Automatic Gain Control for ADC-limited Communication

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***Abstract*— As the date rates and bandwidths of communication systems scale up, the cost and power consumption of high-precision (e.g., 8-12 bits) analog-to-digital converters (ADCs) become prohibitive. One possible approach to relieve this bottleneck is to redesign communication systems with the starting assumption that the receiver will employ ADCs with drastically reduced precision (e.g., 1-4 bits). Encouraging results from information-theoretic analysis in idealized settings prompt a detailed investigation of receiver signal processing algorithms when ADC precision is reduced. In this work, we investigate the problem of automatic gain control (AGC) for pulse amplitude modulation (PAM) signaling over the AWGN channel, with the goal being to align the ADC thresholds with the maximum likelihood (ML) decision regions. The approach is to apply a variable gain to the ADC *input,* fixing the ADC thresholds, with the gain being determined by estimating the signal amplitude from the quantized ADC *output.* We consider a blind approach in which the ML estimate for the signal amplitude is obtained based on the quantized samples corresponding to an unknown symbol sequence. We obtain good performance, in terms of both channel capacity and uncoded bit error rate, at low to moderate SNR, but the performance can actually degrade as SNR increases due to the increased sensitivity of the ML estimator in this regime. However, we demonstrate that the addition of a random Gaussian dither, with power optimized to minimize the normalized mean squared error of the ML estimate, yields performance close to that of an ideal AGC over the entire range of SNR of interest.**

I. I

The economies of scale provided by digital receiver architectures have propelled mass market deployment of cellular and wireless local area networks over the past two decades. An integral component of such receivers is the analog-to-digital converter (ADC), which converts the received analog waveform into the digital domain, typically with a precision of 8-12 bits. As we attempt to extend digital architectures to multiGigabit communication (e.g., emerging wireless systems in the 60 GHz band [1], or more sophisticated signal processing for optical communication), the ADC becomes a bottleneck due to its prohibitive cost and power consumption [2]. One possible approach to relieve this bottleneck is to employ low-precision ADCs. Information-theoretic analysis for the AWGN channel (which is a good approximation for short-range near-line-of-sight 60 GHz links with directional antennas, for example) shows that, even at moderately high signal-to-noise ratio (SNR), the use of 2-3 bit ADCs leads to only a small degradation in channel capacity [3]–[6] This has motivated more detailed investigation of signal processing for key receiver functionalities when ADC precision is reduced, including the problems of carrier synchronization [7], [8] and channel estimation [9]. In this paper, we build on this work, and consider the problem of automatic gain control (AGC) with low-precision ADC.

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The aim of the AGC operation is to ensure that the ADC quantization thresholds are set so as to optimize the performance of the communication link. For system design with low-precision ADC, information-theoretic results [6] show that, for a real AWGN channel model, given a constraint of K-level ADC quantization (i.e., a precision of $\log_2 K$ bits), it is near-optimal to use the strategy of K-point uniform pulse amplitude modulation (PAM) at the transmitter with mid-point quantization at the receiver, irrespective of the SNR. For example, uniform 4-PAM with inputs from $\{-3A, -A, A, 3A\}$, and an ADC with quantization thresholds set at the ML decision boundaries, $\{-2A, 0, 2A\}$, is a near-optimal combination when ADC precision is restricted to 2 bits. We use this system as our running example. The receiver low-noise amplifier brings the signal plus noise power to within a given dynamic range, but the signal power (i.e., the amplitude A) is unknown. Our goal is to determine how to scale the ADC input so that a 2-bit quantizer with uniform thresholds implements the ML decision boundaries. Consequently, the problem of AGC boils down to that of estimating a single parameter A based on the quantized ADC outputs corresponding to the noisy symbol sequence, and then applying the appropriate scale factor to the ADC input.

In order to decouple the AGC problem from that of frame synchronization, we consider blind estimation of the signal amplitude; that is, the symbol sequence used for estimation is unknown, with symbols picked uniformly from a PAM constellation. While the actual values of the symbols are not used by our estimator, we nevertheless use the term "training sequence" for the symbol sequence used for estimation, since reliable data reception cannot occur until the AGC setting is appropriate. The maximum likelihood (ML) estimator of the signal amplitude is obtained as the minimizer of the Kullback-Leibler (KL) divergence between the expected probability distribution and the empirical probability distribution of the quantized output. It is observed that, depending on the

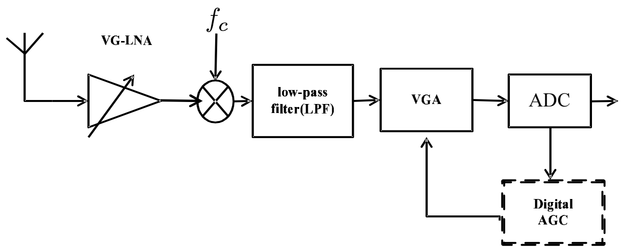

Fig. 1. A typical receiver front-end.

true signal amplitude, the performance of the ML estimator can degrade with increase in the SNR. To alleviate this problem, we investigate the role of dithering, which has been found useful in compensating the severe nonlinearity induced by low-precision quantization in prior work on parameter estimation problems [10]–[12]. Specifically, we add a Gaussian dither signal prior to quantization, with power chosen so as to minimize the normalized mean squared error of the amplitude estimate. For uniform 4-PAM with 2-bit ADC, numerical results are presented to show that, for training sequences of reasonable length, the performance of our dither-based AGC scheme, in terms of both channel capacity and uncoded bit error rate (BER), is close to that with ideal AGC.

The rest of the paper is organized as follows. In Section II, we introduce the receiver architecture, and outline the system model we consider. Section III presents an analysis of our AGC scheme. Numerical results are provided in Section IV followed by the conclusion in Section V.

II. R A S M

A. Receiver Architecture

A typical receiver front-end is illustrated in Fig. 1. It consists of a variable gain low-noise amplifier (VG-LNA) operating at RF, a down-conversion stage, and a variable gain amplifier (VGA) with a digital AGC at the baseband. The power of the incoming RF signal can vary significantly due to path loss and fading. The VG-LNA adjusts its gain so to bring the power level within a smaller dynamic range, while the digital AGC sets the fine-grained scaling implemented using the VGA at the ADC input.

B. System Model and Parameters

Consider uniform M-PAM signaling with uniform M-bin quantization. The signal constellation $\mathcal{X} := \{\alpha_i \mid \alpha_i = (2i - M - 1)A\ , i = 1, 2, \cdots, M\}$, so that there is a single amplitude scaling parameter A. Similarly, the set of M-1 quantizer thresholds is given by $\mathcal{T} := \{t_i \mid t_i = \left(\frac{2i-M}{2}\right)T, i = 1, 2, \cdots, M-1\}$, with a single scaling parameter T. For notational convenience, we also define $t_o := -\infty$ and $t_M := \infty$. For any SNR, we know that it is near-optimal to have the quantizer thresholds to be the mid-points of the constellation points. Without loss of generality, we fix the ADC thresholds $\mathcal{T}$, and scale the VGA gain after estimating A so as to attain this near-optimal setting.

For concreteness, let us assume that the power P_r of the incoming received signal (which is a function of the parameter A) can fluctuate in a 40 dB range. For instance, for an indoor WPAN link, this might correspond to a variation of 0.1m to 10m in the distance between transmitter and receiver. The thermal noise power ${\sigma_i}^2$ at the input to the LNA, which is a function of the bandwidth and the receiver noise figure, is assumed to be known. Fixing ${\sigma_i}^2 = 1$ and P_r to vary between 1-10^4, we thus have a SNR range of 0-40 dB. For a fixed set of thresholds $\mathcal{T}$, there is a desired target level A_t for the signal amplitude level (correspondingly a desired level P_t for the signal power). The analog LNA adjusts its gain based on measurement of the received signal power, and is assumed to bring its output power P to within a range of the desired level. Again, for concreteness, we assume that this range is $[P_t - 5, P_t + 5]$ dB. The role of the digital AGC block now is to estimate the power P (or equivalently, the parameter A) based on the quantized noisy training sequence.

We assume that the noise power σ^2 at the LNA output is known (assuming that the noise power σ_i^2 at the LNA input, and the LNA gain and noise figure, are known). The observations used for estimation of A are therefore given by

$$Y_n = \mathsf{Q}(X_n + W_n), \quad n = 1, \ldots, N \tag{1}$$

where $\mathbf{X} = \{X_1, \cdots, X_n\}$ are i.i.d. samples from an M-PAM constellation with power P, $\{W_n\}$ are i.i.d. $\sim \mathcal{N}(0, \sigma^2)$, Q denotes the quantizer operation, $\mathbf{Y} = \{Y_1, \cdots, Y_n\}$ are the quantized output samples, and N is the length of the training sequence. Each of the quantized output samples $Y_n \in \{y_1, y_2 \ldots, y_M\}$, with $Y_n = y_j$ if $X_n + W_n \in [t_{j-1}, t_j]$.

Running example: While the ML estimator of A we obtain in the next section is valid for general M, most of the subsequent analysis is restricted to the case of 4-PAM input with 2-bit ADC. In this special case, the constellation $\mathcal{X} = \{-3A, -A, A, 3A\}$ and the set of threshold $\mathcal{T} = \{-T, 0, T\}$. The power P is related to the parameter A as $P = 0.5(A^2 + 9A^2) = 5A^2$. Without loss of generality, fix $\sigma^2 = 1$, so that for a 40dB SNR range, P can vary between 1-10^4. With the set of thresholds fixed to be $\mathcal{T} = \{-1, 0, 1\}$, the desired amplitude $A_t = 0.5$ and the desired signal power $P_t = 1.25 = 0.96$ dB. The signal power P at the LNA output can therefore lie in $[-4, 6]$ dB, corresponding to the parameter $A \in [0.23, 0.89]$. For any choice of A in this range, the aim of the AGC block is to obtain an estimate $\hat{A}$ using the quantized samples $\{Y_n\}$.

III. S A E

We first obtain the ML estimator of A based on the quantized samples $\{Y_n\}$. The training sequence $\{X_n\}$ is assumed to be drawn in an i.i.d. manner from a uniform M-PAM distribution $\{\alpha_1(A), \ldots, \alpha_M(A)\}$. We have

$$\hat{A}_{ML} = \arg\max_A \mathrm{P}(\mathbf{Y}|A) = \arg\max_A \prod_{n=1}^{N} \mathrm{P}(Y_n|A), \tag{2}$$

where each $Y_n \in \{y_1, y_2 \ldots, y_M\}$. Combining the terms corresponding to the same output indices, and taking the log-likelihood, we get

$$\hat{A}_{ML} = \arg\max_{A} \sum_{j=1}^{M} N_j \log \mathrm{P}(y_j|A), \tag{3}$$

where N_j is the number of occurrences of y_j in the set $\mathbf{Y}$, and

$$q_j(A) := \mathrm{P}(y_j|A) = \frac{1}{M}\sum_{i=1}^{M}\left(Q\left(\frac{t_{j-1}-\alpha_i(A)}{\sigma}\right) - Q\left(\frac{t_j-\alpha_i(A)}{\sigma}\right)\right), \tag{4}$$

with $Q(x)$ denoting the complementary Gaussian distribution function $\frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)dt$.

Denoting the empirical estimate of $q_j(A)$ as $\hat{q}_j := \frac{N_j}{N}$, we get

$$\hat{A}_{ML} = \arg\max_{A} \sum_{j=1}^{M} \hat{q}_j \log q_j(A). \tag{5}$$

We now add $\sum_{j=1}^{M} \hat{q}_j \log \frac{1}{\hat{q}_j}$ (this is a constant independent of A that does not change the maximizing value of A) to bring the cost function into the following suggestive form:

$$\begin{aligned}\hat{A}_{ML} &= \arg\max_{A}\left(\sum_{j=1}^{M}\hat{q}_j \log q_j(A) + \sum_{j=1}^{M}\hat{q}_j \log\frac{1}{\hat{q}_j}\right)\\ &= \arg\max_{A} -\left(\sum_{j=1}^{M}\hat{q}_j \log\frac{\hat{q}_j}{q_j(A)}\right).\end{aligned} \tag{6}$$

Let $\hat{Q} := \{\hat{q}_1, \hat{q}_2, \cdots, \hat{q}_M\}$ and $Q(A) := \{q_1(A), q_2(A), \cdots, q_M(A)\}$. Therefore, $\mathrm{D}_{\mathrm{KL}}(\hat{Q}\|Q(A)) = \sum_{j=1}^{M}\hat{q}_j \log\frac{\hat{q}_j}{q_j(A)}$ is the Kullback-Leibler (KL) divergence between the distributions Q and $\hat{Q}$, so that the ML estimator is the minimizer of this KL divergence.

In general, the minimum possible divergence of 0 may not be achievable, since there may not exist a choice of A that ensures $q_j(A) = \hat{q}_j \ \forall j$. For ADC precision greater than 2 bits, we cannot obtain a simple expression for the ML estimator. It can be computed numerically by minimizing the KL divergence. A simpler suboptimal solution can be obtained by solving one of the equations $q_j(A) = \hat{q}_j$ for some j. For our running example of 2-bit quantizer with 4-PAM input, the latter approach gives the exact ML estimate. Since $q_1(A) = q_4(A)$ and $q_2(A) = q_3(A)$ for all A, we define $\hat{q} = (\hat{q}_2 + \hat{q}_3)/2$ and $q(A) = q_2(A) = q_3(A)$. Then $(\hat{q}_1 + \hat{q}_4)/2 = 1/2 - \hat{q}$ and $q_1(A) = q_4(A) = 1/2 - q(A)$. Therefore, (6) can be simplified as

$$\begin{aligned}\hat{A}_{ML} &= \arg\max_{A}\left(2\left(\frac{1}{2}-\hat{q}\right)\log\left(\frac{1}{2}-q(A)\right) + 2\hat{q}\log q(A)\right)\\ &= \arg\max_{A} -\left(2\left(\frac{1}{2}-\hat{q}\right)\log\frac{\frac{1}{2}-\hat{q}}{\frac{1}{2}-q(A)} + 2\hat{q}\log\frac{\hat{q}}{q(A)}\right).\end{aligned} \tag{7}$$

In this case, the divergence is minimized by picking A such that $q(A) = \hat{q}$, we therefore do have a simplified expression for the ML estimator

$$\hat{A}_{ML} = q^{-1}(\hat{q}). \tag{8}$$

The inverse function q^{-1} cannot be stated explicitly, but can easily be computed numerically.

Fig. 3 plots the desired inverse function (with the threshold $T = 1$) giving the ML estimate for our running example. We can see that when the noise variance σ^2 is small, the curve is very steep near $q \approx 0.25$, which implies that a small deviation in the empirical probability $\hat{q}$ (from its expected value $q(A)$) can result in a large error in estimating A. The steepness of the curve is simply understood by looking at Fig. 2, which shows the conditional pdf of $\alpha_i + W_n$ for different i. For $\sigma^2 \to 0$, any choice of A in the range $[\frac{T}{3}, T]$ results in $q(A) = 0.25$.

It is clear that when σ^2 is small, the ML estimator can result in poor performance. A simple strategy to improve the performance therefore is to increase the noise variance by deliberate addition of a random Gaussian dither signal prior to quantization. Looking at Fig. 3, this makes the curve smoother around $q = 0.25$; however, it also makes it steeper around other values of q. In order to obtain design guidelines for the dither variance, we next analyze the mean squared error of the ML estimator.

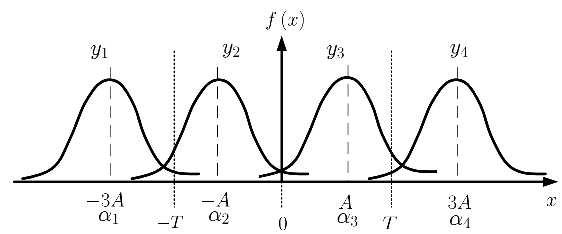

Fig. 2. Conditional probability density functions of 4-PAM and 2-bit quantizer.

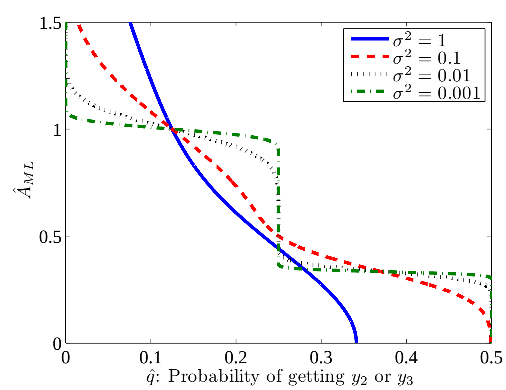

Fig. 3. 2-bit quantizer with 4-PAM: The ML estimation of the amplitude A vs empirical probability of getting y_2 or y_3 (Threshold $T = 1$).

A. NMSE Analysis

Note first that for 2-bit quantizer and 4-PAM input with N training symbols, $N_2 + N_3$ is a random variable following the binomial distribution with parameters N and $2q(A)$, i.e., $N_2 + N_3 \sim \mathrm{B}(N, 2q(A))$. Therefore, $\hat{q}$ is a random variable with mean $E[\hat{q}] = q(A)$ and variance $E\left[(\hat{q} - q(A))^2\right] = q(A)(1 - 2q(A))/2N$.

$$\frac{\partial q}{\partial A} = \frac{1}{4\sqrt{2\pi\sigma^2}}\left(3\exp\left(-\frac{(T-3A)}{2\sigma^2}\right)+\exp\left(-\frac{(T-A)}{2\sigma^2}\right)-\exp\left(-\frac{(T+A)}{2\sigma^2}\right)-3\exp\left(-\frac{(T+3A)}{2\sigma^2}\right)\right). \tag{9}$$

For fixed A, the error in the estimation of A, denoted $\Delta A = \hat{A}_{ML} - A$, due to the difference Δq between the empirical probability $\hat{q}$ and its expected valued $q(A)$, is given by

$$\Delta A \approx \Delta q \left.\frac{\partial A}{\partial q}\right|_{q(A)} = \frac{\Delta q}{\left.\frac{\partial q}{\partial A}\right|_A}, \tag{10}$$

where the function $\frac{\partial q}{\partial A}$ is as expressed in (9). Then the normalized mean square (NMSE) in A can be written as

$$\text{NMSE}(A){=}\text{E}\left[\left(\frac{\Delta A}{A}\right)^2\right] \approx \frac{\text{E}\left[(\Delta q)\right]}{A^2}\left(\left.\frac{\partial q}{\partial A}\right|_A\right)^2 {=} \frac{q(1-2q)}{2NA^2\left(\left.\frac{\partial q}{\partial A}\right|_A\right)^2}. \tag{11}$$

In Fig. 4, we plot theNMSE in A for different choices of A, as a function of σ^2, with the threshold $T = 1$. As explained before, for small σ^2, the error can be excessive for certain values of A.

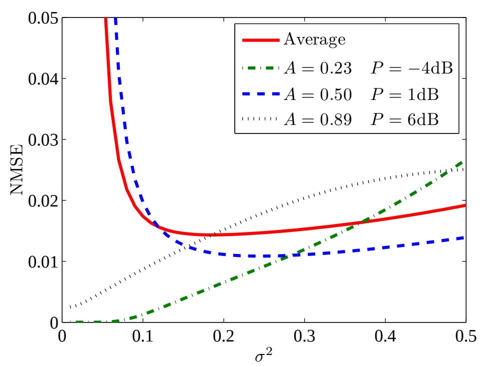

Fig. 4. 2-bit quantizer with 4-PAM: NMSE in estimating A, as a function of σ^2 ($T = 1$, $N = 100$).

B. Dithering

To alleviate the performance degradation in estimation of A in the high SNR regime, we add a Gaussian distributed dither signal prior to quantization, so that our channel model becomes

$$Y_n = \text{Q}(X_n + D_n + W_n), \tag{12}$$

where $D_n \sim \mathcal{N}(0, \sigma_d^2)$ are the i.i.d. dither signals, picked independently of X_n and W_n[1]. As a result, $D_n + W_n$ is $\sim \mathcal{N}(0, \sigma_d^2 + \sigma^2)$.

We now compute the average NMSE (averaged over the prior distribution of A), and pick the dither variance σ_d^2 so that the average NMSE is minimized. In other words, $\sigma_d^2 = \bar{\sigma}^2 - \sigma^2$, where $\bar{\sigma}^2 = \arg\min_{\sigma^2} \text{E}_\text{A}\left[\text{NMSE}(A, \sigma^2)\right]$. Note that the dither signal is added only if $\sigma^2 < \bar{\sigma}^2$.

[1]Note that the dither signal is to be added only during the training period to estimate A, and not during actual data transmission

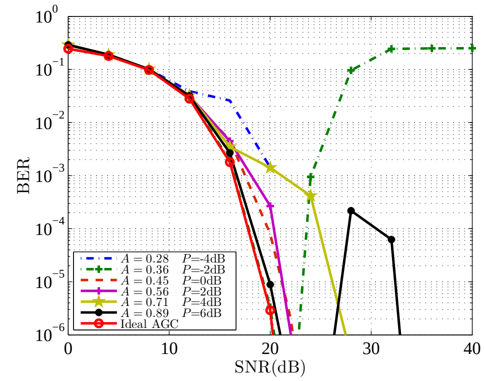

Fig. 5. 2-bit quantizer with 4-PAM: BER vs SNR curves for different signal powers. (Training length $N = 100$)

IV. N R

We now present results for 4-PAM input with 2-bit ADC. To obtain the results, we generated training sequences by picking samples in an i.i.d. manner from the 4-PAM input. In order to optimize the performance, we picked "balanced" training sequences in which all amplitudes occur equally often.

Fig. 5 shows the results without dithering. We plot the uncoded BER versus SNR, for different values of the signal amplitude A (corresponding to different values of the signal power P) with the training set length $N = 100$. It is seen that for some values of A, the BER increases with SNR in the high SNR regime. As explained before, this happens because of large errors in estimation of A at high SNR values.

Next, we consider the performance with dithering. Fig. 6 shows the BER versus SNR curves for different A with the training length $N = 100$. We can see that the addition of the dither signal eliminates the high SNR performance degradation that occurs without dithering. At BER=10^{-3}, the loss compared to ideal AGC is about 1-3 dB, for different A. At BER=10^{-6}, the loss varies between 3-6 dB. We also show the corresponding plots for the input-output mutual information in Fig. 7. Compared to ideal AGC, our proposed estimator incurs a negligible loss. Fig. 8 shows the NMSE versus SNR curves for different A. We can see that the addition of the dither signal prevents the NMSE from shooting up at high SNR.

Finally, Fig. 9 shows the BER performance curves for different lengths of the training sequence N. Each of the curves here is by obtained by averaging the BER over all values of the amplitude A. While for large values of N, the performance approaches that achieved with ideal AGC, for smaller values, we observe an error floor. This is attributed to the fact that for a small training sequence length, there can be certain values of the parameter A for which the estimation error may be large enough to cause both input levels $\{A, 3A\}$ to fall in the same quantization bin after VGA scaling, which results in a significant error probability irrespective of SNR.

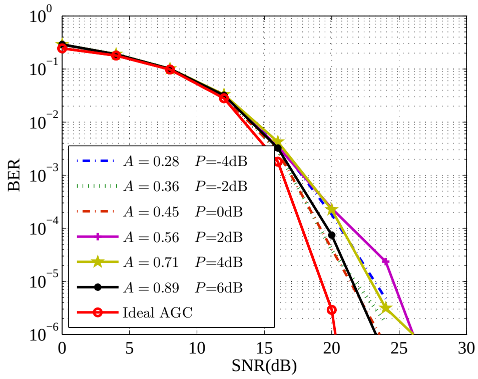

Fig. 6. 2-bit quantizer with 4-PAM: BER vs SNR curves with optimal dithering for different input signal powers. (Training length $N = 100$)

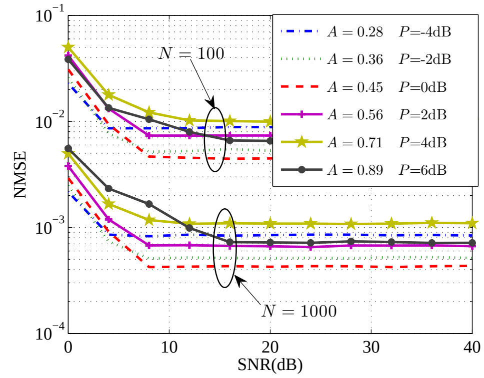

Fig. 8. 2-bit quantizer with 4-PAM: NMSE of estimation of A with optimal dithering for different input signal powers. (Training length $N = 100$)

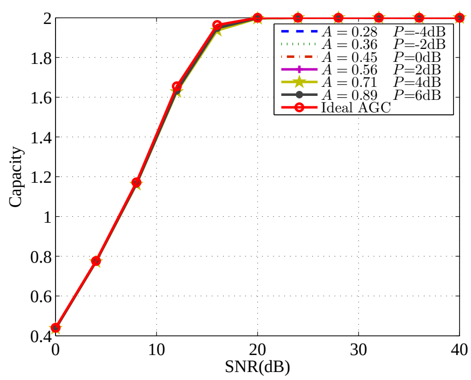

Fig. 7. 2-bit quantizer with 4-PAM: Channel capacity with optimal dithering for different input signal powers. (Training length $N = 100$)

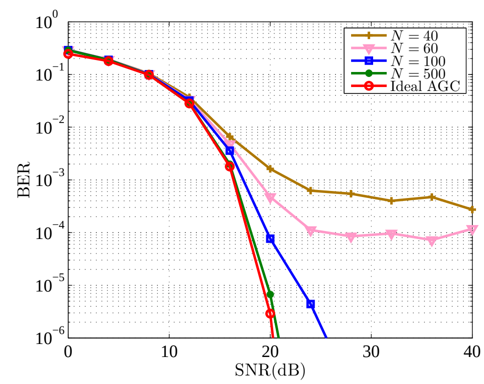

Fig. 9. 2-bit quantizer with 4-PAM: Average BER vs SNR curves for different lengths of the training sequence.

V. C

We have investigated the problem of automatic gain control when ADC precision is constrained at the receiver. As has been observed in the prior parameter estimation problems, dithering is found to be essential in order to obtain a good performance in the face of drastic quantization. Problems for future analysis include a more detailed analytical and numerical analysis for larger constellations (e.g., 8-PAM with 3-bit ADC), as well as investigation of the achievable performance over fading and dispersive channels. Another important topic for future research is whether the required training period can be reduced by adapting the VGA scale factor on the fly, rather than computing it based on one-shot estimation of the signal amplitude.

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