# BAD: Bidirectional Arbitrated Decision-Feedback Equalization

J. K. Nelson, *Student Member, IEEE*, A. C. Singer, *Member, IEEE*, U. Madhow, *Senior Member, IEEE*, and C. S. McGahey, *Member, IEEE*

***Abstract*—The bidirectional arbitrated decision-feedback equalizer (BAD), which has bit-error rate performance between a decision-feedback equalizer (DFE) and maximum *a posteriori* (MAP) detection, is presented. The computational complexity of the BAD algorithm is linear in the channel length, which is the same as that of the DFE, and significantly lower than the exponential complexity of the MAP detector. While the relative performance of BAD to those of the DFE and the MAP detector depends on the specific channel model, for an error probability of $10^{-2}$, the performance of BAD is typically 1–2 dB better than that of the DFE, and within 1 dB of the performance of MAP detection.**

***Index Terms*—Arbiter, decision-feedback equalizers (DFEs), digital communications, equalizers, multipath channels, time-reversal diversity.**

## I. Introduction

THE computational complexity of optimum equalization of an intersymbol interference (ISI) channel is prohibitive for many applications, growing as $M^L$, where $M$ is the alphabet size and $L$ the channel length [1], [2]. As a result, many suboptimal equalization techniques have been proposed. Perhaps the most popular is the decision-feedback equalizer (DFE) [3], which has complexity linear in filter length (typically proportional to $L$) and independent of $M$. For typical channels at modest bit-error rates (BER), the performance of the DFE is about 2–3 dB away from optimal maximum *a posteriori* (MAP) or maximum-likelihood (ML) performance. The bidirectional arbitrated DFE (BAD) proposed in this letter closes this gap to about 1 dB, while incurring complexity on the same order as that of the DFE.

The principle behind ML or MAP detection is to choose, from among all possible candidate data sequences, the one that best explains the received sequence. In the BAD algorithm, we drastically reduce the number of candidate sequences and arbitrate among them based on which candidate best explains the received sequence in a window around the symbol of interest. The two candidate sequences employed in the BAD algorithm are generated by running both a forward and a reverse DFE over a full block of data.

Bidirectional DFE processing has been previously proposed in [4] and [5], using an arbitration mechanism significantly different than the one we consider. In [4] and [5], the mean-squared error (MSE) between the input and output of the DFE decision device is used as the criterion to choose between the results of forward and reverse processing. The MSE criterion is applied to an entire frame of data. In contrast to the preceding *global* MSE criterion, the arbitration mechanism in BAD can be interpreted as a *local* MAP decision between two candidate sequences, since each decision is based only on a window around the bit of interest. Another related class of suboptimum equalizers is that of reduced state sequence estimation [6], [7], or delayed decision feedback [8]. These techniques, which use the DFE to navigate a pruned trellis, approximate ML equalization to varying degrees, but unlike BAD, they require complexity per demodulated symbol that is exponential in the length of the truncated channel.

We compare the simulated performance of BAD with that of the DFE and of MAP detection. While BAD is of primary interest in applications with large symbol alphabets and/or long channels, for which ML and MAP detection are infeasible, many of our simulation results are for binary phase-shift keying (BPSK) transmission over channels of moderate lengths, in order to enable comparison with BER optimal equalization. We do demonstrate, however, that BAD gives similar performance gains over the DFE for 8-ary phase-shift keying (8-PSK) constellations as well. We consider postequalization error probabilities of $10^{-3}$ to $10^{-1}$ for these simulation-based comparisons, since, depending on the strength of the error-correction code used, this is often a range of great practical interest.

This paper is organized as follows. Section II describes the standard equalization problem, the DFE, the BAD algorithm, and the computational complexity of various equalization schemes. Section III provides numerical results for stationary and fading channels. Section IV summarizes our conclusions.

## II. System Model and Algorithm

Consider linear modulation over a real baseband, discrete-time, symbol-spaced channel corrupted by additive white Gaussian noise (AWGN). The transmitted frame of data is

Paper approved by R. A. Kennedy, the Editor for Data Communications Modulation and Signal Design of the IEEE Communications Society. Manuscript received August 29, 2001; revised March 25, 2003. This work was supported in part by the National Science Foundation under Grants NCR96-24008 (CAREER), CCR-0092598 (CAREER), CCR99-79381, ITR00-85929, and ITR02-20118, in part by the Army Research Office under Grant DAAD19-00-1-0567, in part by by the Office of Naval Research under Grant N00014-01-1-0117, and in part by the Motorola Center for Communications. This paper was presented in part at the 34th Conference on Information Science and Systems, Princeton, NJ, March 2000.

J. K. Nelson and A. C. Singer are with the Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: jnelson@ifp.uiuc.edu; acsinger@uiuc.edu).

U. Madhow is with the Department of Electrical and Computer Engineering, University of California, Santa Barbara, CA 93106 USA (e-mail: madhow@ece.ucsb.edu).

C. S. McGahey is with the School of History, Technology, and Society, Georgia Institute of Technology, Atlanta, GA 30332-0345 USA (e-mail: chris.mcgahey@hts.gatech.edu).

Digital Object Identifier 10.1109/TCOMM.2004.841980

TABLE I
BAD

| |
|---|
| (1) Bi-Directional Processing<br>$\hat{b}_f[n] =$ output of forward DFE, $n = 1 \dots N$<br>$\hat{b}_r[n] =$ time-reversed output of reverse DFE, $n = 1 \dots N$ |
| (2) Data Reconstruction<br>$\hat{r}_f[n] = \sum_{k=-L_1}^{L_2} h[k]\hat{b}_f[n-k],\ n = 1 \dots N$<br>$\hat{r}_r[n] = \sum_{k=-L_1}^{L_2} h[k]\hat{b}_r[n-k],\ n = 1 \dots N$ |
| For each $n$ from 1 to $N$<br>If $\hat{b}_f[n] = \hat{b}_r[n]$<br>Then $\hat{b}_{\text{BAD}}[n] = \hat{b}_f[n]$<br>Else<br>(3) Symbol Arbitration<br>$\gamma_f[n] = \sum_{k=-W_1}^{k=W_2} \lvert r[n+k] - \hat{r}_f[n+k]\rvert^2$<br>$\gamma_r[n] = \sum_{k=-W_1}^{k=W_2} \lvert r[n+k] - \hat{r}_r[n+k]\rvert^2$<br>where $W = W_1 + W_2 + 1$ is the length of the arbitration window.<br>$\hat{b}_{BAD}[n] = \begin{cases} \hat{b}_f[n], \gamma_f[n] < \gamma_r[n] \\ \hat{b}_r[n], \gamma_f[n] > \gamma_r[n] \end{cases}$ |

denoted by $\mathbf{b} = \{b[n]\}_{n=1}^{N}$. The channel output at time $n$ is given by

$$r[n] = \sum_{k=-L_1}^{L_2} h[k]b[n-k] + w[n] \tag{1}$$

where $\mathbf{h} = \{h[k], -L_1 \le k \le L_2\}$ is the channel impulse response (CIR), and $w[n]$ is AWGN. The channel is assumed to be time-invariant for the duration of the data sequence. It is assumed that the noise variance $\sigma_w^2$ and the CIR $\mathbf{h}$ are known to the receiver. While the preceding model is used for simplicity of exposition, the BAD algorithm applies in general to any setting in which the classical DFE can be employed. For example, it applies to complex baseband, fractionally spaced channels with complex symbol alphabets and colored noise.

### A. The Classical DFE

The classical DFE consists of a feedforward filter $\mathbf{c}_{\text{ff}}$, which takes the received data $\mathbf{r}[n]$ as input and linearly suppresses precursor ISI, and a feedback filter $\mathbf{c}_{\text{fb}}$, which takes as input hard decisions $\hat{\mathbf{b}}[n]$ on past symbols, and subtracts the estimated postcursor ISI from the output of the feedforward filter.

For implementation of the standard DFE and the BAD algorithm, we consider the minimum mean-squared error (MMSE) DFE, for which the filter coefficients $\mathbf{c}_{\text{ff}}$ and $\mathbf{c}_{\text{fb}}$ are computed to minimize the MSE between the input and output of the decision device. In our numerical results, we assume that the number of feedback coefficients equals the number of past symbols falling in the observation interval. In practice, fewer feedback taps may be used if the channel length is long, relying on the feedforward filter to suppress both the future symbols and a subset of the past symbols.

### B. The BAD Algorithm

The BAD algorithm, which is summarized in Table I, has three stages: 1) bidirectional processing with MMSE-DFEs; 2) data reconstruction; and 3) symbol arbitration. The bidirectional-processing stage involves processing the received sequence with a standard DFE, and processing the time reversal of the received sequence with a DFE designed for the time reversal of the channel. In this manner, two estimates of the transmitted block of data are produced. In the reconstruction stage, each estimate of the transmitted data block is convolved with the channel response to form a noise-free estimate of the received sequence.

If the two estimates of a particular symbol do not agree, arbitration between the estimates must be employed. In the final stage, the BAD algorithm arbitrates between symbol decisions to produce final estimates of the data. The arbitration criterion is the quality of the local match (in a window around the bit of interest) produced with the received sequence. The arbitrated symbol estimate is the one for which the received sequence estimate is closest in Euclidean distance to the true received sequence. Ties are of zero probability, and can be handled by enlarging the arbitration window until the metrics differ.

At modest signal-to-noise ratio (SNR) levels, one of the most significant contributors to the differences in performance between the DFE and ML detection is error propagation. The BAD algorithm reduces the effects of error propagation by arbitrating between candidate sequences with low error correlation. The two estimates $\hat{b}_f[n]$ and $\hat{b}_r[n]$ can differ substantially, providing diversity that is exploited by the BAD algorithm. By having two opportunities to avoid error propagation, we essentially require two error-propagation events to ensue for BAD to have an error, while the standard DFE requires only one. This gives rise to a nearly 3 dB improvement in performance. More elaborate analysis can provide additional insight into the mechanisms that induce errors at high SNR [9].

TABLE II
COMPUTATIONAL COMPLEXITY OF EQUALIZATION METHODS

| Equalizer | Calculations | Operation |
|---|---|---|
| MMSE-DFE | $K_{ff} + K_{fb}$ | ×'s |
| | $K_{ff} + K_{fb} - 1$ | +'s |
| BAD Algorithm | $2(K_{ff} + K_{fb} + (\frac{N+L-1}{N})L + W)$ | ×'s |
| | $2(K_{ff} + K_{fb} - 1 + (\frac{N+L-1}{N})(L-1) + 2W - 1)$ | +'s |
| MAP (BCJR) | $M^{2L}$ | ×'s |
| | $M^{2L}$ | +'s |

### C. Computational Complexity

Table II gives the computational complexity of the BAD algorithm along with the complexity of the MMSE-DFE and MAP detection. $K_{\text{ff}}$ is the length of the feedforward filter, $K_{\text{fb}}$ the length of the feedback filter, $L$ the length of the channel, $W$ the length of the arbitration window, $N$ the length of the data sequence, and $M$ the size of the symbol alphabet. The table clearly illustrates that for larger channel lengths and constellation sizes, BAD complexity is only slightly greater than MMSE-DFE complexity, and far smaller than MAP complexity. Note that the complexity of the BAD algorithm is a linear function of the filter order, channel length, and arbitration window size. Simulation results show that increasing $K_{\text{ff}}$, $K_{\text{fb}}$, and $W$ generally results in lower BER. Hence, in seeking to minimize complexity, one must balance the competing concern of performance.

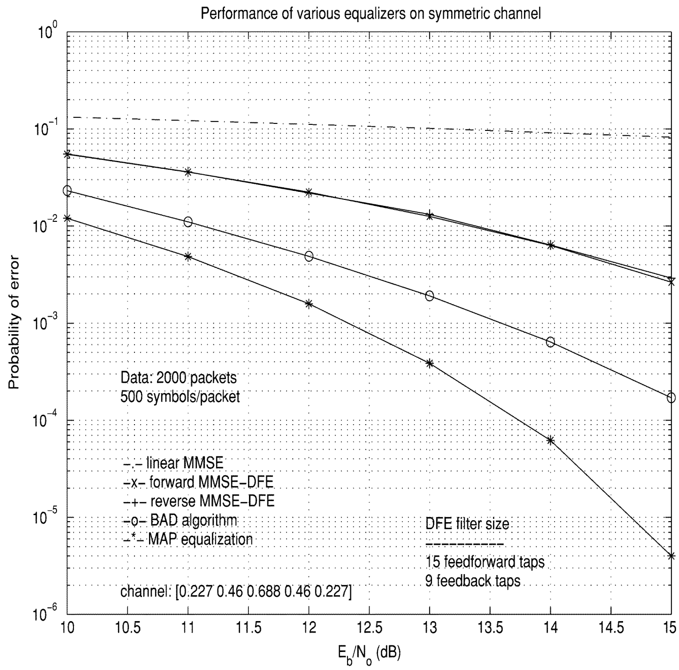


Fig. 1. Performance of the BAD algorithm on a symmetric channel. Shown for comparison are the linear MMSE equalizer, forward/reverse MMSE-DFE, and optimal MAP equalizer performances. Channel: $H_{\mathrm{sym}}(z)$; filter orders: $K_{\mathrm{ff}} = 15, K_{\mathrm{fb}} = 9$; window size: $W = 15$; simulation size: 2000 packets of 500 bits each.

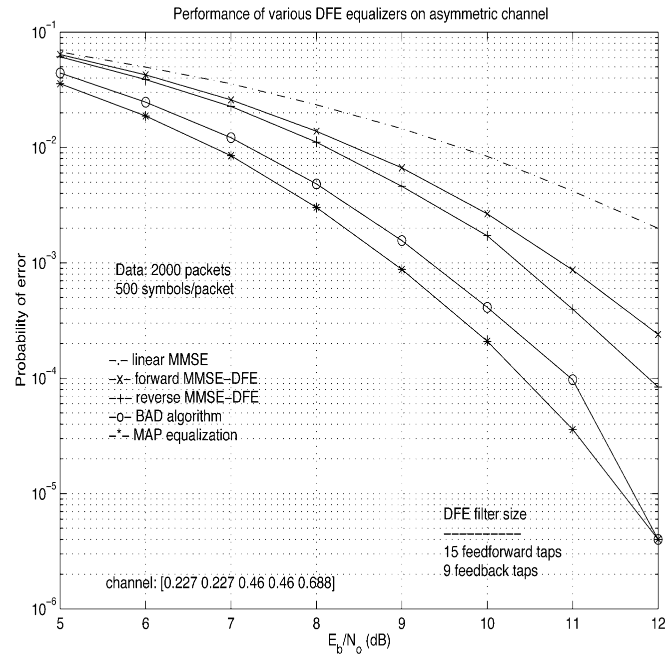


Fig. 2. Performance of the BAD algorithm on a maximum-phase channel. Shown for comparison are the linear MMSE equalizer, forward/reverse MMSE-DFE, and optimal MAP equalizer performances. Channel: $H_{\max}(z)$; filter orders: $K_{\mathrm{ff}} = 15, K_{\mathrm{fb}} = 9$; window size: $W = 15$; simulation size: 2000 packets of 500 bits each.

It should be noted that all three equalization schemes considered require knowledge of the channel over which data is transmitted, and hence, will incur the additional complexity (not included in the table) required to perform channel estimation.

## III. Simulation Results

In Section III-A–III-C, we consider BPSK modulation in order to compare BAD with both DFE and MAP in a variety of settings. However, in Section III-D, we provide simulations for 8-PSK that confirm that BAD provides similar performance gains over the DFE for larger constellations, as well. We plot the BER versus $E_b/N_0$, where $E_b$ is the energy per transmitted bit, and the AWGN has variance $\sigma_w^2 = N_0/2$. For simulations over fading channels, we plot the BER versus average $E_b/N_0$. For the BPSK simulations, we use 2000 packets, each of length 500 symbols, for each point on the curve. For the 8-PSK simulations in Section III-D, it is necessary to use 20 000 packets per simulation point to obtain a smooth curve. Unless otherwise specified, the length of the arbitration window for simulation of the BAD algorithm is $W = 15$, and the orders of the DFE filters are 15 for the feedforward and 9 for the feedback.

### *A. Stationary Channel Simulations*

The simulations presented in Figs. 1 and 2 consider two channels: a symmetric channel $H_{\mathrm{sym}}(z)$, and a maximum-phase channel, $H_{\max}(z)$, both given below

$$H_{\mathrm{sym}}(z) = 0.227 + 0.46z^{-1} + 0.688z^{-2} + 0.46z^{-3} + 0.227z^{-4} \quad (2)$$

$$H_{\max}(z) = 0.227 + 0.227z^{-1} + 0.46z^{-2} + 0.46z^{-3} + 0.688z^{-4}. \quad (3)$$

Fig. 1 compares the performance of several equalization methods on $H_{\mathrm{sym}}(z)$, which nearly contains a spectral null. Linear equalizers typically perform poorly on such channels due to noise enhancement, and indeed, the BER achieved by the linear MMSE equalizer is high (approximately $10^{-1}$) and nearly flat over the range of SNR considered. The forward and reverse DFE processors yield identical performance, which can be attributed to the symmetry of the channel. As Fig. 1 shows, the BAD algorithm improves upon the performance of the DFE by approximately 2 dB, and is within about 1 dB of the optimal MAP performance.

The maximum-phase channel $H_{\max}(z)$, obtained by rearranging the taps of the symmetric channel, was selected for the difficulty it presents to causal equalization methods. The ideal zero-forcing equalization filter, $H_{\max}^{-1}(z)$, has poles outside the unit circle, and hence, an impulse response that is anticausal and infinite in length. If equalization is limited to finite-length designs, $H_{\max}^{-1}(z)$ may be only roughly approximated. As discussed in [5], bidirectional processing is particularly beneficial in such a scenario, because the reverse processor sees a minimum-phase channel. The performance of various equalizers on the maximum-phase channel is shown in Fig. 2. The linear MMSE equalizer shows a greater improvement with increasing SNR in this case, but the DFE still gives significantly better performance. As expected, the reverse DFE performance is slightly better than that of the forward DFE. For the maximum-phase channel, the BAD algorithm performs at least 1 dB better than the reverse DFE, and within 0.5 dB of the MAP detector.

### *B. Multipath Fading Channel Simulations*

A multipath fading channel $H_{\mathrm{mp}}(z)$ was chosen to simulate practical performance in a time-varying environment. The

TABLE III
MULTIPATH FADING CHANNEL SIMULATION PARAMETERS

| Parameter | Value |
|---|---|
| Model | Hilly Terrain (12 tap setting) [11] |
| Model tap delays ($\mu$s) | 0.0, 0.1, 0.3, 0.5, 0.7, 1.0 |
| Model tap delays ($\mu$s) - cont. | 1.3, 15.0, 15.2, 15.7, 17.2, 20.0 |
| Model tap average rel. powers (dB) | -10, -8, -6, -4, 0, 0, -4, -8, -9, -10, -12, -14 |
| Vehicle speed | 100 km/hr |
| Transmission freq. | 900 MHz |
| Symbol period | 3.692 $\mu$s |
| Oversampling rate | 1x |
| Channel length | 7 taps |
| Channel sampling period | 4.615 ms |

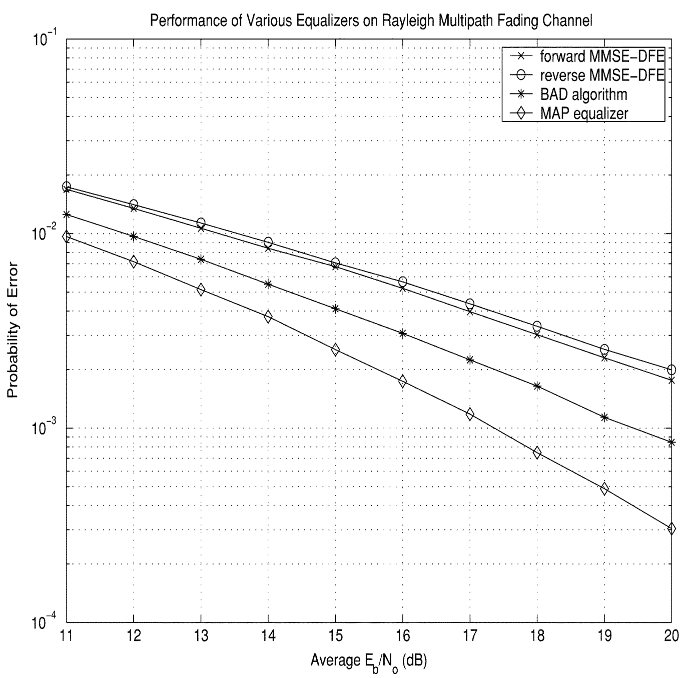


Fig. 3. Multipath fading channel performance of the BAD algorithm. Shown for comparison are the forward/reverse MMSE-DFE and optimal MAP equalizer performances. Channel: $H_{\mathrm{mp}}(z)$; filter orders: $K_{\mathrm{ff}} = 7, K_{\mathrm{fb}} = 6$; window size: $W = 7$; simulation size: 6760 packets of 500 bits each. See Table III for parameters used in generating $H_{\mathrm{mp}}(z)$.

unique difficulty of a fading channel is that its response may vary in time from minimum to maximum phase, and may also enter a deep fade. For the simulations presented here, the envelope of each individual multipath component varies according to a Rayleigh distribution. The rate of fading is assumed to be slow enough that the channel response may be considered constant over the duration of one packet. The parameters used in generating the multipath fading channel are shown in Table III. Fig. 3 shows the performance of the forward DFE, reverse DFE, BAD algorithm, and optimal MAP equalizer on the multipath fading channel. At BER $= 4 \times 10^{-3}$, the BAD algorithm performs approximately 2 dB better than the forward MMSE-DFE and 1 dB worse than MAP detection. These results are representative of the performance gains that may be expected in wireless applications with multipath when significant ISI is present.

### C. *Effect of Parameter Variation on Performance*

Variation of the DFE filter order and arbitration window length can have a dramatic impact on the performance of

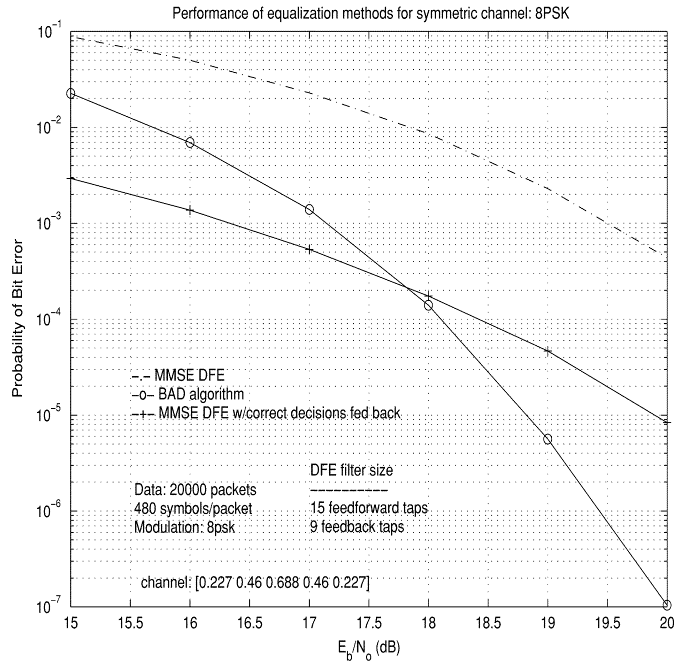


Fig. 4. BAD algorithm performance for 8-PSK modulation. Shown for comparison are performances of the forward MMSE-DFE and the forward MMSE-DFE with error propagation artificially removed. Channel: $H_{\mathrm{sym}}(z)$; filter orders: $K_{\mathrm{ff}} = 15, K_{\mathrm{fb}} = 9$; window size: $W = 15$; simulation size: 20 000 packets of 480 symbols each.

the BAD algorithm. This impact is most readily seen on a maximum-phase channel, such as $H_{\max}(z)$. Results show that forward DFE performance improves substantially as filter order is increased from 5 to 15 for the feedforward filter, and from 4 to 9 for the feedback filter. However, the reverse DFE, whose filter coefficients were chosen to equalize the minimum-phase channel $z^{-4}H_{\max}(1/z)$, shows no performance improvement over this range of filter orders. This is dominated by the substantial difference in the unconstrained-complexity MMSE-DFEs (forward and reverse). The BAD algorithm shows performance improvement that, while less than that of the forward DFE, is nevertheless significant. Another strategy for improving the performance of the BAD algorithm is to increase the arbitration window size. Results show that variation in window size has a noticeable effect on performance for window lengths similar to the length of the channel, but improvement becomes marginal as window size increases beyond $W = 9$ for the five-tap channel $H_{\max}(z)$.

### D. *8-PSK Modulation*

Figs. 1–3 have shown the performance of the BAD algorithm over a variety of channels when BPSK modulation is employed. To demonstrate that the performance gains of BAD over the DFE hold for larger constellations, results of an 8-PSK simulation are shown in Fig. 4. A MAP equalizer was not included in this simulation because of the high complexity of the Bahl–Cocke–Jelinek–Raviv (BCJR) algorithm for larger constellation sizes. Comparing Fig. 4 with Fig. 1, we see that while the performance curves are translated to the right due to the lower power efficiency of 8-PSK, the gains of BAD over the DFE are roughly the same in both cases.

## IV. Conclusion

We have presented a new algorithm for arbitrating between the symbol estimates generated by forward and reverse DFEs. The BAD algorithm makes each symbol decision by determining which DFE output sequence best explains the sequence received from the channel locally around the symbol of interest. This arbitration mechanism exploits the different error distributions at the outputs of the forward and reverse DFEs. Simulation results show that the BAD algorithm outperforms the DFE by 1–2 dB, and is within 1 dB of optimal MAP performance for a variety of channels.

## References


[1] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal decoding of linear codes for minimizing symbol error rate,” *IEEE Trans. Inf. Theory*, vol. IT-20, pp. 284–287, Mar. 1974.

[2] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1995.

[3] M. E. Austin, *Decision-Feedback Equalization for Digital Communication Over Dispersive Channels*. Cambridge, MA: MIT Press, Aug. 1967.

[4] A. Higashi and H. Suzuki, “Dual-mode equalization for digital mobile radio,” in *Proc. IEICE*, vol. 74-B-II, Mar. 1991, pp. 91–100.

[5] S. Ariyavisitakul, “A decision feedback equalizer with time-reversal structure,” *IEEE J. Sel. Areas Commun.*, vol. 10, pp. 599–613, Apr. 1992.

[6] V. M. Eyuboglu and S. U. H. Qureshi, “Reduced-state sequence estimation with set partitioning and decision feedback,” *IEEE Trans. Commun.*, vol. 36, pp. 13–20, Jan. 1988.

[7] P. R. Chevillat and E. Eleftheriou, “Decoding of trellis encoded signals in the presence of intersymbol interference and noise,” *IEEE Trans. Commun.*, vol. 37, pp. 669–676, Jul. 1989.

[8] A. Duel-Hallen and C. Heegard, “Delayed decision-feedback sequence estimation,” *IEEE Trans. Commun.*, vol. 37, pp. 428–436, May 1989.

[9] J. K. Nelson, A. C. Singer, and U. Madhow, “Asymptotic efficiency of the bi-directional arbitrated DFE,” in *Proc. IEEE Workshop Statist. Signal Process.*, Sep. 2003, pp. 86–89.

[10] G. D. Forney, “Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interference,” *IEEE Trans. Inf. Theory*, vol. IT-18, pp. 363–378, May 1972.

[11] *Digital Cellular Telecommunications System (Phase 2+); Radio Transmission and Reception*, Draft GSM 05.05 V8.2.0, Dec. 1999.