. Regardless of the mechanism used to reduce timing uncertainty, it would be necessary to use reasonably stable clocks sharing time-of-day information at the transmitters and receivers.

Assuming that the length of the acquisition preamble is fixed, the performance criterion for the acquisition scheme is taken to be the probability of successful acquisition of the desired transmission within a time $T_{\max}$ determined by the length of the preamble. Numerical results based on an approximate analysis show that, for a desired value of the probability of successful acquisition, the required value of $T_{\max}$ for our scheme is much smaller than for a serial search scheme. Also, while the required $T_{\max}$ for our scheme is larger than for a passive matched filter scheme, our scheme is much less complex for the same probability of successful acquisition, the length of the filter used in our scheme can be much smaller than the length required for a passive matched filter scheme.

The system model is given in Section II. In Section III, we approximate the acquisition process by a renewal process, and give a recursive formula for the probability of successful acquisition, together with a nonrecursive approximation for this probability which is easier to compute. In order to compare our scheme with existing schemes, we also provide an approximate analysis for serial search and passive matched filter acquisition schemes. Numerical results are provided in Section IV to illustrate this comparison, and our conclusions are given in Section V.

II. System Model

The model for the desired and interfering transmissions is as in [5]. It is assumed that there is a preamble for acquisition, so that the desired transmission has no data modulation, and is given by

$$r_0(t) = 2^{1/2}\left(\sum_{j=-\infty}^{\infty} a_j \psi(t - jT_c - \tau T_c)\right)\cos(\omega_c t + \phi)$$

where ψ is a unit-amplitude rectangular pulse of duration T_c and (a_j) is the signature sequence for the desired transmission. The delay is expressed as a multiple τ of the chip interval T_c. It is assumed that τ is a positive integer; this enables us to establish a discrete-time model for the acquisition process. The effect of noninteger τ is discussed later. Because multiple-access interference is the dominant impairment even for moderate values of signal-to-noise ratio, we ignore thermal noise. We note, however, that since we use a Gaussian approximation for analyzing the effect of the multiple-access interference, the effect of such noise could be incorporated simply by increasing the variance of the interference. We assume that there are J interfering transmissions at the same carrier frequency. The total number of transmissions is thus $K = J + 1$. The jth interfering transmission $1 \leq j \leq J$ is given by

$$r_j(t) = (2P_j)^{1/2} \sum_{k=-\infty}^{\infty} x_k^{(j)} \psi(t - kT_c - \tau_j T_c) \cdot \cos(\omega_c + \phi + \theta_j)$$

where P_j is the power relative to that of the desired transmission and θ_j is the carrier phase relative to that of the desired transmission. The sequence $(x_k^{(j)})$ results from multiplying the data sequence (if present) and the signature sequence of the interfering transmission, and the delay is expressed modulo the chip interval, so τ_j takes values in the interval [0, 1]. The additive interference is thus given by $X(t) = \sum_{j=1}^{J} r_j(t)$, and the net received signal is $r(t) = r_0(t) + X(t)$.

The sequences (a_k) and $(x_k^{(j)})$ are modeled as random and independent. For each of these sequences, the elements are assumed to be independent and identically distributed, taking values +1 or −1 with equal probability. This model is referred to as the *random sequence model*, and has been used previously in [5] to evaluate the acquisition performance of a passive matched filter scheme, in bit error probability evaluations [4], [6], and in modeling a different aspect of the acquisition problem [2]. Although signature sequences are deterministic in practice, the random sequence model is useful in obtaining performance estimates in terms of a few key system parameters before detailed design choices (such as choosing the signature sequences) have been made.

If the receiver can acquire the carrier frequency and the phase of the target transmission perfectly, it is possible to use *coherent* processing to compute the following statistics:

$$Z_k = \frac{2^{1/2}}{T_c}\int_{kT_c}^{(k+1)T_c} r(t)\cos(\omega_c t + \phi)\, dt = a_{k-\tau} + X_k \quad (1)$$

where the additive interference X_k is given by

$$X_k = \sum_{j=1}^{J} P_j^{1/2} \cos\theta_j [(1 - \tau_j) x_k^{(j)} + \tau_j x_{k-1}^{(j)}]. \quad (2)$$

In practice, the carrier phase ϕ of the target transmission may not be known to the receiver prior to acquisition, and *noncoherent* processing may be required. The framework developed in this paper applies to both coherent and noncoherent processing. For ease of presentation, however, we focus on coherent processing for the most part, indicating the changes required for noncoherent processing where appropriate. In particular, the receiver operations for noncoherent processing (see also [10]) are specified by (7)–(8) in Section III.

We consider an *asynchronous* multiple-access system, in which the relative phases θ_j and the relative delays τ_j are modeled as independent random variables: the θ_j are uniformly distributed over $[0, 2\pi]$ and the τ_j are uniformly distributed over [0,1]. This system is difficult to analyze due to the dependencies introduced by the asynchronism. Most of these dependencies can be eliminated by conditioning the acquisition process on the relative carrier phases θ_j and the relative delays τ_j appearing in (2), computing the performance measure of interest, and subsequently removing the conditioning. Such methods have been used previously in bit and packet error probability computations [4], [6]. Although we indicate how such methods apply to our problem, our numerical results are based on approximations which are less computationally intensive. For simplicity in presentation, we assume that all received signals have equal power ($P_j = 1,\ 1 \le j \le J$ in (2)). Our approximations are based on an application of the central limit theorem to yield an "improved Gaussian approximation" (in the terminology of [6]) to the effect of each interfering signal, and subsequently simplifying it further to yield the "standard Gaussian approximation." A detailed study of the effect of unequal powers can probably be carried out using the improved Gaussian approximation, and is worth exploring in future work. However, the equal power assumption and the standard Gaussian approximation suffice for our present purpose of obtaining a preliminary comparison of various acquisition schemes.

Our task is to estimate τ based on the sequence of statistics (Z_k). We assume that the acquisition process starts at time zero, and that τ is known to take an integer value in the interval $[1, T]$ where the initial timing uncertainty T is a small integer. The Z_k are the input to a discrete-time filter of length N. At any given time in the process of acquisition, the filter is matched to a section $a_{r-N+1}, a_{r-N+2}, \cdots, a_r$ of the signature sequence of the desired transmission; that is, the filter coefficients are given by $h_i = a_{r-i}$, $i = 0, 1, \cdots, N-1$. The integer r is called the *target phase*, and may be changed during the process of acquisition by reprogramming the filter. Referring to (1), *current phase* at time k is defined to be $k - \tau$. This is the index of the element of the desired signature sequence that contributes to Z_k. The matched filter output (W_n) is given by

$$W_n = \sum_{i=0}^{N-1} h_i Z_{n-i}.$$

We describe the process starting from time r; the target phase is r, and the current phase is $r - \tau$. Note that this condition is satisfied for the initial value of $r = 0$. The matched filter output is monitored at intervals of unit length. The desired transmission contributes a peak of height N to the matched filter output at time $r + \tau$ (when the current phase equals the target phase). Since $1 \le \tau \le T$, we expect a peak by time $r + T$, and try to detect it by means of a threshold rule. Specifically, for $\alpha \in [0, 1]$, a *hit* is said to occur when the matched filter output W_n exceeds the threshold αN for some n in the range $r + 1 \le n \le r + T$. If there is no hit by time $r + T$, we know that we have failed to detect the peak at time $r + \tau$ (since $\tau \in [1, T]$). We reset the matched filter coefficients to accommodate the resulting delay, replacing r by $r + T$, and continue monitoring the matched filter output. This preserves the initial timing uncertainty.

For $t \ne r + \tau$, a hit at time t is called a *false alarm*. A hit at time $t = r + \tau$ is called a *correct hit*, and the absence of a hit at that time is called a *miss*. If a hit does occur, we stop monitoring the matched filter output and start verifying the hit by correlating the matched filter input with the appropriate section of the desired signature sequence over an interval C. Starting from phase r, a hit occurs at time t if the phase corresponding to the first time the threshold is exceeded is $r + t$, i.e., if

$$t = \min\{n : r + 1 \le n \le r + T,\ W_n > \alpha N\}.$$

Following the hit, we correlate Z_k, $t + 1 \le k \le t + C$ with $a_{r+1}, \cdots, a_{r+C}$ to obtain the verification statistic

$$V = \sum_{i=1}^{C} a_{r+i} Z_{t+i}.$$

The verification statistic is compared to a threshold to determine whether a hit is authenticated. For $\beta \in [0, 1]$, if $V > \beta C$, then the hit is authenticated, and the acquisition process terminates (typically, tracking and demodulation are initiated at this point). If the hit is correct, then we have *successful acquisition* upon authentication. On the other hand, if the hit is a false alarm and is authenticated, then we have an *overall false alarm*. We assume that the opportunity to acquire the desired signal is lost by the time the receiver recovers from the tracking and demodulation procedures initiated by an overall false alarm. This is typical of applications in which the acquisition time is critical, such as a packet radio network. If $V \le \beta C$, the hit is rejected, and we resume monitoring the output of the matched filter. However, in order to account for the delay in verification, the filter coefficients are matched to a later section of the desired signature sequence. Specifically, we set $r = r + t + C$, so that a peak is expected again after a time τ (when the current phase equals r), thus preserving the initial timing uncertainty. A block diagram of the acquisition scheme is given in Fig. 1.

In both situations under which the matched filter is reset, we revert to the conditions at the beginning of the acquisition process by preserving the relative delay $\tau \in [1, T]$ between the target phase and the current phase of the desired signature sequence. If neither of these two situations occur, the acquisition process terminates in either successful acquisition or overall false alarm. The foregoing description suggests viewing the acquisition process as a renewal process with absorbing states, and our analysis is based on this view. We note that a renewal process model is an approximation even for the random sequences considered in this paper. This is because the portion of the acquisition process that starts after resetting the matched filter is not independent of the past, since some elements of the input sequence (Z_k) contribute both to past decisions and to the matched filter output for the new process. However, the dependence is weak, and is ignored in our analysis.

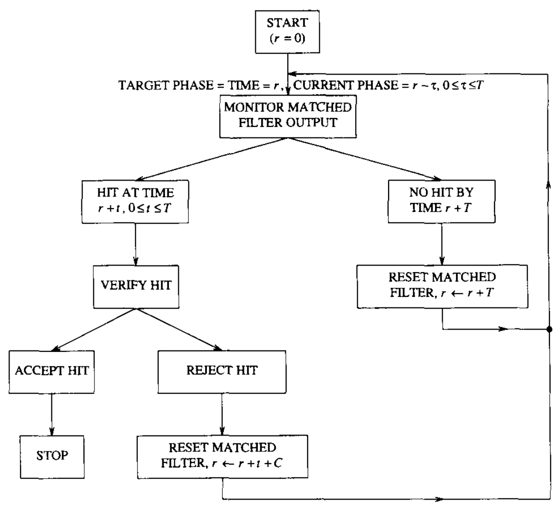

Fig. 1. Block diagram of two-stage acquisition scheme.

III. Mathematical Model and Analysis

For the rest of this paper, it is convenient to make the worst-case assumption $\tau = T$. A generalization to $\tau \le T$ is straightforward, and is omitted. We first describe the renewal process model for the acquisition process, and then give approximate formulas based on the central limit theorem for the transition probabilities for the renewal model. Based on this (approximate) renewal model, we develop an exact recursive formula for the probability of successful acquisition $P_{\mathrm{ACQ}}(x)$ within time x. We also obtain an approximation for this probability which is much easier to compute, and is used in Section IV when optimizing the parameters of the acquisition scheme via exhaustive search. Finally, we give approximate formulas for the probability of successful acquisition for two conventional acquisition schemes: serial search and passive matched filter.

A. Development of the Renewal Process Model

It is assumed that there is a renewal whenever the matched filter is reprogrammed. The possible states of the acquisition process are described in the following. State i corresponds to the current phase being $r + i - T$ (since we assume $\tau = T$) when $1 \le i \le T$. There are two absorbing states, ACQ and FA, corresponding to successful acquisition and overall false alarm, respectively. In addition, there are two intermediate states, VER_f and VER_c. The process starts from state 1, and a renewal consists of a return to state 1. For $1 \le i \le T - 1$, a transition from state i can lead either to VER_f or to $i + 1$; a transition to VER_f corresponds to initiation of the verification stage following a false alarm, and a transition to $i + 1$ occurs if there is no false alarm. Rejection of the false alarm leads to a renewal, and authentication to absorption into FA. From the state $\tau = T$ corresponding to the target phase, there is a renewal if there is a miss, otherwise we proceed to VER_c, which corresponds to initiation of the verification stage following a correct hit. From VER_c, authentication of the hit leads to absorption into ACQ, and rejection leads to a renewal. In general, all transitions from VER_f and VER_c take time C, and all other transitions take unit time.

Note that, if the process is in state i, it must be true that there have been no false alarms in the previous $i - 1$ states that have been visited since the last renewal. Thus, transition probabilities out of state i must be computed by conditioning on no such false alarms having occurred. Since the interference is likely to be less harmful subject to such conditioning, we argue that replacing the conditional probability of a false alarm at state i by the unconditional probability of false alarm leads to a pessimistic approximation to the acquisition probability. This independence approximation is further validated by the fact that for large N, application of a multidimensional central limit theorem (see [5] and the Appendix) shows that the matched filter outputs corresponding to different states are asymptotically independent. Similar approximations are made for other transition probabilities as well.

We also make an approximation regarding the contribution of the target transmission to the matched filter output. When the current phase equals the target phase, the desired transmission contributes a peak of height N to the matched filter output. At other instants, we model its contribution to the matched filter output as resulting from an independent interfering transmission; this, together with the J original interfering transmissions, constitutes the *net interference*. Let Y_n be the matched filter output at time n due to this net interference, and let W_n be the net output at time n. Thus, after each renewal (with target phase r), we have that $W_n = Y_n$, $n \ne r + \tau$, and $W_{r+\tau} = N + Y_{r+\tau}$. A similar approximation is made regarding the correlator output V. When a correct hit is being verified, the correlator output $V = C + U_c$, and when a false alarm is being verified, $V = U_f$ where U_c, U_f are the contribution of the net interference of the output of the correlator for a correct hit and a false alarm, respectively.

Using the above approximations, we obtain the signal flow graph representation of the acquisition process shown in Fig. 2. A transition labeled pz^l has probability p and occurs over l time units. The transition probabilities shown are specified as follows. A false alarm at state i ($i \ne \tau$) causes a transition to state VER_f. The probability of this transition, which is independent of i under our approximations, is given by

$$p_{f1} = P[Y_1 > \alpha N] = P[Y_i > \alpha N], \qquad 1 \le i \le T - 1. \quad (3)$$

A miss at state $i = \tau = T$ causes a transition to state 1 through a resetting of the matched filter; the corresponding transition probability is given by

$$p_{m1} = P[N + Y_T \le \alpha N] = P[Y_T \le -(1 - \alpha)N]. \quad (4)$$

Authentication of a false alarm leads to a transition from state VER_f to the absorbing state FA. This transition probability is given by

$$p_{f2} = P[U_f > \beta C]. \quad (5)$$

Rejection of a correct hit leads to a transition from VER_c to state 1 through a resetting of the matched filter, and the

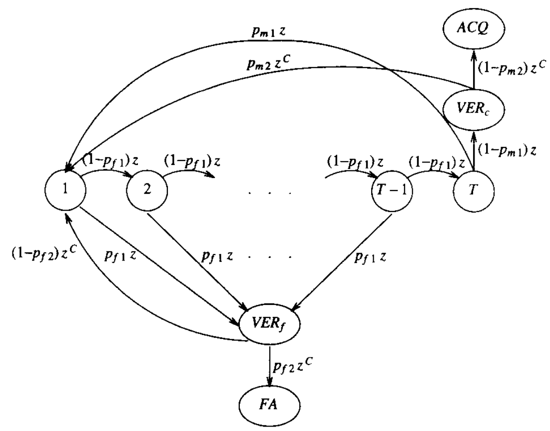

Fig. 2. Signal flow graph representation of acquisition process.

transition probability is given by

$$p_{m2} = P[C + U_c \le \beta C] = P[U_c \le -(1-\beta)C]. \tag{6}$$

So far, we have restricted attention to coherent processing in which the receiver knows the carrier phase ϕ of the desired signal prior to acquisition. If this is not the case, noncoherent processing is required. The receiver computes two orthogonal samples as follows:

$$Z_{k,\,c} = \frac{2^{1/2}}{T_c}\int_{kT_c}^{(k+1)T_c} r(t)\cos(\omega_c t)\,dt = a_{k-\tau}\cos\phi + X_{k,c}, \tag{7}$$

$$Z_{k,\,s} = -\frac{2^{1/2}}{T_c}\int_{kT_c}^{(k+1)T_c} r(t)\sin(\omega_c t)\,dt = a_{k-\tau}\sin\phi + X_{k,s}, \tag{8}$$

where

$$X_{k,\,c} = \sum_{j=1}^{J} P_j^{1/2}\cos\theta_j[(1-\tau_j)x_k^{(j)} + \tau_j x_{k-1}^{(j)}]. \tag{9}$$

$$X_{k,\,s} = \sum_{j=1}^{J} P_j^{1/2}\sin\theta_j[(1-\tau_j)x_k^{(j)} + \tau_j x_{k-1}^{(j)}]. \tag{10}$$

(Note a slight change of notation, in that the carrier phases of the interfering signals are now expressed relative to the receiver's carrier rather than the carrier phase for the desired signal.)

The decision statistic used for the matched filter stage is

$$\tilde{W}_n = \left[\left(\sum_{l=0}^{N-1} h_i Z_{n-i,\,c}\right)^2 + \left(\sum_{l=0}^{N-1} h_i Z_{n-i,\,s}\right)^2\right]^{1/2} \tag{11}$$

and that for the verification stage is

$$\tilde{V} = \left[\left(\sum_{l=1}^{C} a_{r+i} Z_{t+i,\,c}\right)^2 + \left(\sum_{l=1}^{C} a_{r+i} Z_{t+i,\,s}\right)^2\right]^{1/2}. \tag{12}$$

As before, a hit occurs if $\tilde{W}_n > \alpha N$ and is assumed to be correct if $\tilde{V} > \beta C$. The renewal model for the acquisition process is as for coherent processing, but the transition probabilities are different. As mentioned in Section III-B and shown in the Appendix, the central limit theorem can be used to approximate the transition probabilities for both coherent and noncoherent processing.

It is also worth commenting on the effect of noninteger τ on the preceding development. First, if τ is not an integer, the matched filter target phase may never be perfectly aligned with the current phase, so that, instead of a single state corresponding to a hit, there may be several consecutive states in which there is a strong component due to the desired signal. However, the contribution of the desired signal to the matched filter output in these states would, in general, be smaller than N. This problem can be alleviated by sampling the matched filter output at faster than the chip rate, so that there is at least one state for which the desired signal's contribution is close to N. For PN sequences, with chip rate sampling, there may be two consecutive samples for which the desired signal's contribution is roughly $N/2$, whereas sampling at twice the chip rate may lead to two samples with signal contribution approximately $3N/4$ and two with signal contribution approximately $N/4$. On the other hand, sampling at higher rates also increases the number of false alarm states. A detailed examination of the impact of this tradeoff on acquisition performance is an important topic for future work. Note that the problem is additionally complicated by the fact that the magnitude of the signal contribution in the hit states, and hence the state transition probabilities, depends on the offset of the phase from the target phases at the sampling instants. (The range over which the offset can vary decreases with the sampling rate). To first order, however, this problem may be addressed simply by scaling up the values of N, C, $T_{\max}$ obtained for integer τ (by a factor of two for chip-rate sampling, or a factor of 4/3 for sampling at twice at chip rate). Note also that this problem has a similar impact on the conventional acquisition schemes as well, so that the assumption of integer τ is adequate for an initial comparison of acquisition performance.

B. Computation of Transition Probabilities

We approximate the transition probabilities (3)–(6) by applying the central limit theorem as $N \to \infty$ and $C \to \infty$. The reasoning that leads to the formulas below is sketched in the Appendix, and is based on the development in [5]. Previous applications of the central limit theorem in evaluation of bit error probability are found in [4], [6], and, at least implicitly, in [8]. Note that, while we apply these asymptotic approximations for moderate values of N and C, the corresponding numerical results for the bit error probability [6] indicate that the approximations should be fairly accurate.

Let $Q(\cdot)$ be the complementary standard Gaussian distribution function. We obtain that

$$p_{f1} \approx Q\left(\alpha\sqrt{\frac{3N}{J+3}}\right), \quad p_{m1} \approx Q\left((1-\alpha)\sqrt{\frac{3N}{J}}\right) \tag{13}$$

$$p_{f2} \approx Q\left(\beta\sqrt{\frac{3C}{J+3}}\right), \qquad p_{m2} \approx Q\left((1-\beta)\sqrt{\frac{3C}{J}}\right). \tag{14}$$

For noncoherent processing, assuming that the desired signal's carrier phase ϕ is a random variable which is uniformly distributed over $[0, 2\pi]$, we show in the appendix that the central limit theorem approximation implies that the distribution of the decision statistics (11) and (12) is Rician for a hit state and Rayleigh for other states. The transition probabilities for noncoherent processing are therefore obtained as:

$$\tilde{p}_{f1} \approx \exp\left(-\alpha^2 \frac{3N}{2J+3}\right),$$
$$\tilde{p}_{m1} \approx 1 - Q_M\left(\sqrt{\frac{3N}{J}}, \alpha\sqrt{\frac{3N}{J}}\right), \tag{15}$$

$$\tilde{p}_{f2} \approx \exp\left(-\beta^2 \frac{3C}{2J+3}\right),$$
$$\tilde{p}_{m2} \approx 1 - Q_M\left(\sqrt{\frac{3C}{J}}, \beta\sqrt{\frac{3C}{J}}\right) \tag{16}$$

where $Q_M(x, y)$ is the Marcum's Q-function [11], defined by

$$Q_M(x, y) = \int_y^\infty t \exp\left[-(t^2 + x^2)/2\right] I_0(tx)\, dt$$

and where $I_0(\cdot)$ is the modified Bessel function of order zero.

C. Evaluation of the Probability of Successful Acquisition

Let $P_{\text{ACQ}}(x)$ denote the probability of successful acquisition within time x. We give a recursive formula for $P_{\text{ACQ}}(x)$ in the following. The performance criterion of interest is the probability of successful acquisition $P_{\text{SA}} = P_{\text{ACQ}}(T_{\max})$ where $T_{\max}$ is the maximum time allowed for acquisition. We also obtain an approximation and upper and lower bounds for P_{SA} in the form of easily computable nonrecursive formulas.

We need to establish the following notation. For $1 \leq i \leq T-1$, let

$$q_f(i) = (1 - p_{f1})^{i-1} p_{f1} (1 - p_{f2})$$

be the probability that a renewal (after time $i + C$) is caused by a false alarm at state i that is subsequently rejected at the verification stage. Let

$$q_{m2} = (1 - p_{f1})^{T-1} (1 - p_{m1}) p_{m2}$$

be the probability of a renewal (after time $T + C$) because a correct hit at state T is rejected at the verification stage. Let

$$q_{nh} = (1 - p_{f1})^{T-1} p_{m1}$$

be the probability of a renewal (after time T) caused by no hits at states 1 through T. Finally, let

$$q_s = (1 - p_{f1})^{T-1} (1 - p_{m1})(1 - p_{m2})$$

be the probability of occurrence of successful acquisition without any renewal (absorption into ACQ after time $T + C$).

The required recursion now follows from Fig. 2, and is given by

$$\begin{aligned} P_{\text{ACQ}}(x+N) &= q_s + q_{nh} P_{\text{ACQ}}(x - T) \\ &\quad + \sum_{l=1}^{T-1} q_f(i) P_{\text{ACQ}}(x - i - C) \\ &\quad + q_{m2} P_{\text{ACQ}}(x - T - C), \qquad x \geq T + C, \end{aligned} \tag{17}$$

with initial condition

$$P_{\text{ACQ}}(x + N) = 0, \qquad x < T + C.$$

The acquisition time $x + N$ includes the time required to initially clock the signal into the matched filter, and the initial condition reflects the fact that successful acquisition starting from state 1 takes a time of at least $T + C$.

The computational complexity of the above recursion is principally due to the fact that the amount of time required for a renewal via a hit that is subsequently rejected depends on the state at which the hit occurs; a hit at state i corresponds to a duration $i + C$. The computation can be simplified by assuming that all renewals via rejected hits correspond to a duration $a + C$, and we denote the resulting estimate of the probability of successful acquisition by $\tilde{P}(a)$, computed as follows. Starting from state 1, successful acquisition results from any sequence of renewals, followed by absorption into ACQ, which satisfies the property that the net time elapsed is at most $T_{\max} - N$. After the last renewal before absorption into ACQ, we have a correct hit a state T followed by successful verification; these two steps take time $T + C$. Prior to these steps, we index by i the number of renewals due to rejected hits (each such renewal takes time $a + C$ under the approximation) and j the number of renewals due to no hits in a duration T (each such renewal takes time T), so that the estimate $\tilde{P}(a)$ is given by

$$\tilde{P}(a) = q_s \sum_{(i,\, j) \in A} C(i + j, j) q_h^i q_{nh}^j$$

where the set A is given by

$$A = \{(i, j) | i(a + C) + jT + (T + C) \leq T_{\max} - N\},$$

and where $C(n, m) = n!/[m!(n - m)!]$.

In our numerical results in Section IV, we use the approximation $\tilde{P}_{\text{SA}} = \tilde{P}(\overline{a})$ where $\overline{a}$ is the expected value of a, given by

$$\overline{a} = \left\{ T q_{m2} + \sum_{l=1}^{T-1} i q_f(i) \right\} / q_h$$

where $q_h = q_{m2} + \sum_{l=1}^{T-1} q_f(i)$ is the probability of a renewal via a rejected hit. It is worth noting that $P_U = \tilde{P}(1)$ and $P_L = \tilde{P}(T)$ are upper and lower bounds for the acquisition probability, respectively.

D. Analysis of Existing Schemes

We give an approximate analysis for serial search and passive matched filter schemes. A serial search correlates the received signal against a given phase for a duration C^*, and comparing with a threshold $\beta^* C^*$. This is done for each of the T possible phases, so that, assuming that the correct phase is the last one tested, we obtain a worst case acquisition time of TC^*. Whenever a threshold is exceeded, the acquisition process terminates. It is possible to improve this scheme by employing techniques based on the sequential probability ratio test (see [2] and [10], for instance) but such refinements are beyond the scope of this paper. Formulas for the probability $p_f(ss)$ of a false alarm (i.e. of the threshold being exceeded for an incorrect phase) and the probability $p_m(ss)$ of a miss (i.e., the threshold not being exceeded for the correct phase) follow from the same reasoning that leads to (13)–(14), and are given by

$$p_f(ss) \approx Q\left(\beta^* \sqrt{\frac{3C^*}{J+3}}\right),$$
$$p_m(ss) \approx Q\left((1-\beta^*)\sqrt{\frac{3C^*}{J}}\right). \tag{18}$$

Making the worst case assumption that the correct phase is the last to be tested, the probability of successful acquisition is given by

$$P_{\mathrm{SA}}(ss) \approx [1 - p_f(ss)]^{T-1}[1 - p_m(ss)]. \tag{19}$$

A passive matched filter scheme applies a threshold rule at the output of a matched filter of length N^*; the acquisition process terminates if the output exceeds $\alpha^* N^*$. Once the correct phase is missed, no reprogramming of the matched filter is done to restart the search. Assuming again that the correct phase appears last, the acquisition time is $T+N^*$ where we include the time N^* required to initially clock the received signal into the matched filter. The probability of successful acquisition is given by

$$P_{\mathrm{SA}}(mf) \approx [1 - p_f(mf)]^{T-1}[1 - p_m(mf)] \tag{20}$$

where

$$p_f(mf) \approx Q\left(\alpha^* \sqrt{\frac{3N^*}{J+3}}\right),$$
$$p_m(mf) \approx Q\left((1-\alpha^*)\sqrt{\frac{3N^*}{J}}\right). \tag{21}$$

As in the analysis for our scheme, the formula (20) may be viewed as an approximation resulting from a multidimensional version of the central limit theorem, which implies that the matched filter outputs corresponding to different phases are asymptotically independent. Further, this independence approximation is likely to be pessimistic by an argument similar to the one in Section III-B. For the serial search scheme, on the other hand, the correlator outputs for different phases are truly independent under the random sequence model, so that (19) is approximate only in that the expressions for $p_f(ss)$ and $p_m(ss)$ in (18) are approximate. Further, as for the two-stage scheme, the preceding probabilities are easily modified as in (15)–(16) to account for noncoherent processing. These formulas are omitted for brevity.

IV. Numerical Results and Discussion

Given the number of interfering transmissions J and the timing uncertainty T, our design objective is to choose the parameters of each of the acquisition schemes such that the probability of successful acquisition is at least p_{acq}. For our two-stage scheme, we impose an additional constraint on the length N of the matched filter for the first stage. This constraint may arise from the fact that the matched filter employed for acquisition may be the same filter that is subsequently used for demodulation, in which case N equals the number of chips per symbol, or the *processing gain*. Alternatively, there may be other considerations of cost and technology which constrain the length of the matched filter. The desired value of the acquisition probability is fixed at $p_{\mathrm{acq}} = 0.99$ for all the results in this section.

For the serial search scheme, we compute the minimum value of the correlation period C^* such that, when optimized over β^*, the acquisition probability $P_{\mathrm{SA}}(ss) \geq p_{\mathrm{acq}}$. The acquisition time is given by $T_{\max} = TC^*$. Since the formulas for acquisition probability for the passive matched filter are entirely similar, the minimum value of N^* and the corresponding threshold α^*, are equal to C^* and β^* obtained above. The acquisition time, however, is much smaller, and is given by $T_{\max} = N^* + T$ where we count the time for initially clocking the signal into the matched filter. While there are only two parameters to be chosen in the conventional schemes, we must choose four parameters for the two-stage scheme, despite the fact that we have fixed the matched filter length N: the thresholds α and β, the verification period C, and the time $T_{\max}$ allowed for acquisition. This choice must be made to minimize $T_{\max}$, subject to the constraint that $P_{\mathrm{SA}} \geq p_{\mathrm{acq}}$. Since the optimization of the two-stage scheme is so time consuming, we attempt to find the best set of parameters by exhaustive search for coherent processing only, since computation of the transition probabilities for noncoherent processing is more time consuming (there is a good rational approximation [1] for $Q(\cdot)$, but not for $Q_M(\cdot, \cdot)$). For noncoherent processing, we use trial and error to find a suboptimal set of parameters for the two-stage scheme that satisfies the design constraints. Despite this bias against the two-stage scheme (the conventional schemes are optimized for both coherent and noncoherent processing), it is seen to compare very favorably with both the serial search and passive matched filter schemes in all the cases considered.

TABLE I
PARAMETERS FOR ACHIEVING AN ACQUISITION PROBABILITY OF 0.99 FOR A SYSTEM WITH $J = 9$ INTERFERING SIGNALS AND A TIMING UNCERTAINTY $T = 10$.

Type of scheme	Parameters	
	Coherent	Noncoherent
Two-stage	$N = 30$, $\alpha = 1$, $C = 27$, $\beta = 0.48$, $T_{\max} = 184$	$N = 30$, $\alpha = 1$, $C = 50$, $\beta = 0.75$, $T_{\max} = 338$
Serial search	$C^* = 121$, $\beta^* = 0.6$, $T_{\max} = 1210$	$C^* = 134$, $\beta^* = 0.63$, $T_{\max} = 1340$
Passive matched filter	$N^* = 121$, $\alpha^* = 0.6$, $T_{\max} = 131$	$N^* = 134$, $\alpha^* = 0.63$, $T_{\max} = 144$

TABLE II
PARAMETERS FOR ACHIEVING AN ACQUISITION PROBABILITY OF 0.99 FOR A SYSTEM WITH $J = 30$ INTERFERING SIGNALS AND A TIMING UNCERTAINTY $T = 50$

Type of scheme	Parameters	
	Coherent	Noncoherent
Two-stage	$N = 100$, $\alpha = 1$, $C = 112$, $\beta = 0.56$, $T_{\max} = 858$	$N = 100$, $\alpha = 1$, $C = 180$, $\beta = 0.8$, $T_{\max} = 2339$
Serial search	$C^* = 418$, $\beta^* = 0.61$, $T_{\max} = 20900$	$C^* = 472$, $\beta^* = 0.65$, $T_{\max} = 23600$
Passive matched filter	$N^* = 418$, $\alpha^* = 0.61$, $T_{\max} = 468$	$N^* = 472$, $\alpha^* = 0.65$, $T_{\max} = 512$

Table I contains our first set of numerical results, which are for a system with a small number of interfering signals, $J = 9$, and a small timing uncertainty, $T = 10$. We fix the matched filter length for the two-stage scheme at $N = 30$, which is a length which might typically be used for demodulation purposes. We note that the passive matched filter scheme required for the same performance is much more complex than the two-stage scheme ($N^* \gg N$), and that the acquisition time $T_{\max}$ for the two-stage scheme is much smaller than that for serial search, and only moderately larger than for a passive matched filter scheme. The two-stage scheme therefore satisfies its design objective of trading off complexity versus acquisition time.

The value $\alpha = 1$ used for the first stage indicates that it is primarily devoted to rejecting false alarms. The extra time that elapses due to a miss is T, while the time for a false alarm is at least as large as C. Even with a miss probability $p_{m1} = 1/2$, we ultimately get a correct hit with high probability, and β in the second stage is set to ensure that, with high probability, a false alarm gets rejected and a miss accepted. For all the schemes, the complexity and acquisition time are higher for noncoherent processing, as expected.

Our second set of numerical results, shown in Table II, are for a larger system with $J = 30$ and $T = 50$. Since the matched filter length for demodulation typically scales linearly with J, it is reasonable to scale up our constraint for the matched filter length for the first stage of our scheme as well, so that we set $N = 100$. As before, the two-stage scheme is seen to strike a good compromise between the serial search and passive matched filter schemes.

We note that the search for the parameters of the two-stage scheme was carried out using the nonrecursive approximation $\tilde{P}_{\mathrm{SA}}$, and the results were verified using the recursive formula (17). In all the instances considered, the error in the approximation was negligible: the relative error in $(1 - P_{\mathrm{SA}})$ was a few percent.

V. CONCLUSIONS

Numerical results based on approximate analysis for random signature sequences indicate that, for both coherent and noncoherent processing, rapid acquisition in the presence of multiple-access interference is possible by means of a suitably optimized two-stage acquisition scheme. This scheme represents a compromise in terms of performance and complexity between serial search schemes on the one hand, and parallel search or passive matched filter schemes on the other. The key idea consists of using a programmable matched filter front-end to maintain a small timing uncertainty and to reject incorrect phases so that a long correlation is not required for most such phases.

The results of this study, which is based on a number of idealizations and approximations, are meant to serve as motivation for more detailed studies. However, the conclusions from our numerical results are so intuitively reasonable that we expect them to be true, at least at a qualitative level, even for more detailed system models. Some of the issues that need to be addressed in future work are as follows:

1) the effect of the loss in desired signal strength due to mismatch between the phase at sampling instants and the current phase, and the tradeoffs involved in increasing the sampling rate;
2) refining the approximations for the transition probabilities (e.g., by using an improved Gaussian approximation) to obtain better estimates of acquisition performance for the random signature sequence model;
3) verification of the rough system sizing based on the random signature sequence model by simulations for specific choices of signature sequences.

APPENDIX

We use the central limit theorem to develop approximations for the transition probabilities for both coherent and noncoherent processing. The general approach is as in [5]; here, we merely sketch the reasoning behind (13)–(16) employed in our numerical results. We consider coherent processing first. The interference input is given by (2) where we condition on the θ_j and the τ_j, and we assume equal-power signals ($P_j = 1$). For the purpose of computing false alarm probabilities, we model the effect of the desired transmission as that of an independent interferer, resulting in a net interference input of $X_k' = X_k + a_k$ which is independent of the filter coefficients $\{h_i\}$. We set $X_k' = X_k$ for computing miss probabilities. We then have

$$Y_n = \sum_{l=0}^{N-1} h_i X_{n-i}'. \tag{A.1}$$

The individual terms in the above sum are not independent for an asynchronous system. However, as shown in [5, Appendix], Y_n can be expressed as a linear functional of a two-dimensional random vector which can be expressed as a sum of N random vectors. Application of a two-dimensional central limit theorem then yields, as $N \to \infty$, that

$$p_{f1} \approx Q(\alpha\sqrt{N/(\sigma^2+1)}),$$
$$p_{m1} \approx Q((1-\alpha)\sqrt{N/\sigma^2}) \quad \text{(A.2)}$$

where

$$\sigma^2 = \sum_{j-1}^{J} P_j \cos^2(\theta_j)[(1-\tau_j)^2 + \tau_j^2]. \quad \text{(A.3)}$$

Similarly, as $C \to \infty$, we obtain that

$$p_{f2} \approx Q(\beta\sqrt{C/(\sigma^2+1)}),$$
$$p_{m2} \approx Q((1-\beta)\sqrt{C/\sigma^2}). \quad \text{(A.4)}$$

Conditioned on the θ_j and the τ_j, therefore, we can solve for the probability of successful acquisition using the above transition probabilities in the renewal model. The conditioning can subsequently be removed by averaging over the θ_j and τ_j. Expressions similar to the above in the context of computing error probabilities are developed in [6] using a different approach; the resulting approximation is called the "improved Gaussian" approximation. This is contrast to the "standard Gaussian" approximation (a term used in [6] for the formula introduced in [8]), which amounts to replacing σ^2 in (A.2) and (A.4) by its mean $J/3$ (obtained by averaging (A.3) over τ_j and θ_j). This yields (13) and (14), which are used in our numerical results. We have chosen to use this simple approximation in our numerical computations because our purpose is restricted to providing a rough comparison of our scheme with other acquisition schemes; all conclusions based on a random sequence model must in any event be validated through experiments with specific deterministic sequences. Note that the equations (18) and (21) for the other schemes follow from arguments similar to the above.

We now consider noncoherent processing. We analyze the first stage in detail; the analysis for the second stage is entirely similar. It follows from (7)–(11) that the decision statistic for this stage can be written in the following form:

$$\tilde{W} = (A_c^2 + A_s^2)^{1/2}$$

where

$$A_c = m\sqrt{N}\cos\phi + Y_c, \qquad A_s = m\sqrt{N}\sin\phi + Y_s \quad \text{(A.6)}$$

where $m = \sqrt{N}$ when the target phase equals the current phase, and $m = 0$ otherwise, and where Y_c, Y_s are, respectively, the cosine and sine parts of the effective interference. The random variables Y_c and Y_s are uncorrelated, and each has mean zero and variance $NJ/3$ when the target phase equals the current phase, and $N(J/3 + 1/2)$ otherwise. These observations follow from the assumption that the carrier phase ϕ for the desired signal, and the interference carrier phases θ_j, $1 \leq j \leq J$, are independent random variables uniformly distributed over $[0, 2\pi]$. Application of the central limit theorem conditioned on the carrier phases and relative delays, and subsequent simplification calculations by averaging over these quantities, implies that $Y_c/\sqrt{N}$ and $Y_s/\sqrt{N}$ tend to independent Gaussian random variables with mean zero and variance μ^2 where $\mu^2 = J/3$ when the target phase equals the current phase, and $\mu^2 = J/3 + 1/2$ otherwise.

The preceding analysis implies that the normalized decision statistic $D = \tilde{W}/\sqrt{N}$ can be approximately represented as

$$D = [\overline{A}_c^2 + \overline{A}_s^2]^{1/2}$$

where $\overline{A}_c$, $\overline{A}_s$ are independent Gaussian random variables with mean $m\cos\phi$, $m\sin\phi$, and variance μ^2. Thus, D is a Rician random variable with probability density

$$p_D(r) = (r/\mu^2)\exp[-(m^2+r^2)/2\mu^2] I_0(r/\mu^2), \quad r \geq 0,$$

so that $P[D > R] = Q_M(m/\mu, R/\mu)$. For $m = 0$, the decision statistic is Rayleigh and $P[D > R] = \exp(-R^2/2\mu^2)$. Replacing N by C in the preceding analysis, these results apply to the decision statistic $\tilde{V}$ for the second stage as well. The following observations now lead immediately to the formulas (15)–(16) for the transition probabilities:

1) $\tilde{p}_{f1} = P[D > \alpha\sqrt{N}]$ for $m = 0$ and $\mu^2 = J/3 + 1/2$,
2) $\tilde{p}_{m1} = P[D \leq \alpha\sqrt{N}]$ for $m = \sqrt{N}$ and $\mu^2 = J/3$,
3) $\tilde{p}_{f2} = P[D > \beta\sqrt{C}]$ for $m = 0$ and $\mu^2 = J/3 + 1/2$,
4) $\tilde{p}_{m2} = P[D \leq \beta\sqrt{C}]$ for $m = \sqrt{C}$ and $\mu^2 = J/3$.

It remains to comment on some independence approximations made in this paper. In the context of our two-stage scheme, consider the contributions of the net interference to the matched filter output for different states of the renewal process; these are the random variables $Y_1, \cdots, Y_T$. While application of the central limit theorem to the individual Y_n yields (A.1), it is easy to show also that the Y_n are uncorrelated random variables. Application of a multidimensional central limit theorem thus implies that a suitably normalized version of the random vector $(Y_1, \cdots, Y_T)$ tends to a mean zero Gaussian random vector with covariance matrix diag $(\sigma^2+1, \cdots, \sigma^2+1, \sigma^2)$, so that the Y_n are asymptotically independent. This, together with the argument in Section III–A that the independence assumption is actually pessimistic, leads to (3) and (4) for the state transition probabilities. A similar reasoning leads to (20) for $P_{\mathrm{SA}}(\text{pass})$. The preceding arguments are seen to apply to systems with noncoherent processing as well.

References

[1] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions.* New York: Dover, 1970.

[2] K. K. Chawla and D. V. Sarwate, "Coherent acquisition of PN sequences using a random sequence model and the SPRT," in *Proc. Annu. Conf. Inform. Sci. Syst.*, Princeton Univ., Princeton, NJ, Mar. 1990, pp. 52–57.

[3] J. K. Holmes, *Coherent Spread Spectrum Systems.* New York: Wiley, 1982.

[4] J. S. Lehnert and M. B. Pursley, "Error probabilities for binary direct-sequence spread-spectrum communications with random signature sequences," *IEEE Trans. Commun.*, vol. 35, pp. 85–96, Jan. 1987.

[5] U. Madhow and M. B. Pursley, "Acquisition in direct-sequence spread-spectrum communication networks: an asymptotic analysis," *IEEE Trans. Inform. Theory*, vol. 39, pp. 903–912, May 1993.

[6] R. K. Morrow, Jr. and J. S. Lehnert, "Bit-to-bit dependence in DS/SSMA packet systems with random signature sequences," *IEEE Trans. Commun.*, vol. 37, pp. 1052–1061, Oct. 1989.

[7] A. Polydoros and C. L. Weber, "A unified approach to serial search spread-spectrum code acquisition—Part II: A matched filter receiver," *IEEE Trans. Commun.*, vol. COM-32, pp. 550–560, May 1984.

[8] M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication—Part I: System analysis," *IEEE Trans. Commun.*, vol. COM-25, pp. 795–799, Aug. 1977.

[9] S. S. Rappaport and D. L. Schilling, "A two-level coarse code acquisition scheme for spread spectrum radio," *IEEE Trans. Commun.*, vol. 28, pp. 1734–1742, Sept. 1980.

[10] S. Tantarantana and A. Lam, "Noncoherent sequential acquisition for DS/SS systems," in *Proc. 29th Allerton Conf. Commun., Contr. Comput.*, Univ. of Illinois at Urbana-Champaign, Oct. 1991, pp. 370–379.

[11] H. L. Van Trees, *Detection, Estimation, and Modulation Theory—Part I.* New York: Wiley, 1968.

[12] N. D. Wilson, S. S. Rappaport, and M. M. Vasudevan, "Rapid acquisition scheme for spread spectrum radio in a fading environment," *IEE Proc.*, vol. 135, pt. F, pp. 95–104, Feb. 1988.

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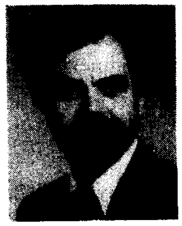

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