Characterizing Outage Rates for Space–Time Communication Over Wideband Channels

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***Abstract*—We provide a compact characterization of outage rates for a wideband wireless communication system whose parameters are chosen to model an outdoor cellular downlink. The base station transmitter is equipped with an antenna array, while the mobile receiver has a single antenna. Our analysis quantifies the effects of frequency and spatial diversity for measurement-based channel models available in the literature. Design prescriptions based on our framework would apply, for example, to fourth-generation cellular systems using orthogonal frequency-division multiplexing. Our information-theoretic computations yield the following findings.**

- **Complex models typically employed in simulations can be replaced by simple, bandwidth-dependent, tap-delay-line models without loss of accuracy.**
- **The spectral efficiency (i.e., the achievable rate, divided by the bandwidth) is well approximated as a Gaussian random variable, so that it is only necessary to specify its mean and variance in order to compute the outage rates. We provide analytical formulas for the mean and variance as a function of the space–time channel model, and verify that the resulting outage rates match closely with simulation.**
- **For a wide class of outdoor channels, the mean spectral efficiency depends only on the spatial diversity, while the variance depends on the spatial and frequency diversity via a product. Our definitions of frequency and spatial diversity have physically motivated interpretations, and do not rely on high signal-to-noise ratio asymptotics, as in prior work.**

***Index Terms*—Diversity methods, fading channels, information rates, multiple-input multiple-output (MIMO) systems.**

I. INTRODUCTION

IN THIS PAPER, we consider a wideband wireless communication system modeling a typical "downlink" for outdoor cellular communication systems, in which the base station (BS) transmitter may be equipped with multiple antennas, but the mobile receiver has a single antenna element, as shown in Fig. 1. We employ empirical outdoor channel models available in the literature [1], [2]. These are based on fitting measured data to the well-known Saleh–Valenzuela channel model [3], in which the channel impulse response (CIR) is obtained as one or more clusters of multipath components, randomly generated according to specified distributions. The contribution of this paper is in providing an information-theoretic framework for system-design choices by characterizing outage rates in terms of the measurement-based channel model, the available bandwidth, and the geometry of the transmit antenna array.

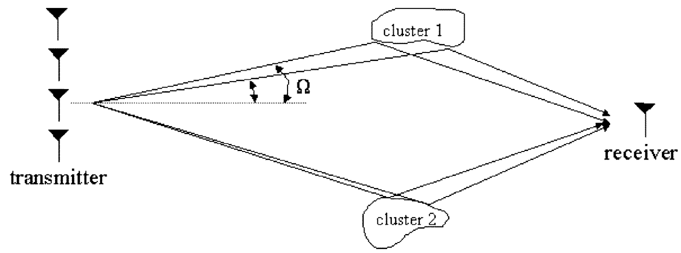

Fig. 1. Channel model setup. Specular rays from the BS to the mobile are grouped together in one or more clusters. The BS may have multiple antennas, while the receiver is restricted to a single antenna element.

The transmitter does not have channel feedback, and employs white Gaussian input over the available bandwidth. This is consistent, for example, with an orthogonal frequency-division multiplexing (OFDM) system employing the same constellation on each subcarrier, with the size of the constellation depending on the design value of the signal-to-noise ratio (SNR). The receiver is assumed to know the channel, which is selected randomly from an ensemble, and then frozen over the transmitted codeword. Outage occurs when the transmission rate is larger than the maximum achievable rate (with white Gaussian input) for the given channel. The outage rate is, therefore, defined as the maximum transmission rate such that the probability of outage is below some specified level.

Our analytical approach is based on two key observations. First, given the statistics of a wideband channel, a classical bandwidth-dependent tap-delay-line (TDL) model suffices to characterize outage rates. Thus, there is no need to obtain channel realizations by generating a large number of paths at spacing closer than the available bandwidth (as is typical in many wideband channel simulators). Second, the spectral efficiency is well approximated as a Gaussian random variable, hence, the outage rates are completely determined by the mean and variance of the spectral efficiency. Since the mean and variance can be expressed as simple functions of system and physical parameters, outage rates can also be compactly characterized in terms of these fundamental parameters.

For the bulk of the paper, we focus on channels modeled by a single cluster of multipath components. This is because

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many outdoor channels are well modeled by a single multipath cluster, and because analysis of a one-cluster channel is a building block for analysis of channels with multiple clusters. For single-cluster systems, analysis of the mean and variance of the spectral efficiency yields physically motivated definitions of frequency diversity D_f and spatial diversity D_s in terms of an equivalent system with D_f independent and identically distributed (i.i.d.) temporal paths, and D_s i.i.d. spatial paths. Note that, in most prior work on diversity [4], [5], d-level diversity corresponds to the error probability at high SNR, decreasing at rate SNR^{-d}. A system with M i.i.d. paths will have $d = M$. Our work does not rely on high SNR asymptotics to define effective diversity, but the definitions are qualitatively similar, in that if a system has M i.i.d. paths, the effective diversity will still be M.

We find that the mean of the spectral efficiency is a function of D_s alone, while the variance scales as $(1/D_f D_s)$. The frequency diversity is a function of the bandwidth and power-delay profile (PDP), while the spatial diversity depends on the power-angle profile (PAP) and the geometry of the transmit antenna array. (The running example in this paper is a linear array with equally spaced elements.)

We also show that, while the variance reduction from spatial diversity can be obtained by simple strategies such as alternating across transmit antennas, nontrivial space–time/frequency codes are required to obtain the increase in the mean spectral efficiency (and hence, the outage rate) available with multiple antennas.

Relation to Previous Work: Our focus on wideband outdoor communication is in contrast to the bulk of the literature on space–time communication, which focuses on narrowband channels with a rich scattering environment typical of indoor applications [6]–[9]. Compared with indoor channels, outdoor channels have higher delay spreads (and hence, provide more frequency diversity for a given bandwidth), but smaller spread in spatial angles (thus providing less spatial diversity). The use of TDL models is common in the literature [10], and our simplified TDL model is similar to the one used in [11].

The work most closely related to ours is that of Bolcskei *et al.* [11], which examines the ergodic capacity and outage rates for OFDM multiple-input multiple-output (MIMO) channels. Our work differs from [11] in two key aspects. First, instead of starting from TDL channel models, we demonstrate that TDL models suffice for performance evaluation by explicitly linking them to, and comparing them with, measurement-based Saleh–Valenzuela-type models. Second, we provide analytical estimates of outage rates that are easy to compute, in contrast to the simulation-based estimates in [11].

It is also worth mentioning recent work [12] on narrowband MIMO systems, in which the capacity is shown to be well approximated as Gaussian. However, while the Gaussianity in our system results from averaging over bandwidth (assumed to be larger than the coherence bandwidth of the channel), Gaussianity for narrowband MIMO systems arises from averaging over the fading coefficients (assumed to be uncorrelated, unlike in this paper) for multiple transmit–receive element pairs.

Paper Outline: Section II describes the channel model, both the path-generation model, and the equivalent simplified TDL model. Sections III and IV focus on single-cluster channels. Section III considers systems with a single transmit antenna, allowing the effects of frequency diversity on outage rates to be isolated and quantified. Transmitters with multiple antennas are considered in Section IV. The outage rates now depend on both spatial and frequency diversity, and are shown to be expressible as a simple function of both quantities. Section V shows how the results for single-cluster channels can be extended to channels with two clusters. Conclusions are found in Section VI.

II. Channel Model

In this section, we show how a typical space–time channel model based on the superposition of specular rays can be simplified to a TDL model with Gaussian taps, if there are sufficient rays for the Central Limit Theorem (CLT) to come into effect. We then show by simulation how outage rates computed using the TDL model match those computed using the more complex ray-generation model. Because the two models give similar results, we choose to use the more analytically tractable TDL model in the remainder of the paper. This section is intended to serve as a justification for our use of the TDL model, as well as to give an overview of the channel model itself.

It is assumed that the BS is far enough away from the mobile such that far-field approximations hold, and that the BS is of much higher altitude. This scenario is typical of suburban settings, where the BS is on a tower, but is also common in many urban environments. In such a setting, there is little to no local scattering around the BS, and signals that reach a particular mobile leave the BS in a fairly narrow spatial cone.

The discrete-ray channel model consists of clusters of specular rays as shown in Fig. 1, where each ray is parameterized by its delay, angle of departure (AOD), and amplitude. The CIR can then be written as

$$\mathbf{h}(\tau, \Omega) = \sum_{j=1}^{M} \sum_{i=1}^{N_j} \alpha_{ij} e^{j\Theta_{ij}} \mathbf{a}(\Omega_{ij}) \delta(\tau - \tau_{ij}) \delta(\Omega - \Omega_{ij}) \quad (1)$$

where M is the number of clusters and N_j is the number of rays in the jth cluster. Here, Ω_{ij} denotes the AOD, τ_{ij} the delay, and α_{ij} the amplitude of the ith ray of the jth cluster. The phase shifts Θ_{ij} for these paths are modeled as i.i.d. and uniformly distributed over $[0, 2\pi]$, as in [3], [13], and [14]. This is motivated by the observation that, for large carrier frequencies, even small variations in the delay lead to large changes in the carrier phase. For our running example of a linear array, the BS array response as a function of AOD is given as follows:

$$\begin{aligned} \mathbf{a}(\Omega) &= [a_1 \ldots a_l \ldots a_{N_T}]^T \\ a_l(\Omega) &= e^{j(l-1)2\pi \frac{d}{\lambda} \sin(\Omega)}, \quad l = 1, \ldots, N_T \end{aligned} \quad (2)$$

where d is the antenna-array spacing, and λ the carrier wavelength. Because the number of clusters in such systems is small (generally one or two), we first restrict our attention to channels with a single cluster, and then consider channels with multiple clusters in Section V.

Let $f_\tau(\cdot)$ and $f_\Omega(\cdot)$ denote the densities of the path delays and the AODs, and let $P_\tau(\cdot)$ and $P_\Omega(\cdot)$ denote the PDP and PAP, respectively. In a typical simulation model based on explicit path generation, a large number of paths are generated with the delays and AODs chosen according to specified distributions (often fitted to measurements). The (un-normalized) amplitudes of these paths are chosen so as to be consistent with the measured PDP and PAP, as follows [1]:

$$\tilde{\alpha}_i = \sqrt{\frac{P_\tau(\tau_i)P_\Omega(\Omega_i)}{f_\tau(\tau_i)f_\Omega(\Omega_i)}}. \tag{3}$$

The amplitudes α_i are then obtained by normalizing the $\tilde{\alpha}_i$ so that $\sum_{i=1}^{N}|\alpha_i|^2 = 1$.

When the system bandwidth is restricted to W, the preceding model can be replaced by a $(1/W)$-spaced TDL model, since paths with closer spacing cannot be resolved. Each tap corresponds to a number of discrete paths, and if there are enough such paths, application of the CLT implies that the taps are complex Gaussian. By virtue of the consistency condition (3), it is easy to see that the statistics of these taps depend only on $P_\tau(\cdot)$ and $P_\Omega(\cdot)$, and not on the delay and angle distributions. Thus, in order to determine the statistics of the TDL model, it suffices to consider a channel model in which the paths form a continuum according to the PDPs and PAPs. This "continuous" channel model is given as

$$\begin{aligned}\mathbf{h}(\tau,\Omega) = \int_0^\infty \int_{-\pi}^{\pi} &\sqrt{P_\tau(\tau')P(\Omega')}e^{j\Theta(\tau',\Omega')} \\ &\times \mathbf{a}(\Omega')\delta(\tau-\tau')\delta(\Omega-\Omega')d\tau' d\Omega'. \end{aligned}\tag{4}$$

Imposing the bandwidth constraint to obtain the discrete TDL model with spacing $\Delta = 1/W$, we have that the ith tap, $\mathbf{h}_W(i/W)$, is found from the continuum of unresolved paths in the time interval $[(i/W), (i/W)+\Delta]$

$$\begin{aligned}\mathbf{h}_W\left(\frac{i}{W}\right) &= \int_{\frac{i}{W}}^{\frac{i}{W}+\Delta}\int_{-\pi}^{\pi}\sqrt{P_\tau(\tau')}\sqrt{P_\Omega(\Omega')}e^{j\Theta(\tau',\Omega')} \\ &\quad\times \mathbf{a}(\Omega')d\tau' d\Omega' &(5)\\ &\approx \sqrt{P_\tau\left(\frac{i}{W}\right)}\int_{\frac{i}{W}}^{\frac{i}{W}+\Delta}\int_{-\pi}^{\pi}\sqrt{P_\Omega(\Omega')}e^{j\Theta(\tau',\Omega')} \\ &\quad\times \mathbf{a}(\Omega')d\tau' d\Omega'. &(6)\end{aligned}$$

By the CLT, the double integral in (6) can then be approximated by a zero-mean complex Gaussian vector with covariance matrix $\mathbf{C}$, where

$$\mathbf{C} = E[\mathbf{a}(\Omega')\mathbf{a}(\Omega')^H] = \int_{-\pi}^{\pi}\mathbf{a}(\Omega')\mathbf{a}(\Omega')^H P_\Omega(\Omega')d\Omega'. \tag{7}$$

Letting $\{\mathbf{v}_i\}$ be i.i.d., where $\mathbf{v}_i \sim CN(0,\mathbf{C})\ \forall i$, this gives

$$\mathbf{h}_W\left(\frac{i}{W}\right) \approx A_i\mathbf{v}_i\delta\left(\tau - \frac{i}{W}\right) \tag{8}$$

where the A_i are proportional to the square root of the PDP

$$A_i \propto \sqrt{P_\tau\left(\frac{i}{W}\right)} \tag{9}$$

but have been normalized such that

$$\sum_{i=0}^{\infty} A_i^2 = 1. \tag{10}$$

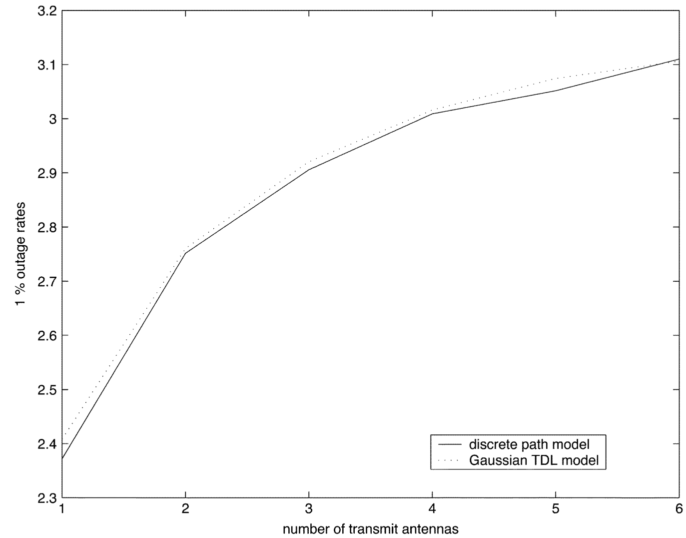

Fig. 2. 1% outage rates (b/sHz) versus the number of transmit antennas using the discrete-path model and the TDL model. The bandwidth is 25 MHz, and SNR $=$ 10 dB. The antenna spacing over the wavelength, (d/λ), is set to 3. The discrete-path model has 100 paths.

This yields the following vector TDL channel response:

$$\mathbf{h}_W(\tau) = \sum_{i=0}^{\infty} A_i\mathbf{v}_i\delta\left(\tau - \frac{i}{W}\right). \tag{11}$$

Following the measurement-based observations in [2] and [15], we employ exponential PDPs and Laplacian PAPs throughout this paper, i.e.,

$$P_\tau(\tau') = \frac{1}{\tau_{\text{rms}}}e^{-\frac{\tau'-\tau_e}{\tau_{\text{rms}}}} \quad \text{for } \tau' \geq \tau_e \tag{12}$$

$$P_\Omega(\Omega') = \frac{1}{2\Omega_{\text{spread}}}e^{-\left|\frac{\Omega'-\overline{\Omega}}{\Omega_{\text{spread}}}\right|}. \tag{13}$$

We will use the notation $L(a,b)$ to refer to a Laplacian profile with $\overline{\Omega} = a$ and $\Omega_{\text{spread}} = b$.

Of course, the bandwidth-dependent TDL model (11) and the discrete-ray model (1) are only equivalent if there are enough paths with resolution less than $(1/W)$ to ensure Gaussian statistics. However, we find that for a modest number of discrete paths, the two models give the same outage rates, even at high bandwidths. For example, Fig. 2 shows 1% outage rates obtained from the discrete-ray model with 100 paths, along with 1% outage rates obtained from the TDL model, for a bandwidth of 25 MHz. [Outage rates are explicitly defined in (17)]. In this simulation, the parameters in (12) and (13) are: $\tau_e = 0$ s, $\tau_{\text{rms}} = 0.5\ \mu$s, $\overline{\Omega} = 0°$, and $\Omega_{\text{spread}} = 10°$. The SNR is 10 dB. It can be seen that the outage rates match well for the two models. Since we will not be considering bandwidths higher than 25 MHz in this paper, the TDL model is a good approximation to the more complex ray-based model for our purposes, and we will thus exclusively use the simpler model for the remainder of the paper.

III. Single-Input Single-Output (SISO) System

To understand the effects of delay spread and bandwidth, we begin by considering a system with one transmit element. In this case, the TDL model (11) simplifies to the following:

$$h_W(\tau) = \sum_{i=0}^{\infty} A_i v_i \delta\left(\tau - \frac{i}{W}\right) \tag{14}$$

where the v_i are $CN(0,1)$.

The spectral efficiency is given by

$$I_W = \frac{1}{W}\int_{\frac{-W}{2}}^{\frac{W}{2}} \log(1+\mathrm{SNR}|H(f)|^2)\, df \tag{15}$$

where $H(f)$ is the channel frequency response. Using the "continuous" channel model (4), specialized to a single transmit antenna, we obtain that

$$H(f) = \int_0^{\infty} \sqrt{P_\tau(\tau')} e^{j\Theta(\tau')} e^{-j2\pi f\tau'} d\tau'. \tag{16}$$

We are interested in computing the outage rates, which depend solely on the distribution of I_W. The outage rate $R(x)$ is defined as the largest transmission rate R such that the following condition holds:

$$P[I_W < R] = x. \tag{17}$$

In other words, "outage" occurs when the spectral efficiency falls below the transmission rate, and sending at rate $R(x)$ ensures that the probability of outage is below the specified level x. In order to characterize the outage rate $R(x)$, we must first characterize the distribution of the spectral efficiency, I_W.

We propose that the spectral efficiency can be accurately modeled as a Gaussian random variable because the squared frequency response $|H(f)|^2$ decorrelates fast enough such that the CLT applies. In order to see this, we first note that $H(f)$ are i.i.d. proper complex Gaussian random variables [16]

$$H(f) \sim CN(0,1). \tag{18}$$

This follows from applying the CLT to the integral in (16). The phases $\Theta(\tau)$ are modeled as independent and uniform over $[0, 2\pi]$, so that the integral is a sum over a continuum of independent, zero mean, random variables. The variance of $H(f)$ is given by

$$E[|H(f)|^2] = \int_0^{\infty} P_\tau(\tau') d\tau' = 1. \tag{19}$$

Using similar arguments, we also infer that $H(f_1)$ and $H(f_2)$ are jointly proper complex Gaussian random variables for any two frequencies f_1 and f_2. Thus, $\{H(f), -\infty < f < +\infty\}$ is a zero mean, proper complex Gaussian random process. Computation of its autocorrelation function shows that $H(f)$ is wide-sense stationary (WSS), with autocorrelation function given by

$$R_H(\nu) = \int_0^{\infty} P_\tau(\tau') e^{-j2\pi\nu\tau'} d\tau'. \tag{20}$$

For an exponential PDP as in (12), we obtain

$$|R_H(\nu)|^2 = \frac{1}{|1+j2\pi\nu\tau_{\mathrm{rms}}|^2} \tag{21}$$

which approaches zero as $|\nu| \to \infty$. We infer the Gaussianity of I_W in the following theorem, using a CLT for dependent random variables.

Theorem 1: Suppose that $R_H(\nu)$ decays as fast as $(1/|\nu|)$. (This condition is satisfied by exponential PDPs). Then

$$\lim_{W\to+\infty} \sqrt{W}(I_W - E[I_W]) = N\left(0, \sigma_{I_W}^2\right) \tag{22}$$

for some finite value of $\sigma_{I_W}^2$, where the limit is approached in distribution. In other words, we can approximate I_W as a Gaussian random variable when W is large.

Proof: Due to space constraints, we provide here only a sketch of the proof that conveys the basic idea. Details are available from the authors upon request. We first approximate I_W as a Riemann sum

$$I_W \approx \frac{1}{S_W}\sum_{i=1}^{S_W} \log(1+\mathrm{SNR}|H(f_i)|^2) \tag{23}$$

where the $\{f_i\}$ are spaced Δf apart, and $S_W = (W/\Delta f)$. Let $X(f_i) \equiv \log(1+\mathrm{SNR}|H(f_i)|^2)$. $\{X(f_i)\}$ is stationary, since $\{H(f_i)\}$ is a WSS Gaussian random process, and therefore stationary. Thus, we can use [17, Corollary 4.4.1] to show the Gaussianity of I_W. The latter corollary requires that

$$E|X(f_i)|^{2+\delta} \le N\ \forall i \tag{24}$$

for some $\delta > 0$ and $N < \infty$, and requires that for certain functions of the $\{X(f_i)\}$, the conditional mean, when conditioned on past events, approaches the unconditional mean sufficiently fast, as the past events recede further into the past. Condition (24) is easily seen to hold for $\delta = 3$. Since $H(f_i) \sim CN(0,1)\ \forall i$ [by (18)], it can be seen that $E[(|H(f_i)|^2)^3] \approx 6\ \forall i$. Thus

$$\begin{aligned} E[X(f_i)^3] &= E[(\log(1+\mathrm{SNR}|H(f_i)|^2))^3] \\ &\le E[(\mathrm{SNR}|H(f_i)|^2)^3] \\ &\approx \mathrm{SNR}^3 * 6\ \forall i \end{aligned} \tag{25}$$

which is finite for finite SNRs, and so condition (24) holds. We omit the details on why, for certain functions of the $\{X(f_i)\}$, the conditional mean approaches the unconditional mean sufficiently fast, however, the basic idea is that the $\{X(f_i)\}$ need to decorrelate at a fast enough rate. If $H(f_i)$ and $H(f_j)$ decorrelate as $(1/|i-j|)$, then for moderate-to-high SNR, $X(f_i)$ and $X(f_j)$ decorrelate as $(1/|i-j|^2)$, which is sufficient in this scenario for [17, Corollary 4.4.1] to hold. □

Since I_W is approximately Gaussian, the outage rates are then fully determined by the mean and variance of I_W, and we can, therefore, approximate the 1% outage rate $R(0.01)$ as follows:

$$\hat{R}(0.01) = E[I_W] - \sqrt{\mathrm{var}[I_W]} * Q^{-1}(0.01) \tag{26}$$

where Q denotes the complementary cumulative distribution function of a standard Gaussian. In order to corroborate the Gaussianity assumption, we simulate both $R(0.01)$ and $\hat{R}(0.01)$ as a function of W for a system with an exponential PDP (12) having $\tau_{\mathrm{rms}} = 0.5\ \mu$s and SNR = 10 dB. Fig. 3 shows the simulated outage rates $R(0.01)$ along with the estimated outage rates $\hat{R}(0.01)$, where $E[I_W]$ and $\mathrm{var}[I_W]$ in (26) have been obtained

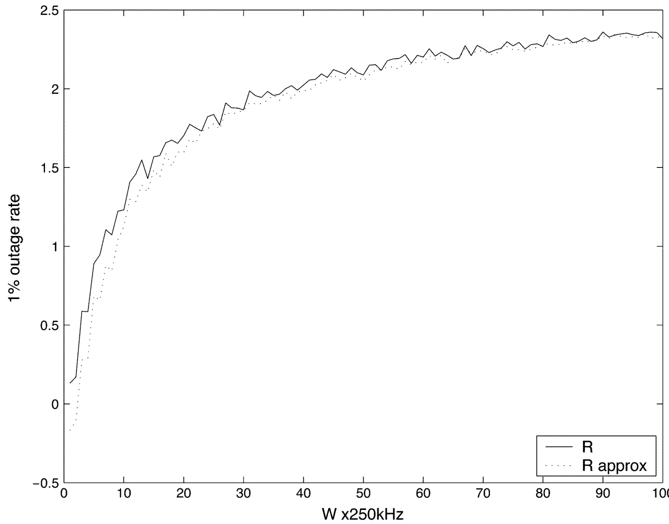

Fig. 3. $R(0.01)$ and $\hat{R}(0.01)$ (b/sHz) where $E[I_W]$ and $\mathrm{var}[I_W]$ are obtained from simulated values.

from the simulation statistics. The curves are well matched, indicating the accuracy of the Gaussian approximation, even at low bandwidths. This approximation was also found to hold over a wide range of SNRs.

We now give a rule of thumb for determining the bandwidth needed for the Gaussian approximation to hold, based upon the following reasoning. The Gaussianity of I_W relies on the fact that the variables $X(f)$ decorrelate quickly with frequency, where $X(f) = \log(1 + \mathrm{SNR}|H(f)|^2)$. Recall that $H(f)$ is a stationary Gaussian random process, and that $H(f) \sim CN(0,1)\ \forall f$. We have verified that, when $H(f_1)$ and $H(f_2)$ have correlation coefficient $\rho \leq 0.5$, then the mutual information between $X(f_1)$ and $X(f_2)$ is close to zero, meaning that $X(f_1)$ and $X(f_2)$ are approximately independent. For an exponential PDP, the autocorrelation function is given in (20), and $\rho \leq 0.5$ when (approximately) $|f_1 - f_2| \geq (1/4\tau_{\mathrm{rms}})$. Noting that the coherence bandwidth, W_C, is equal to $(1/\tau_{\mathrm{rms}})$, this means that $X(f_1)$ and $X(f_2)$ are nearly independent for frequency differences greater than $W_C/4$. Now, note that I_W is an average of random variables $X(f_i)$, and that the CLT comes into effect quite quickly when adding up (nearly) independent random variables, say when about 10 such random variables are added up. This leads to the rough rule of thumb that, for exponential PDPs, we expect I_W to be approximately Gaussian for $W \approx 10W_C/4 = 2.5W_C$. For the above example, $W_C = 2$ MHz, and indeed the Gaussian approximation works well for bandwidths of 5 MHz or more.

In order to characterize the outage rates in terms of system and physical parameters, the mean and variance of the spectral efficiency must now be expressed as functions of these parameters. To this end, we provide the following proposition.

Proposition 1: For a SISO system, the mean of the spectral efficiency can be characterized as follows:

$$\begin{aligned} E[I_W] &= E[\log(1+\mathrm{SNR}|X|^2)] \quad X \sim CN(0,1) \\ &= \int_0^\infty \log(1+\mathrm{SNR}t)e^{-t}\,dt \\ &= C_{\mathrm{Rayleigh}}(\mathrm{SNR}) \end{aligned} \tag{27}$$

where C_{Rayleigh} is the ergodic capacity of a Rayleigh fading channel.

The variance of the spectral efficiency can be characterized as

$$\mathrm{var}[I_W] \approx \gamma^2 \left(\sum_{i=0}^{\infty} A_i^4\right) \tag{28}$$

where $\gamma \approx (\mathrm{SNR}/\mathrm{SNR}+1)$ and the A_i are proportional to the PDP, as defined in (9) and (10).

Proof: Equation (27) follows directly from the definition of spectral efficiency, (15), and the fact that $H(f) \sim CN(0,1)$ (18).

We now make the following approximation of I_W in order to estimate the variance:

$$\begin{aligned} I_W &= \frac{1}{W}\int_{\frac{-W}{2}}^{\frac{W}{2}} \log(1+\mathrm{SNR}|H(f)|^2)\,df \\ &= \frac{1}{W}\int_{\frac{-W}{2}}^{\frac{W}{2}} \log(1+\mathrm{SNR}) + \log\left(1+\zeta(|H(f)|^2-1)\right)\,df \\ &\approx \log(1+\mathrm{SNR}) + \gamma\frac{1}{W}\int_{\frac{-W}{2}}^{\frac{W}{2}} (|H(f)|^2-1)\,df \end{aligned} \tag{29}$$

where $\zeta = (\mathrm{SNR}/\mathrm{SNR}+1)$. Using the approximation $\log(1+x) \approx x$ for x small, we expect $\gamma = \zeta$ to yield a good estimate in (29). However, a somewhat better estimate is obtained by choosing γ to minimize the mean square approximation error, d, given by

$$d = \log(1+\zeta(|H(f)|^2-1)) - \gamma(|H(f)|^2-1). \tag{30}$$

The value of γ turns out to be approximately equal to ζ. We can now estimate the variance of I_W as

$$\begin{aligned} \mathrm{var}[I_W] &\approx \gamma^2 \mathrm{var}\left[\frac{1}{W}\int |H(f)|^2 df\right] \\ &\approx \gamma^2 \mathrm{var}\left[\sum_{i=0}^{\infty} A_i^2 |v_i|^2\right] \qquad (31) \\ &= \gamma^2 \sum_{i=0}^{\infty} A_i^4 \qquad (32) \end{aligned}$$

where the last equality follows from Parseval's Theorem. □

A. Effective Frequency Diversity

Since the mean spectral efficiency is independent of bandwidth, the variance measures the available frequency diversity. We say that a system has *effective frequency diversity* D_f if the variance of its spectral efficiency equals that of a system with D_f i.i.d. paths. For D i.i.d. paths, $\sum_{i=0}^{D-1} A_i^4 = (1/D)$ [since $\sum_i A_i^2 = 1$, we have $A_i^2 \equiv (1/D)$]. This motivates defining the effective frequency diversity as

$$D_f \equiv \frac{1}{\sum_{i=0}^{\infty} A_i^4}. \tag{33}$$

For an exponential PDP, we can simplify this expression by letting $\beta_W \equiv \exp(-(1/W\tau_{\mathrm{rms}}))$, and noting that $\sum_{i=0}^{\infty} A_i^2 = (1-\beta_W)\sum_{i=0}^{\infty} \beta_W^i = 1$. Thus

$$\sum_{i=0}^{\infty} A_i^4 = (1-\beta_W)^2 \sum_{i=0}^{\infty} \beta_W^{2i} = \frac{1-\beta_W}{1+\beta_W} \tag{34}$$

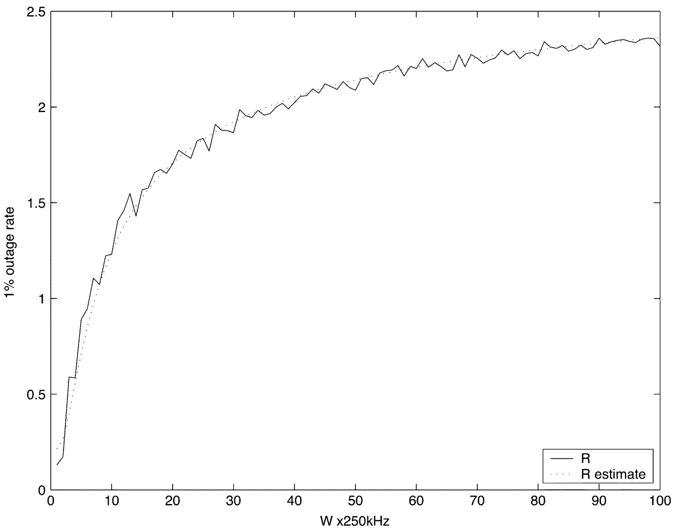

Fig. 4. $R(0.01)$ and $\hat{R}(0.01)$ (b/sHz) where $E[I_W]$ and $\mathrm{var}[I_W]$ are obtained using *Proposition 1*.

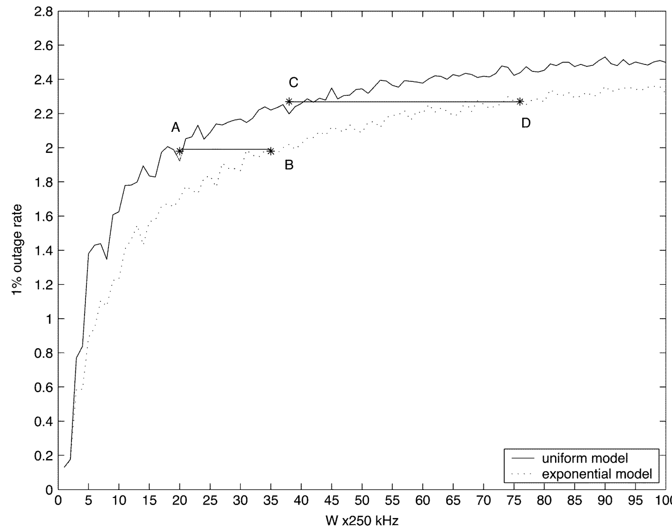

Fig. 5. $R(0.01)$ (b/sHz) for an exponential PDP with $\tau_{\mathrm{rms}} = 0.5\ \mu$s, and a uniform PDP from 0 to 2 μs.

giving

$$D_f = \frac{1+\beta_W}{1-\beta_W}. \tag{35}$$

If the bandwidth is large compared with the channel coherence bandwidth (i.e., $W\tau_{\mathrm{rms}}$ large), this reduces to $D_f \approx 2W\tau_{\mathrm{rms}}$. We can now rewrite *Proposition 1* in terms of the effective frequency diversity.

Proposition 1b: The mean and variance of the spectral efficiency of a SISO system can be written in terms of the effective frequency diversity, as follows:

$$E[I_W] = C_{\mathrm{Rayleigh}}(\mathrm{SNR})$$
$$\mathrm{var}[I_W] \approx \gamma^2\left(\frac{1}{D_f}\right)$$

where $\gamma \approx (\mathrm{SNR}/\mathrm{SNR}+1)$.

Fig. 4 compares the simulated 1% outage rates with the estimated 1% outage rates calculated using the equations in *Proposition 1b* for $E[I_W]$ and $\mathrm{var}[I_W]$ and the approximation for D_f given in (35). The parameters are the same as those used to generate Fig. 3. The estimated outage rates clearly match those obtained by simulation.

In order to further substantiate our definition of effective frequency diversity, we show that a system with an exponential PDP and diversity D_f has the same outage rates as a system with D_f i.i.d. paths. Fig. 5 shows the outage rates for a system with an exponential PDP, where $\tau_{\mathrm{rms}} = 0.5\ \mu$s, along with the outage rates for a system with uniformly distributed power from 0 to 2 μs (98% of the energy of the exponential PDP is contained within this time window). The number of discernible paths increases with increasing bandwidth for both systems. When the uniform model has 10 i.i.d. paths, the 1% outage rate is at point A. Similarly, when the exponential model has effective diversity $D_f = (1+\beta_W/1-\beta_W) = 10$, the 1% outage rate is at point B. It can be seen from the figure that these outage rates do indeed match. Points C and D correspond to 19 i.i.d. paths and $D_f = 19$, respectively. The outage rates again match, corroborating our assumption that D_f measures the effective number of i.i.d. resolvable paths.

IV. Multiple-Input Single-Output (MISO) System

In this section, we consider a transmitter with N_T antennas sending to a single-antenna receiver. The channel response is as given in (11). We show that, whereas the outage rates for a SISO system only depends on the effective frequency diversity and SNR, the outage rates in this case depends on the effective spatial diversity as well.

A. Effective Spatial Diversity

In keeping with the definition of D_f, we define the spatial diversity D_s to be the effective number of i.i.d. paths in space. (We will need this quantity when quantifying outage rates for MISO systems). We first consider the mutual information for a single frequency bin

$$I(f) = \log\left(1 + \frac{\mathrm{SNR}}{N_T}||\mathbf{H}(f)||^2\right) \tag{36}$$

where $\mathbf{H}(f) \sim CN(0, \mathbf{C})$.

Let $\lambda_1, \ldots, \lambda_{N_T}$ denote the eigenvalues of the normalized covariance matrix $\tilde{\mathbf{C}} = (1/N_T)\mathbf{C}$, and let

$$Y \equiv \frac{||\mathbf{H}(f)||^2}{N_T} \tag{37}$$

so $I(f) = \log(1 + \mathrm{SNR} * Y)$. Decomposing $\mathbf{H}(f)$ along the eigenvectors $\{\mathbf{u}_i, i = 1, \ldots, N_T\}$ of $\mathbf{C}$, we have $\mathbf{H}(f) = \sum_{i=1}^{N_T} \alpha_i \mathbf{u}_i$, where α_i are independent, zero mean, complex Gaussian random variables with $(1/N_T)E[|\alpha_i|^2] = \lambda_i, i = 1, \ldots, N_T$. Thus, $||\mathbf{H}(f)||^2$ equals the sum of the energies along the orthonormal eigenvectors, and

$$Y = \frac{||\mathbf{H}(f)||^2}{N_T} = \sum_{i=1}^{N_T} X_i \tag{38}$$

where $X_i = (1/N_T)|\alpha_i|^2$ are independent exponential random variables with means $\lambda_i, i = 1, \ldots, N_T$. Note that we have normalized such that $\sum_{i=1}^{N_T} E[X_i] = \sum_{i=1}^{N_T} \lambda_i = 1$.

Using the same procedure as in *Proposition 1*, we can show that the variance of $I(f)$ is proportional to $\mathrm{var}[Y] = \sum_{i=1}^{N_T} \lambda_i^2$. We use the variance of $I(f)$ as a measure of the available spatial diversity, and say that a system has effective spatial diversity D_s if the variance equals that of a system with D_s i.i.d. paths. For D i.i.d. spatial paths, $\lambda_i \equiv (1/D)$, so that $\mathrm{var}[Y] = \sum_{i=1}^{D} \lambda_i^2 = (1/D)$. This motivates defining the spatial diversity as

$$D_s \equiv \frac{1}{\mathrm{var}[Y]} \tag{39}$$

$$= \frac{1}{\sum_{i=1}^{N_T} \lambda_i^2}. \tag{40}$$

Note that D_s depends on the PAP as well as the antenna spacing, and increasing the antenna spacing increases D_s up to its maximum value of N_T.

B. Outage Rates for MISO System

The spectral efficiency of a wideband channel with multiple transmit elements is given by

$$I_W = \frac{1}{W} \int_{-\frac{W}{2}}^{\frac{W}{2}} \log\left(1 + \frac{\mathrm{SNR}}{N_T} \|\mathbf{H}(f)\|^2\right) df. \tag{41}$$

Just as for SISO systems, I_W for a MISO system can be approximated as a Gaussian random variable. Consequently, the outage rates for a MISO system can also be characterized by the mean and variance of I_W. The following proposition shows how to approximate these quantities in terms of the system parameters.

Proposition 2: The mean and variance of the spectral efficiency in a MISO system are given as follows:

$$E[I_W] = E\left[\log\left(1 + \mathrm{SNR} \sum_{i=1}^{N_T} X_i\right)\right]$$
$$\mathrm{var}[I_W] \approx \gamma^2 \left(\frac{1}{D_f}\right)\left(\frac{1}{D_s}\right)$$

where X_i are independent exponential random variables with means λ_i, and $\gamma \approx (\mathrm{SNR}/\mathrm{SNR}+1)$.

Proof: The expected value of I_W follows from the definition of spectral efficiency (41), and from (37) and (38). To calculate the variance, we proceed as in *Proposition 1*. Analogous to (29), we have

$$I_W \approx \log(1 + \mathrm{SNR}) + \gamma \frac{1}{W} \int_{\frac{-W}{2}}^{\frac{W}{2}} \left(\frac{\|\mathbf{H}(f)\|^2}{N_T} - 1\right) df \tag{42}$$

where γ is chosen to minimize the approximation error and is approximately equal to ζ, with $\zeta = (\mathrm{SNR}/\mathrm{SNR}+1)$. This gives

$$\mathrm{var}[I_W] \approx \gamma^2 \mathrm{var}\left(\frac{1}{W} \int_{\frac{-W}{2}}^{\frac{W}{2}} \frac{\|\mathbf{H}(f)\|^2}{N_T} df\right) \tag{43}$$

$$= \gamma^2 \mathrm{var}\left(\sum_{i=0}^{\infty} A_i^2 \frac{\|\mathbf{v}_i\|^2}{N_T}\right) \tag{44}$$

$$= \gamma^2 \sum_{i=0}^{\infty} A_i^4 \mathrm{var}\left(\frac{\|\mathbf{v}\|^2}{N_T}\right) \tag{45}$$

$$= \gamma^2 \sum_{i=0}^{\infty} A_i^4 \sum_{l=1}^{N_T} \lambda_l^2 \tag{46}$$

$$= \gamma^2 \left(\frac{1}{D_f}\right)\left(\frac{1}{D_s}\right). \tag{47}$$

The first equality follows from Parseval's Theorem and the second comes about because the $\mathbf{v}_i$ are independent. Equation (47) uses the definitions of effective spatial diversity and effective frequency diversity. □

Note that $E[I_W]$ depends on the individual values of the eigenvalues $\{\lambda_i\}$, rather than only on D_s. Thus, two systems with the same frequency and spatial diversity may have different outage rates, even though the variance of I_W is the same in both cases.[1] This is contrast to a SISO system, in which the mean is independent of the frequency diversity, so that the variance completely characterizes the outage rate. However, we find in our numerical experiments that a useful approximation to the mean in a MISO system, which depends only on D_s, can be found by computing $E[I_W]$ for a system with D_s i.i.d. spatial paths: $E[I_W] \approx E[\log(1 + \mathrm{SNR} * Y_{D_s})]$, where $Y_{D_s} = \sum_{i=1}^{D_s} X_i$ is a Gamma random variable obtained as a sum of D_s i.i.d. exponentials with mean $(1/D_s)$ each. Thus, the outage rates can be approximated using only the effective spatial diversity, the effective frequency diversity, and the SNR.

C. Maximizing D_s

If the eigenvalues of $\mathbf{C}$ are all equal, D_s will have its maximum value N_T, giving optimum outage rates for a fixed PDP and bandwidth. Clearly, D_s depends on both the PAP as well as the antenna spacing. In this section, we show how the antenna spacing required for maximum spatial diversity scales with the angular spread of the PAP.

The eigenvalues of $\mathbf{C}$ are equal when $\mathbf{C} = \mathbf{I}_{N_T} \times N_T$. By definition, $\mathbf{C}_{mm} = 1\{m = 1, \ldots N_T\}$, and $\mathbf{C}_{mn} = E[e^{j2\pi(d/\lambda)\sin(\Omega)(m-n)}]$. Letting $\varrho(\Omega) \equiv (d/\lambda)\sin(\Omega)$ and $\varsigma(\Omega) \equiv \mathrm{mod}(\varrho(\Omega), 1)$, it can be seen that $\mathbf{C} = \mathbf{I}_{N_T} \times N_T$ when ς is a uniform random variable over [0, 1]. We are interested in finding conditions on how Ω should be distributed so that ς is distributed approximately uniformly over [0, 1], leading to full spatial diversity.

Fig. 6 shows $\varsigma(\Omega)$ as Ω varies over $[(-\pi/2), (\pi/2)]$ for $(d/\lambda) = 3$ and $(d/\lambda) = 5$. It can be seen that $\varsigma(\Omega)$ is approximately linear around $\Omega = 0°$ and is parabolic around $\Omega = (\pi/2)$. For simplicity, we restrict our attention to values of Ω where $\varsigma(\Omega)$ is approximately linear. Let Ω_0 be any value of Ω within this region. The variation in ϱ due to a small variation in Ω is given as follows:

$$\varrho(\Omega_0 + \delta) - \varrho(\Omega_0) \approx \frac{d}{\lambda} \cos(\Omega_0)\delta. \tag{48}$$

Thus, if Ω is uniformly distributed over $[\Omega_0 - (\Delta\Omega/2), \Omega_0 + (\Delta\Omega/2)]$, where $\Delta\Omega$ is small, we may approximate ϱ as uniform over $[\varrho(\Omega_0) - (\Delta\varrho/2), \varrho(\Omega_0) + (\Delta\varrho/2)]$, where $\Delta\varrho \approx$

[1]The difference between the outage rate and ergodic capacity would, however, be the same in both cases.

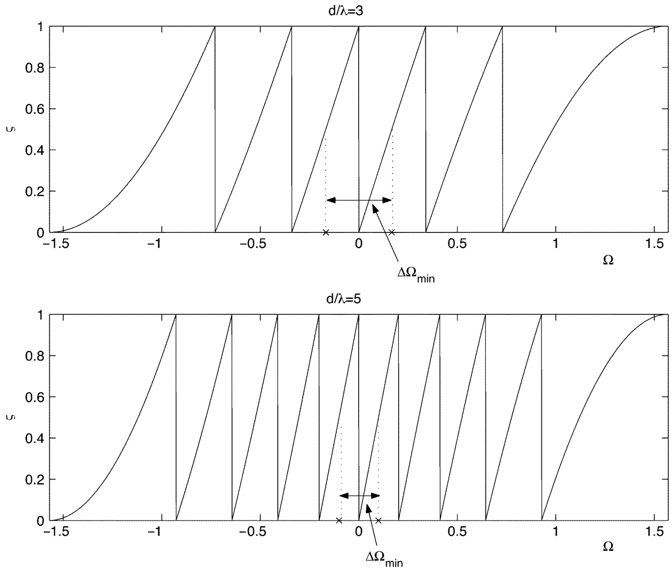

Fig. 6. $\varsigma(\Omega) = \mathrm{mod}((d/\lambda)\sin(\Omega), 1)$.

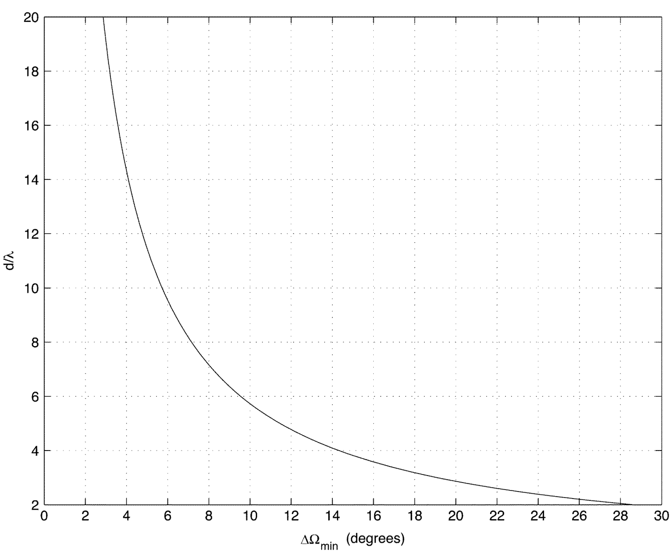

Fig. 7. Minimum antenna spacing which gives optimal performance versus $\Delta\Omega_{\min}$ when Ω is uniformly distributed in $[-(\Delta\Omega_{\min}/2), (\Delta\Omega_{\min}/2)]$.

$(d/\lambda)\cos(\Omega_0)(\Delta\Omega)$. If $\Delta\varrho = 1$, then ς is uniform over [0, 1]. This implies that for full spatial diversity, the minimum angular spread needed is

$$\Delta\Omega \approx \frac{1}{\frac{d}{\lambda}\cos(\Omega_0)}. \tag{49}$$

Thus, the smallest angular spread necessary occurs when $\Omega_0 = 0^\circ$, giving

$$\Delta\Omega_{\min} \approx \frac{\lambda}{d}. \tag{50}$$

Fig. 7 shows $\Delta\Omega_{\min}$ versus (d/λ). For a given angular spread, one can use the curve to find the minimum antenna spacing necessary in order to achieve $D_s = N_T$. As the angular spread of the system decreases, the required antenna spacing becomes increasingly large. For example, an angular spread of four degrees requires the antenna spacing to be 14 times the carrier wavelength. Of course, this curve was generated using the assumption that Ω is uniformly distributed. For a more realistic scenario, where the PAP is Laplacian as in (13), we find that this curve serves as a rough estimate of the antenna spacing necessary to maximize D_s if $\Delta\Omega_{\min}$ is replaced by $2\Omega_{\text{spread}}$, where $2\Omega_{\text{spread}}$ measures the spread of the Laplacian PAP.

D. Alternating Scheme

Given the frequency diversity available in a wideband system, it is natural to ask if coding in space can be simplified. To this end, we consider a frequency-alternation scheme in which all the energy is sent from one antenna for each frequency bin, but the sending antenna alternates with frequency. For example, if there were two transmit antennas, the first would send in frequency bins $\{1, 3, 5, \ldots\}$, while the second would send in bins $\{2, 4, 6, \ldots\}$. This scheme capitalizes on the fact that, in many cases, the coherence bandwidth of the channel is large enough such that the frequency response of a particular antenna remains essentially constant over several frequency bins. There is, therefore, no loss of diversity in frequency if only one of these frequency bins is used, while spatial diversity is achieved by sending from different antennas over consecutive frequency bins. The mutual information for this scheme is given as

$$I_{\text{alt}} \approx \sum_{j=1}^{N_T} \frac{1}{N_T}\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} \log(1 + \mathrm{SNR}|H_j(f)|^2)\right) df \tag{51}$$

where the approximation is due to the fact that there is some small loss in frequency diversity which we are neglecting. Since I_{alt} is again approximately Gaussian, we characterize it via its mean and the variance.

Proposition 3: The mean and variance for a MISO system using the alternation scheme can be approximated as follows:

$$E[I_{\text{alt}}] = E[I_W(N_T = 1)] = C_{\text{Rayleigh}}$$
$$\mathrm{var}[I_{\text{alt}}] \approx \mathrm{var}[I_W].$$

Proof: Letting $H_j(f)$ denote the channel response at the jth transmit antenna, we have

$$E[I_{\text{alt}}] = E\left[\sum_{j=1}^{N_T} \frac{1}{N_T}\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} \log(1 + \mathrm{SNR}|H_j(f)|^2) df\right)\right] \tag{52}$$

$$= \sum_{j=1}^{N_T} \frac{1}{N_T}\left(\frac{1}{W}\int_{-\frac{W}{2}}^{\frac{W}{2}} E[\log(1 + \mathrm{SNR}|H_j(f)|^2)] df\right) \tag{53}$$

$$= E[\log(1 + \mathrm{SNR}|H_1(f)|^2)] \tag{54}$$

which is the mean spectral efficiency for a SISO system. To approximate the variance, we proceed as before

$$I_{\text{alt}} \approx \sum_{j=1}^{N_T} \frac{1}{N_T}\left[\log(1 + \mathrm{SNR}) + \gamma\frac{1}{W}\int_{\frac{-W}{2}}^{\frac{W}{2}} (|H_j(f)|^2 - 1) df\right] \tag{55}$$

$$= \log(1 + \mathrm{SNR}) + \gamma\frac{1}{W}\int_{\frac{-W}{2}}^{\frac{W}{2}} \left(\frac{\|\mathbf{H}(f)\|^2}{N_T} - 1\right) df. \tag{56}$$

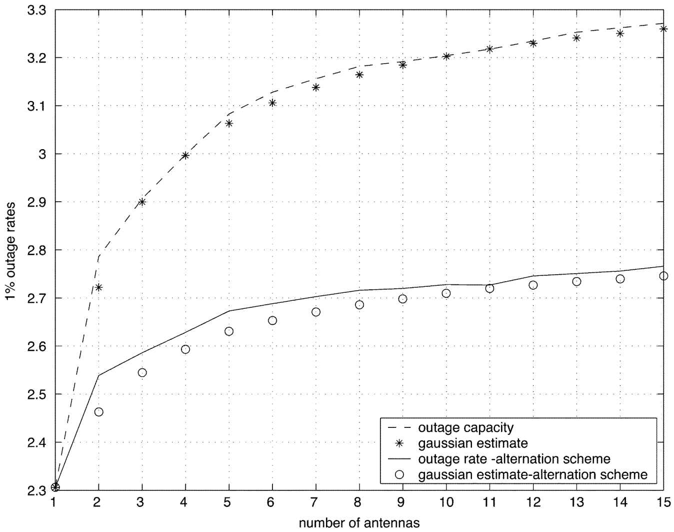

Fig. 8. 1% outage rates (b/sHz): simulated and estimated values. $W = 25$ MHz, SNR $= 10$ dB, and $\tau_{\rm rms} = 0.5\ \mu$s. $(d/\lambda) = 3$ and Ω is Laplacian with $\Omega_{\rm spread} = 10°$, giving the optimal eigenvalue distribution.

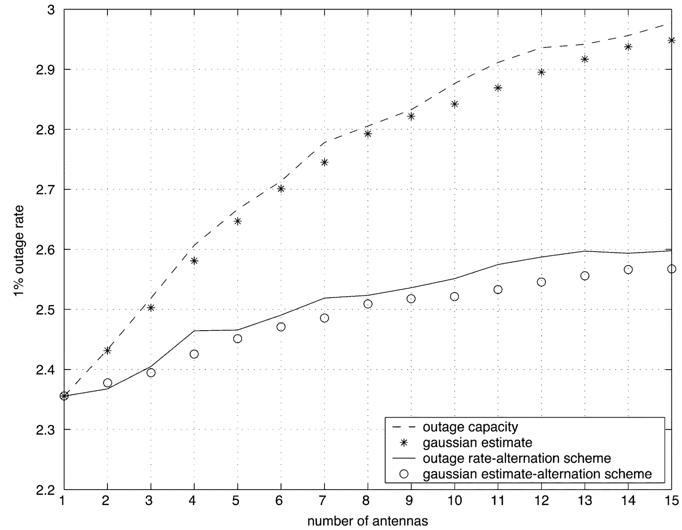

Fig. 9. 1% outage rates (b/sHz): simulated and estimated values. $W = 25$ MHz, SNR $= 10$ dB, and $\tau_{\rm rms} = 0.5\ \mu$s. $(d/\lambda) = 0.5$ and Ω is Laplacian with $\Omega_{\rm spread} = 5°$.

This is the same as in (42), meaning that the variance of $I_{\rm alt}$ is approximately the same as the variance of I_W for a MISO system with the same number of transmit antennas. □

Because $\mathrm{var}[I_{\rm alt}] \approx \mathrm{var}[I_W]$, the alternation scheme reduces the variance of the spectral efficiency as much as a full-blown space–time code, However, alternation cannot increase the mean spectral efficiency, unlike more complex space–time codes. Hence, outage rates are smaller than those achieved by nontrivial space–time codes, as seen in the numerical results below.

E. Simulation Results

We show by simulation that the spectral efficiency can be accurately modeled as a Gaussian distribution with mean and variance given in *Proposition 2*. We also verify that spectral efficiency for the alternating scheme is approximately Gaussian with mean and variance, as specified in *Proposition 3*. Figs. 8 and 9 demonstrate the accuracy of these approximations. The simulated 1% outage rates for both a full-blown space–time code and the alternating scheme are shown along side their Gaussian approximations. For both figures, $W = 25$ MHz, SNR $= 10$ dB, and $\tau_{\rm rms} = 0.5\ \mu$s. In Fig. 8, the antenna spacing is chosen large enough so that $D_s = N_T$, and hence, the outage rates in this figure are the maximum possible, given the PDP and the bandwidth. [$(d/\lambda) = 3$ and $\Omega \sim L(0, 10°)$.] In Fig. 9, both the antenna spacing and the angular spread are reduced, resulting in an effective spatial diversity much smaller than N_T. [$(d/\lambda) = 0.5$ and $\Omega \sim L(0, 5°)$.] It can be seen that the estimated values of the outage rates match the simulated values in both cases.

Note that, compared with single-antenna transmission, the alternation scheme obtains less than half the gains achieved by a full-blown space–time code because of its significantly smaller ergodic capacity. Thus, even at large bandwidths, simple variance-reducing strategies, such as alternation, will be significantly inferior to such space–time/frequency codes.

V. Two-Cluster Systems

In this section, we consider outage rates when there are two clusters. The channel model (4) becomes

$$\begin{aligned}\mathbf{h}(\tau,\Omega) = &\int_0^\infty \int_{-\pi}^{\pi} \sqrt{P_{\tau 1}(\tau')P_{\Omega 1}(\Omega')}e^{j\Theta_1(\tau',\Omega')}\\ &\times \mathbf{a}(\Omega')\delta(\tau-\tau')\delta(\Omega-\Omega')d\tau' d\Omega'\\ &+\int_0^\infty \int_{-\pi}^{\pi} \sqrt{P_{\tau 2}(\tau')P_{\Omega 2}(\Omega')}e^{j\Theta_2(\tau',\Omega')}\\ &\times \mathbf{a}(\Omega')\delta(\tau-\tau')\delta(\Omega-\Omega')d\tau' d\Omega' \end{aligned} \quad (57)$$

where the power profiles of the first and second clusters have been normalized such that

$$\int_0^\infty \int_{-\pi}^{\pi} P_{\tau 1}(\tau')P_{\Omega 1}(\Omega') + P_{\tau 1}(\tau')P_{\Omega 2}(\Omega')d\tau' d\Omega' = 1. \quad (58)$$

We denote the total power in cluster j by ρ_j, and the new PDP by $P_\tau(\cdot)$, which is simply $P_{\tau 1}(\cdot) + P_{\tau 2}(\cdot)$.

For a given bandwidth W, the ith tap $\mathbf{h}_W(i/W)$ is again complex Gaussian with zero mean. However, the covariance of the ith tap is now $A_{1i}^2\mathbf{C}_1 + A_{2i}^2\mathbf{C}_2$ where

$$\mathbf{C}_j \equiv \int_{-\pi}^{\pi} \mathbf{a}(\Omega')\mathbf{a}(\Omega')^H P_{\Omega j}(\Omega')d\Omega'. \quad (59)$$

As before, the A_{ji} are proportional to $\sqrt{P_{\tau j}(i/W)}$ and are normalized so $\sum_{i=0}^\infty A_{ji}^2 = \rho_j$. We can rewrite the covariance slightly to read

$$\mathrm{cov}\left(\mathbf{h}_W\left(\frac{i}{W}\right)\right) = \left(A_{1i}^2 + A_{2i}^2\right)\left(\alpha_{1i}\mathbf{C}_1 + \alpha_{2i}\mathbf{C}_2\right) \quad (60)$$

where $\alpha_{ji} = (A_{ji}^2/A_{1i}^2 + A_{2i}^2)$. The term on the left in (60) is simply the power in the ith tap of the PDP $P_\tau(\cdot)$, so letting $A_i^2 \equiv A_{1i}^2 + A_{2i}^2$, we can now write the TDL model as follows:

$$\mathbf{h}_W(t) = \sum_{i=0}^\infty A_i \mathbf{v}_i \delta\left(t - \frac{i}{W}\right) \quad (61)$$

where

$$\mathbf{v}_i \sim CN(0, \alpha_{1i}\mathbf{C}_1 + \alpha_{2i}\mathbf{C}_2) \equiv CN(0, \mathbf{V}_i). \tag{62}$$

Because the $\mathbf{v}_i$ are not identically distributed and independent of the PDP, *Propositions 1–3* do not hold when there are multiple clusters, unless $\mathbf{C}_1 = \mathbf{C}_2$ or there is only one transmit antenna. However, the outage rates can still be easily computed, since I_W is approximately Gaussian. Noting that $\mathbf{H}(f) \sim CN(0, \mathbf{V})$ where $\mathbf{V} = \sum_{i=0}^{\infty} A_i^2 \mathbf{V}_i$, we have the following proposition.

Proposition 4: The mean and variance of the spectral efficiency for a MISO system with two clusters can be approximated as

$$E[I_W] = E\left[\log\left(1 + \mathrm{SNR}\sum_{i=1}^{N_T} X_i\right)\right]$$
$$\mathrm{var}[I_W] \approx \gamma^2 \sum_{i=0}^{\infty} A_i^4 \mathrm{var}\left(\frac{\|\mathbf{v}_i\|^2}{N_T}\right)$$

where X_i are independent exponential random variables with means ν_i, ν_i denoting the eigenvalues of $\mathbf{V}/N_T$. As before, $\gamma \approx (\mathrm{SNR}/\mathrm{SNR}+1)$.

Proof: The proof is similar to that of *Proposition 2*, and so only the main points are given. The expected value of I_W follows from the definition of spectral efficiency and the distribution of $\mathbf{H}(f)$, while the variance of I_W follows directly from (45). The variances of the $\|\mathbf{v}_i\|$ can be found from the eigenvalues of the $\mathbf{V}_i$. □

Thus, for multiple-cluster systems, the outage rates are not separable functions of frequency and spatial diversity, but can still be easily computed from the PDPs and PAPs. This has been verified by simulation.

VI. Conclusion

We have shown that spectral efficiency in wideband systems is well modeled as Gaussian, and thus outage rates can be characterized simply by computing the mean (i.e., the ergodic capacity) and variance of the spectral efficiency. While this can be used to speed up simulation-based computations of outage rates, we have provided analytical formulas for the mean and variance in terms of the bandwidth, SNR, and the channel model (i.e., number of clusters, PDP, PAP, and array geometry). Key to the analysis was the replacement of the Saleh–Valenzuela channel model by a simpler TDL model, which we showed could be done without loss of accuracy for the channel models of interest.

For the important special case of channels with a single cluster of multipath components, the analytical formulas have particularly attractive interpretations, captured by the notions of frequency and spatial diversity defined in the paper. Frequency diversity increases outage rates by reducing the variance of the spectral efficiency, which scales as the inverse of the bandwidth. Spatial diversity (in MISO systems) increases outage rates both by increasing the mean and by reducing the variance of the spectral efficiency. The increase in the mean requires the use of nontrivial space–time/frequency codes, while the decrease in variance is easily achieved by suboptimal diversity strategies. However, since the increase in the mean accounts for more than half the gain in outage rates over SISO systems, the system designer should consider the use of space–time/frequency codes even in wideband systems with significant amounts of frequency diversity.

The transmitter is assumed not to have channel feedback in this paper. It is known [18], [19], however, that even imperfect feedback can significantly enhance space–time communication in narrowband systems. For example, feedback permits the use of transmit beamforming at the BS, providing better performance than open-loop space–time/frequency codes, while incurring smaller decoding complexity at the mobile receiver. The significant performance improvements obtained from "implicit" feedback in wideband space–time communication are investigated in a companion paper [20].

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