

Blind Adaptive Interference Suppression for the Near-Far Resistant Acquisition and Demodulation of Direct-Sequence CDMA Signals

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Abstract— Two key operations required of a receiver in a direct-sequence (DS) code division multiple access (CDMA) system are the *timing acquisition* of transmissions that are starting up or have lost synchronization, and the *demodulation* of transmissions that have been acquired. The reliability of both these operations is limited by multiple-access interference, especially for conventional matched filter-based methods, whose performance displays an interference floor and is vulnerable to the near-far problem. Recent work has shown that, provided timing information is available for a given transmission, it can be demodulated reliably using blind or training-sequence-based adaptive interference suppression techniques. These techniques are near-far resistant, unlike the matched filter demodulator, and do not require explicit knowledge of the interference parameters, unlike nonadaptive multiuser detectors. In this paper, we present a blind adaptive interference suppression technique for *joint* acquisition and demodulation, which has the unique feature that the output of the acquisition process is not simply the timing of the desired transmission, but a near-far resistant demodulator that implicitly accounts for knowledge of the timing and amplitudes of *all* transmissions to suppress the multiple-access interference. The *only* knowledge required by the scheme is that of the desired transmission's signature sequence, so that it is amenable to a decentralized implementation. On the other hand, it can be efficiently implemented as a centralized scheme in which the bulk of the computations for the adaptation are common to all transmissions that need to be acquired or demodulated.

I. INTRODUCTION

THE major limitation on the performance and capacity of direct-sequence (DS) code division multiple access (CDMA) is the multiple-access interference due to simultaneous transmissions. In particular, the conventional matched filter demodulator, which ignores the structure of the interference, suffers from an *interference floor* (i.e., its error probability does not go to zero as the background noise level vanishes), and from the *near-far problem* (i.e., its error probability can deteriorate significantly if an interfering transmission has a much higher power than the desired transmission). In recent work [1], [8], [10], [14], it has been shown that min-

imum mean squared error (MMSE) receivers can be used to suppress multiple-access interference. The MMSE receiver can be implemented adaptively, e.g., by using a training sequence of symbols for the desired transmission for initial adaptation, followed by decision-directed adaptation. The MMSE detector can also be implemented via *blind* adaptation [3], in which knowledge of the desired transmission's timing and spreading waveform is used instead of a training sequence. The results of [8] imply that the adaptive demodulators in [1], [8], [10], [14], and in [3] do not exhibit an interference floor and are near-far resistant. Further, these demodulators do not require explicit knowledge of the interference parameters and have relatively low complexity, unlike the near-far resistant centralized multiuser detectors proposed in the past (see [17] for a survey of the latter).

Demodulation of a CDMA signal must, however, be preceded by *acquisition*, in which the receiver acquires the timing of a transmission that is starting up or has lost synchronization. The adaptive demodulators in [1], [8], [10], [14], and in [3] all assume that some form of timing information regarding the desired transmission is available. While this information could conceivably be obtained using conventional acquisition techniques based on matched filters or correlators, the latter techniques also suffer from the near-far problem, and are at least as interference-limited [9] as conventional demodulation methods.

In this paper, we use the blind adaptive demodulator in [3] as a building block for a blind adaptive interference suppression scheme for *joint* acquisition and demodulation. The only knowledge assumed by the receiver is a knowledge of the spreading sequence of the desired transmission. The key idea underlying joint acquisition and demodulation is to choose an observation interval for demodulation that is large enough so that one complete symbol of the desired transmission falls into it, regardless of the timing uncertainty. For a system with multipath spread that is small compared to the bit interval T , choosing an observation interval of $2T$ suffices for this purpose. We then quantize the timing uncertainty into a finite set of hypotheses, run an adaptive demodulator under each hypothesis, and pick the demodulator that performs the best according to information derived from receiver statistics. Thus, a unique feature of our method is that it results not only in an explicit estimate of the desired signal's timing, but also in a near-far resistant receiver which automatically accounts for the delays and amplitudes of the interfering

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transmissions. The latter can be directly used for subsequent demodulation as well as for continuing blind or decision-directed adaptation as proposed in [3] or [8]. Our approach is different from more complex *simultaneous* timing estimation and demodulation schemes such as [13], since demodulation in our method occurs *after* a near-far resistant demodulator has been computed based on the timing acquisition algorithm.

Near-far resistant estimation of the timing of the desired transmission using the eigendecomposition-based MUSIC algorithm [11] has been proposed in [2], and [15]. However, since the observation interval used is of length T and is not necessarily aligned with the bit interval for the desired transmission, this algorithm only yields the timing of the desired transmission, which is not sufficient for near-far resistant demodulation. This method can be extended to obtain a near-far resistant demodulator in the following two different ways. (a) Use an observation interval of length T , estimate the delays for *all* transmissions, and then compute a near-far resistant demodulator based on these estimates. Delay estimation for all transmissions would require knowledge of all the spreading sequences in the method proposed in [2], [15]. (b) Apply the MUSIC method with a $2T$ observation interval, in which case computation of a near-far resistant demodulator based on the estimated timing can be done exactly as in this paper (see (12)). Method (b) would therefore yield a blind joint acquisition and demodulation scheme, albeit with a complexity that is somewhat higher than that of the scheme presented here. The MUSIC algorithm as presented in [2] and [15] also requires knowledge of the number of users, although this requirement could be removed by estimating the number of significant users using a number of methods (e.g., see [18]). A detailed comparison of suitably optimized versions of the MUSIC method and the scheme presented here, especially recursive versions for time-varying channels, is an important topic for further investigation.

A simpler approach to acquisition, based on MMSE adaptation using an all-one training sequence, has been proposed in [12]. Again, since the observation interval is of length T in [12], this method does not yield information sufficient to compute a near-far resistant demodulator. Modification of the method to use a $2T$ observation interval removes this problem. However, the use of an all-one training sequence means that different transmissions in acquisition mode cannot be distinguished in a near-far resistant manner. If the timing uncertainty is finite, this second problem can be addressed by using different, and “sufficiently random” training sequences for different users in acquisition mode, and by running adaptive demodulators for each of a finite number of timing hypotheses, as in this paper. Since our purpose in this paper is to develop and understand the blind method in detail, we refer the reader to [7] for a training based method for joint acquisition and demodulation that incorporates the preceding modifications, and to [6] for a numerical comparison of early versions of our blind and training based schemes.

Section II contains background material, including the system model and a review of the blind demodulator. Section III provides a description of our joint acquisition and demodulation scheme. Section IV contains performance analysis of

the scheme. Numerical results are given in Section V, and Section VI contains our conclusions.

II. BACKGROUND

A. Asynchronous CDMA

We consider an asynchronous DS CDMA system with K simultaneous antipodal transmissions over an additive white Gaussian noise (AWGN) real baseband channel. In principle, the scheme proposed here extends trivially to a complex baseband model that encompasses two-dimensional signaling and multipath fading. However, the *performance* of adaptive methods over time-varying complex baseband channels is an open issue that is currently under study. The received signal due to the k th transmission ($1 \leq k \leq K$) is given by

$$r^{(k)}(t) = \sum_{n=-\infty}^{\infty} b_{k,n} A_k s_k(t - nT - \tau_k) \quad (1)$$

where T is the bit interval, $b_{k,n} \in \{-1, 1\}$ is the n th bit of the k th transmission, A_k is its amplitude, τ_k is its relative delay with respect to the receiver, and $s_k(t)$ is its spreading waveform, given by

$$s_k(t) = \sum_{j=0}^{N-1} a_k[j] \psi(t - jT_c). \quad (2)$$

Here, $a_k[j] \in \{-1, 1\}$ is the j th element of the spreading sequence for the k th transmission, $\psi(t)$ is the chip waveform (typically assumed to be of duration T_c), and $N = T/T_c$ is the processing gain.

The net received signal is given by

$$r(t) = \sum_{k=1}^K r^{(k)}(t) + n(t) \quad (3)$$

where $n(t)$ is AWGN. The bits $\{b_{k,n}\}$ are assumed to be uncorrelated for all k and n . Taking the first transmission to be the *desired* transmission, our objective is to demodulate its bit sequence $\{b_{1,n}\}$. The only knowledge assumed is that of the desired signature sequence. The delays and amplitudes for all transmissions are unknown, as are the signature sequences for the $K - 1$ *interfering* transmissions. The adaptive algorithm for joint acquisition and demodulation will result in a linear receiver that implicitly accounts for these unknown parameters.

B. The Equivalent Synchronous Discrete Time Model

Since the digital signal processing required for interference suppression occurs in discrete time, we restrict attention to an *equivalent synchronous discrete time* model obtained by chip matched filtering the received signal, sampling at a multiple of the chip rate, and limiting attention to a finite observation interval for each bit decision. All results in this paper are for a rectangular chip waveform and chip rate sampling. Generalizations to other chip waveforms and sampling rates

is straightforward.¹ The length of the observation interval is chosen to be $2T$, which is the minimum length such that one complete bit of the desired transmission falls in the interval regardless of the relative delay τ_1 .² The latter property is crucial to the design of our acquisition algorithm. If more than one transmission is being acquired or demodulated by the receiver, this property also enables the use of common observation intervals for all transmissions. For least squares or recursive least squares (RLS) adaptation, this will imply that the computation of the inverse of the empirical cross-correlation matrix can be used for all transmissions, so that an efficient centralized implementation of the decentralized adaptive method proposed here is possible.

The l th discrete time sample can be written as

$$r[l] = \int_{lT_c}^{(l+1)T_c} r(t) dt. \quad (4)$$

Each observation interval corresponds to $2N$ samples, and the vector of received samples for the n th observation interval is $\mathbf{r}_n = (r[nN], r[nN+1], \dots, r[nN+2N-1])^T$. The number of samples used for each bit decision is $2N$, where N is the *processing gain*, or the number of chips per bit, and we may henceforth consider an *equivalent synchronous system* with received vector $\mathbf{r}_n \in \mathcal{R}^{2N}$ of $2N$ samples for the n th observation interval.

We will now express the observation vector \mathbf{r}_n in terms of the parameters of the asynchronous CDMA model (1)–(3). Without loss of generality, let $b_{k,n}$ denote the bit of the k th transmission that falls completely in the n th observation interval, and let τ_k denote the delay of this bit relative to the left edge of the n th observation interval. Since $\tau_k \in [0, T)$ we may write it as a multiple of the chip interval T_c as follows: $\tau_k = (n_k + \delta_k)T_c$, where n_k is an integer between 0 and $N-1$, and $\delta_k \in [0, 1)$. Let \mathbf{a}_k denote a vector of length $2N$ consisting of the N elements of the signature sequence of the k th transmission followed by N zeroes, i.e., $\mathbf{a}_k = (a_k[0], \dots, a_k[N-1], 0, \dots, 0)^T$. Let \mathbf{T}_L denote the acyclic left shift operator, and \mathbf{T}_R denote the acyclic right shift operator, both operating on vectors of length $2N$. Thus, for a vector $\mathbf{x} = (x_0, \dots, x_{2N-1})^T$, we have $\mathbf{T}_L \mathbf{x} = (x_1, \dots, x_{2N-1}, 0)^T$ and $\mathbf{T}_R \mathbf{x} = (0, x_0, \dots, x_{2N-2})^T$.

For each asynchronous transmission, three consecutive bit intervals overlap with a given observation interval of length $2T$. Furthermore, since the system is chip-asynchronous, two adjacent chips contribute to each chip sample. The contribution of the k th transmission to the received vector $\mathbf{r}_n \in \mathcal{R}^{2N}$ of $2N$ samples for the n th observation is therefore given by

$$\mathbf{r}_n^{(k)} = b_{k,n-1} \mathbf{v}_k^{-1} + b_{k,n} \mathbf{v}_k^0 + b_{k,n+1} \mathbf{v}_k^1$$

where

$$\begin{aligned} \mathbf{v}_k^{-1} &= A_k [(1 - \delta_k) \mathbf{T}_L^{N-n_k} \mathbf{a}_k + \delta_k \mathbf{T}_L^{N-n_k-1} \mathbf{a}_k] \\ \mathbf{v}_k^0 &= A_k [(1 - \delta_k) \mathbf{T}_R^{n_k} \mathbf{a}_k + \delta_k \mathbf{T}_R^{n_k+1} \mathbf{a}_k] \\ \mathbf{v}_k^1 &= A_k [(1 - \delta_k) \mathbf{T}_R^{n_k+N} \mathbf{a}_k + \delta_k \mathbf{T}_R^{n_k+N+1} \mathbf{a}_k]. \end{aligned} \quad (5)$$

¹While sampling at twice the chip rate would preserve most of the information in the continuous-time signal, it would also lead to a larger number of adaptive taps for a fixed observation interval.

²Longer observation intervals result in receivers with better steady state performance, but with higher complexity and slower adaptation speed.

Thus, the contribution due to the k th transmission consists of *three* signal vectors, each modulating a different bit. Each of these signal vectors are linear combinations of two adjacent shifts of the signature sequence for the k th transmission. The net received vector is given by

$$\mathbf{r}_n = \sum_{k=1}^K \mathbf{r}_n^{(k)} + \mathbf{w}_n \quad (6)$$

where \mathbf{w}_n is white Gaussian noise with covariance $\sigma^2 \mathbf{I}$. Our task is to demodulate bit $b_{1,n}$ based on the observation vector \mathbf{r}_n . The observation vectors $\{\mathbf{r}_n\}$ are identically distributed, which means that adaptive mechanisms that exploit the structure of \mathbf{r}_n for demodulation can be devised. Note that the observation vectors are not independent, since a given bit appears in three consecutive observation intervals, and since the first N noise samples for \mathbf{r}_{n+1} are the same as the last N noise samples of \mathbf{r}_n . This is irrelevant in determining the structure of the adaptive algorithm, however.

Since the model (5)–(6) is notationally cumbersome, it is convenient to consider the following *generic* equivalent synchronous model:

$$\mathbf{r}_n = b_0[n] \mathbf{u}_0 + \sum_{j=1}^J b_j[n] \mathbf{u}_j + \mathbf{w}_n \quad (7)$$

where $b_0[n]$ is the *desired bit* that we wish to demodulate, \mathbf{u}_0 is the vector modulating it, and, for $1 \leq j \leq J$, $b_j[n]$ are interfering bits due to intersymbol interference and multiple-access interference, and \mathbf{u}_j are interference vectors modulating these bits. Recalling that the first transmission is the desired transmission, the correspondence between (7) and (5)–(6) is as follows: $b_0[n] = b_{1,n}$, the desired signal vector $\mathbf{u}_0 = \mathbf{v}_1^0$, and the interfering vectors $\{\mathbf{u}_j, 1 \leq j \leq J\}$ are the set of vectors $\{\mathbf{v}_k^{-1}, \mathbf{v}_k^0, \mathbf{v}_k^1, 2 \leq k \leq K\}$ due to multiple-access interference together with the vectors $\{\mathbf{v}_1^{-1}, \mathbf{v}_1^1\}$ due to intersymbol interference. Thus, the number of interference vectors is given by $J = 3(K-1)+2$, three for each interfering transmission and two for the adjacent bits of the desired transmission. The interfering bits $b_j[n]$ are simply the bits modulating the interfering vectors, as specified in (5). The bits $b_j[n], 0 \leq j \leq J$ are uncorrelated by virtue of our assumption that a given bit is uncorrelated with different bits of the same transmission, as well as with bits of other transmissions.

For the remainder of this paper, we find it notationally convenient to hide the fine structure of the equivalent synchronous model and work with (7).

C. Blind Demodulation

In this section, we supply background material adapted from [3] together with some additional definitions and formulas that will be required in our acquisition algorithm. Letting \langle, \rangle denote inner product, the blind minimum output energy (MOE) demodulator [3] for the equivalent synchronous system (7) corresponds to an estimate $\hat{b}_0[n] = \text{sgn}(\langle \mathbf{c}_{MOE}, \mathbf{r}_n \rangle)$, where the correlator \mathbf{c}_{MOE} is chosen to minimize the output energy $E\{(\langle \mathbf{c}_{MOE}, \mathbf{r}_n \rangle)^2\}$, subject to the constraint

$$\langle \mathbf{c}_{MOE}, \hat{\mathbf{u}}_0 \rangle = 1. \quad (8)$$

Here $\hat{\mathbf{u}}_0$ is a *nominal signal vector*, which is the receiver's estimate of the *direction* of the desired signal vector \mathbf{u}_0 . We will assume that $\|\hat{\mathbf{u}}_0\| = 1$ without loss of generality.

The norm squared of \mathbf{c}_{MOE} will be referred to as the *detector energy* β , and is a measure of the amount of noise enhancement at the output. If $\hat{\mathbf{u}}_0$ has a nonzero component $\hat{\mathbf{p}}_I$ orthogonal to the space \mathcal{S}_I spanned by the interference vectors $\mathbf{u}_1, \dots, \mathbf{u}_J$, complete cancellation of the interference is obtained by setting $\mathbf{c} = \mathbf{c}_I = \alpha_I \hat{\mathbf{p}}_I$, where α_I is chosen to satisfy (8), and the resulting choice of \mathbf{c} satisfies

$$\beta_I = \|\mathbf{c}_I\|^2 = 1/\|\hat{\mathbf{p}}_I\|^2 = 1/\eta_I \quad (9)$$

where $\eta_I = \|\hat{\mathbf{p}}_I\|^2$ is the energy of the component of the nominal orthogonal to the interference space (this equals the relative energy of the orthogonal component by virtue of the normalization $\|\hat{\mathbf{u}}_0\|^2 = 1$). Smaller values of η_I correspond to larger β_I and, hence, to more noise enhancement at the output of the detector.

Mismatch between the nominal $\hat{\mathbf{u}}_0$ and the signal vector \mathbf{u}_0 can occur due to errors in the timing estimate or due to the presence of multipath components not accounted for in the nominal. This can result in signal loss. In particular, complete signal cancellation is possible by setting $\mathbf{c} = \mathbf{c}_0 = \alpha_0 \hat{\mathbf{p}}_0$, where $\hat{\mathbf{p}}_0$ denotes the projection of the nominal $\hat{\mathbf{u}}_0$ orthogonal to the space spanned by the desired signal vector \mathbf{u}_0 , and where α_0 is chosen to satisfy (8). The resulting choice of \mathbf{c} satisfies

$$\beta_0 = \|\mathbf{c}_0\|^2 = 1/\|\hat{\mathbf{p}}_0\|^2 = 1/\eta_0; \quad (10)$$

where η_0 is the energy of the component of the nominal orthogonal to the space spanned by the signal vector \mathbf{u}_0 , i.e., η_0 is the near-far resistance of a hypothetical system in which the nominal is the desired signal vector and the interference consists solely of the desired signal vector \mathbf{u}_0 .

Equations (9) and (10) imply that, if the nominal $\hat{\mathbf{u}}_0$ is closer to the space spanned by the desired signal vector \mathbf{u}_0 than to the interference subspace \mathcal{S}_I (i.e., if $\eta_0 = \|\hat{\mathbf{p}}_0\|^2 < \|\hat{\mathbf{p}}_I\|^2 = \eta_I$), then interference suppression can be achieved while avoiding excessive signal cancellation by constraining the norm of \mathbf{c} , i.e., by constraining the detector energy β . Ideally, the constraint should be such that $\beta = \|\mathbf{c}\|^2$ is allowed to exceed β_I but not β_0 . Using Lagrange multipliers to reflect the norm constraint and (8), the cost function to be minimized becomes

$$E\{(\langle \mathbf{c}, \mathbf{r}_n \rangle)^2\} + \lambda \langle \mathbf{c}, \hat{\mathbf{u}}_0 \rangle + \nu \|\mathbf{c}\|^2. \quad (11)$$

While λ must be chosen to satisfy (8), it is convenient to define an *unscaled MOE solution*, which corresponds to $\lambda = -2$ as follows:³

$$\mathbf{c} = (\mathbf{R} + \nu \mathbf{I})^{-1} \hat{\mathbf{u}}_0 \quad (12)$$

where $\mathbf{R} = E\{\mathbf{r}_n \mathbf{r}_n^T\}$ is the statistical correlation matrix for the observation vector. This solution must be scaled down by $\langle \mathbf{c}, \hat{\mathbf{u}}_0 \rangle$ in order to satisfy the constraint (8), and the

³Note that demodulation performance does not depend on scaling for a constant modulus constellation.

resulting MOE is given by

$$\begin{aligned} \zeta &= E\left[\left\langle \frac{\mathbf{c}}{\langle \mathbf{c}, \hat{\mathbf{u}}_0 \rangle}, \mathbf{r}_n \right\rangle^2\right] = \frac{\mathbf{c}^T \mathbf{R} \mathbf{c}}{\langle \mathbf{c}, \hat{\mathbf{u}}_0 \rangle^2} \\ &= \frac{\hat{\mathbf{u}}_0^T (\mathbf{R} + \nu \mathbf{I})^{-1} \mathbf{R} (\mathbf{R} + \nu \mathbf{I})^{-1} \hat{\mathbf{u}}_0}{(\hat{\mathbf{u}}_0^T (\mathbf{R} + \nu \mathbf{I})^{-1} \hat{\mathbf{u}}_0)^2} \end{aligned} \quad (13)$$

Similar, the detector energy β for the appropriately scaled MOE solution is given by

$$\beta = \frac{\|\mathbf{c}\|^2}{\langle \mathbf{c}, \hat{\mathbf{u}}_0 \rangle^2} = \frac{\hat{\mathbf{u}}_0^T (\mathbf{R} + \nu \mathbf{I})^{-2} \hat{\mathbf{u}}_0}{(\hat{\mathbf{u}}_0^T (\mathbf{R} + \nu \mathbf{I})^{-1} \hat{\mathbf{u}}_0)^2}. \quad (14)$$

Recalling that the bits $b_j[n]$ in (7) are uncorrelated, the correlation matrix \mathbf{R} is given by

$$\mathbf{R} = \sum_{j=0}^J \mathbf{u}_j \mathbf{u}_j^T + \sigma^2 \mathbf{I}. \quad (15)$$

From (12) and (15), it is clear that the Lagrange multiplier ν for the constraint on $\|\mathbf{c}\|^2$ plays the same role as additional noise variance in determining the MOE solution. We will henceforth call ν the *fictitious noise variance*. Large ν leads to less signal loss, and hence to greater tolerance to mismatch at the cost of less interference suppression.

Our acquisition scheme coarsely quantizes the delay into a number of hypotheses, computes the MOE solution for each corresponding nominal, and attempts to choose the best hypothesis based on the resulting MOE's. If all shifts of the desired spreading sequence have reasonably low cross correlations with the interference, the amount of interference suppression under different hypotheses should be comparable. However, it should be harder to suppress the desired signal for nominals that are close to the direction of the desired signal vector. For the same value of detector energy β , therefore, one would expect a larger MOE for the better hypotheses. However, constraining β to be the same under different hypotheses requires, in general, that different values of ν are used. This is computationally cumbersome, since the inverse of $\mathbf{R} + \nu \mathbf{I}$ must be computed for each ν . In order to avoid this, we assume that the same fictitious noise variance ν is used for finding the MOE solutions under all delay hypotheses. We then compare the MOE in (13) for different hypotheses, and choose the one that is the largest.

The MOE ζ computed in (13) is normalized such that $\langle \mathbf{c}, \hat{\mathbf{u}}_0 \rangle = 1$. A different normalization that will be useful in our acquisition algorithm is $\|\mathbf{c}\|^2 = 1$, which yields the *normalized MOE* ξ defined by

$$\begin{aligned} \xi &= E\left[\left\langle \frac{\mathbf{c}}{\|\mathbf{c}\|}, \mathbf{r}_n \right\rangle^2\right] = \frac{\mathbf{c}^T \mathbf{R} \mathbf{c}}{\|\mathbf{c}\|^2} \\ &= \frac{\hat{\mathbf{u}}_0^T (\mathbf{R} + \nu \mathbf{I})^{-1} \mathbf{R} (\mathbf{R} + \nu \mathbf{I})^{-1} \hat{\mathbf{u}}_0}{\hat{\mathbf{u}}_0^T (\mathbf{R} + \nu \mathbf{I})^{-2} \hat{\mathbf{u}}_0}. \end{aligned} \quad (16)$$

The utility of the normalized MOE ξ is as follows: Once a coarse estimate of the delay has been obtained, a local search for refining this estimate can be performed by maximizing ξ . This is because, if there is a nominal with no mismatch, the signal is not suppressed, leading to a large output energy ζ .

Further, in the absence of mismatch, the detector energy β is expended only on suppressing interference, so that β should be smaller at the correct delay. Thus, $\xi = \frac{\zeta}{\beta}$ should be large at the true delay. We will use this idea to interpolate between two coarse delay hypotheses to estimate the true delay.

The reason that the MOE ζ is not used to interpolate between delay hypotheses is that maximizing it locally is difficult. On the other hand, the normalized MOE ξ is not used for deciding among the coarse delay hypotheses because, in the presence of mismatch, the detector energy β under the good delay hypotheses can be large at low noise levels (since the MOE criterion is trying to take advantage of the mismatch to suppress the signal as well as the interference, especially when the interference is weak). Thus, even though ζ is larger for the good hypotheses, so is β , which may cause $\xi = \zeta/\beta$ to be smaller than for some incorrect delay hypothesis. Thus, testing the hypotheses based on ξ breaks down in a low noise, single-user regime, where we would like to obtain the best performance. This phenomenon has been verified numerically, although we do not include those results here. Note that the problem occurs due to the mismatch due to coarse delay quantization, and is not an issue when we are trying to choose among a set of nominals such that one corresponds to the true delay. We will therefore use the MOE ζ for the coarse delay estimate and the normalized MOE ξ for refining the estimate.

III. BLIND ACQUISITION AND DEMODULATION

In the acquisition phase, the receiver does not know either of the parameters n_1 or δ_1 specifying the delay $\tau_1 = (n_1 + \delta_1)T_c$ for the first transmission, and therefore does not know the desired signal vector \mathbf{u}_0 . However, we do know that \mathbf{u}_0 is a linear combination of two shifts of the desired transmission's spreading sequence, given by

$$\mathbf{u}_0 = \mathbf{v}_1^0 = A_1[(1 - \delta_1)\mathbf{T}_R^{n_1}\mathbf{a}_1 + \delta_1\mathbf{T}_R^{n_1+1}\mathbf{a}_1]. \quad (17)$$

We must now determine the level of delay discretization needed to translate the uncertainty in n_1 and δ_1 into a finite number of hypotheses.

A. Number of Delay Hypotheses Needed

The delay discretization should be such that the mismatch, and hence the signal cancellation, for the best hypothesis is not excessive, regardless of the true delay τ_1 . This is true (see the previous section) if, for each $\tau_1 \in [0, T)$, there is a hypothesis for which the distance of the nominal from the signal space is smaller than its distance from the interference space, i.e., if $\eta_0 < \eta_I$, where the inequality is preferably satisfied by a wide margin. For signature sequences with good cross correlation properties, any given shift of the desired signature sequence will have a nonzero component orthogonal to the interference space, so that the near-far resistance η_I of the nominal corresponding to each delay hypothesis can be expected to satisfy some designed lower bound, e.g., $\eta_I \geq 0.5$. For the best hypothesis, η_0 must be smaller than this lower bound.

Consider \mathbf{u}_0 given by (17). Suppose the hypothesized delays are integer multiples of the chip interval, so that the nomi-

nals corresponding to the two closest hypotheses are (scalar multiples of) $\hat{\mathbf{u}}_0 = \mathbf{T}_R^{n_1}\mathbf{a}_1$ and $\hat{\mathbf{u}}_0 = \mathbf{T}_R^{n_1+1}\mathbf{a}_1$, respectively. To obtain a rough idea of the number of delay hypotheses needed, it suffices to assume that these two nominals are orthogonal, since shifts of typical signature sequences are nearly orthogonal, in practice. We then obtain that the near-far resistances of these nominals relative to the signal vector \mathbf{u}_0 in (17) are, respectively,

$$\eta_0^{(1)} = \frac{\delta_1^2}{\delta_1^2 + (1 - \delta_1)^2} \quad (18)$$

$$\eta_0^{(2)} = \frac{(1 - \delta_1)^2}{\delta_1^2 + (1 - \delta_1)^2}. \quad (19)$$

For $\delta_1 = 0.5$, we have $\eta_0^{(1)} = \eta_0^{(2)} = 0.5$, which is comparable to typical values of the near-far resistance η_I relative to the interference space. Thus, if the delay hypotheses are spaced by the chip interval, the amount of signal loss could be large even under the closest hypothesis.

Consider now an intermediate delay hypothesis with nominal (a scalar multiple of) $\hat{\mathbf{u}}_0 = \frac{1}{2}(\mathbf{T}_R^{n_1}\mathbf{a}_1 + \mathbf{T}_R^{n_1+1}\mathbf{a}_1)$, which corresponds to a hypothesized delay of $(n_1 + \frac{1}{2})T_c$. The near-far resistance of this nominal relative to the signal space can be shown to be

$$\eta_0^{(3)} = 1 - \frac{1}{2[\delta_1^2 + (1 - \delta_1)^2]} \quad (20)$$

For a given δ_1 , the best hypothesis is the one with the smallest value of η_0 . From (18)–(20), it can be shown that the worst-case value of η_0 for the best hypothesis is $\frac{3-2\sqrt{2}}{4-2\sqrt{2}} \approx 0.146$. The value of η_I should therefore be larger than this value for near-far resistant timing acquisition. In contrast, when the delay is perfectly known, the value of η_I is, in theory, only required to be nonzero. The penalty in terms of near-far resistance for not knowing the delay can be reduced by using a finer delay discretization, which in turn implies a larger number of delay hypotheses and thus greater complexity.

B. Acquisition Algorithm

For $i = 0, 1, \dots, N - 1$, the delay hypotheses and the corresponding nominals are given by: Hypothesis H_{2i} : Delay $\tau_1^{(i)} = iT_c$, Nominal $\hat{\mathbf{u}}_0^{(i)} = \mathbf{T}_R^i\mathbf{a}_1/||\mathbf{T}_R^i\mathbf{a}_1||$ Hypothesis H_{2i+1} : Delay $\tau_1^{(i)} = (i + \frac{1}{2})T_c$, Nominal $\hat{\mathbf{u}}_0^{(i)} = (\mathbf{T}_R^i\mathbf{a}_1 + \mathbf{T}_R^{i+1}\mathbf{a}_1)/||\mathbf{T}_R^i\mathbf{a}_1 + \mathbf{T}_R^{i+1}\mathbf{a}_1||$. All nominals are normalized to unit energy to enable a fair comparison of the MOE's ζ for different hypotheses.

Step 1: Main Computations

For the i th hypothesis, $i = 0, 1, \dots, 2N - 1$, compute the MOE solution, as follows, as in (12):

$$\mathbf{c}^{(i)} = (\mathbf{R} + \nu\mathbf{I})^{-1}\hat{\mathbf{u}}_0^{(i)} \quad (21)$$

and the MOE and normalized MOE as in (13) and (16), respectively, as follows:

$$\zeta_i = \frac{\mathbf{c}^{(i)T}\hat{\mathbf{R}}\mathbf{c}^{(i)}}{(\mathbf{c}^{(i)T}\hat{\mathbf{u}}_0^{(i)})^2} \quad (22)$$

$$\xi_i = \frac{\mathbf{c}^{(i)T} \hat{\mathbf{R}} \mathbf{c}^{(i)}}{\mathbf{c}^{(i)T} \mathbf{c}^{(i)}}. \quad (23)$$

Step 2: Finding the Best Hypotheses

Let i_{max} denote the index of the hypothesis with the largest MOE, i.e.,

$$\zeta_{i_{max}} = \max_i \zeta_i.$$

Similarly, let i_{next} denote the index of the best adjacent hypothesis, i.e.,

$$\zeta_{i_{next}} = \max\{\zeta_{i_{max}-1}, \zeta_{i_{max}+1}\}$$

except for $i_{max} = 2N - 1$, for which we set $i_{next} = 2N - 2$, and $i_{max} = 0$, for which $i_{next} = 1$. This is because it is assumed (without loss of generality) that the delay $\tau_1 \in [0, T]$.

Step 3: Combining Rule

Define the *interpolated nominal*

$$\hat{\mathbf{u}}_0(\lambda) = \lambda \hat{\mathbf{u}}_0^{(i_{max})} + (1 - \lambda) \hat{\mathbf{u}}_0^{(i_{next})} \quad (24)$$

where $0 \leq \lambda \leq 1$, and let $\xi(\lambda)$ denote the normalized MOE corresponding to this nominal. Let $\lambda_{max} = \operatorname{argmax}\{\xi(\lambda) : 0 \leq \lambda \leq 1\}$ denote the value of $\lambda \in [0, 1]$ that maximizes the normalized MOE. As shown in Appendix A, the computation of λ_{max} is simple, involving solution of a quadratic equation for λ and comparison of $\xi(1) = \xi_{i_{max}}$ and $\xi(0) = \xi_{i_{next}}$ with the values of $\xi(\lambda)$ evaluated at the solutions to the quadratic equation.

Step 4: Algorithm Outputs

The demodulator produced by the acquisition algorithm is given by

$$\mathbf{c}_{out} = \lambda_{max} \mathbf{c}^{(i_{max})} + (1 - \lambda_{max}) \mathbf{c}^{(i_{next})} \quad (25)$$

which is the (unscaled) MOE solution for the *maximizing interpolated nominal*

$$\hat{\mathbf{u}}_0^{(max)} = \hat{\mathbf{u}}_0(\lambda_{max}) = \lambda_{max} \hat{\mathbf{u}}_0^{(i_{max})} + (1 - \lambda_{max}) \hat{\mathbf{u}}_0^{(i_{next})}. \quad (26)$$

For the delay estimate, applying the definitions of the $\hat{\mathbf{u}}_0^{(i)}$ and using (24), the maximizing interpolated nominal is rewritten as

$$\hat{\mathbf{u}}_0(\lambda_{max}) = a \mathbf{T}_R^n \mathbf{a}_1 + b \mathbf{T}_R^{n+1} \mathbf{a}_1$$

where n , a and b have a straightforward dependence on λ_{max} , i_{max} and i_{next} (see Appendix A). Referring to (5), the delay estimate is given by

$$\hat{\tau}_1 = \left(n + \frac{b}{a+b}\right) T_c. \quad (27)$$

We will illustrate the operation of our algorithm via a least squares implementation, which follows the steps 1 through 4 described previously, except that the cross correlation matrix \mathbf{R} is replaced by the empirical crosscorrelation matrix $\hat{\mathbf{R}}$ computed over M_{LS} bit intervals

$$\hat{\mathbf{R}} = (1/M_{LS}) \sum_{n=1}^{M_{LS}} \mathbf{r}_n \mathbf{r}_n^T \quad (28)$$

The MOE corresponding to hypothesis H_i is denoted by $\hat{\zeta}_i$, in order to distinguish it from its steady state equivalent.

In practice, an RLS implementation might be preferred in order to track time variations. It is also possible to use a stochastic gradient algorithm, but the convergence of such algorithms for blind adaptation has been found to be slow [3]. Furthermore, the complexity of running $2N$ stochastic gradient algorithms corresponding to the $2N$ hypotheses is comparable to that of $2N$ RLS algorithms that share the most significant part of the computation, i.e., the recursive computation of the inverse of the empirical crosscorrelation matrix.

Choice of ν : The fictitious noise variance ν must be chosen to optimize performance: a small value allows more interference suppression, but also more signal loss. Numerical results in [3] and [5] indicate that a *fictitious* SNR of $\|\mathbf{u}_0\|^2/(\sigma^2 + \nu)$ of 10 dB appears to work well over most ranges of relative amplitudes.⁴ However, since such a choice of ν cannot be implemented without knowing the amplitude of the desired transmission, we consider a more practical choice of ν , which scales according to the net power in the received signal, estimated as $\operatorname{trace}(\hat{\mathbf{R}})$. Thus, we set $\nu = \alpha \operatorname{trace}(\hat{\mathbf{R}})$. As discussed in Section V, the performance is sensitive to the choice of α . While one fairly complex method for automatically choosing α is provided in Appendix B, finding more satisfactory solutions is left as an open problem.

IV. PERFORMANCE ANALYSIS

Steady state benchmarks for the performance of the algorithm are established by running the algorithm using the statistical cross correlation matrix \mathbf{R} . The error probability P_e for the resulting demodulator \mathbf{c}_{out} , and the cosine κ of the angle between \mathbf{c}_{out} and the ideal MMSE solution, $\mathbf{c}_{MMSE} = \mathbf{R}^{-1} \mathbf{u}_0$, are computed.⁵ The error probability $P_{e, MF}$ for the matched filter $\mathbf{c} = \mathbf{u}_0$ is computed as a benchmark that any adaptive scheme should be able to beat when the interference powers are significant. This comparison is biased in favor of the matched filter, since we assume a perfect delay estimate in this case. The latter would require a separate acquisition scheme in practice. The steady state performance measures are compared with corresponding results obtained for each simulation run of a least squares implementation. The bias and variance of the delay estimate $\hat{\tau}_1$ are other performance measures of interest associated with the least squares implementation.

In addition to the preceding, a key performance measure is the acquisition error probability, defined as the event that the best delay hypothesis is not one of the two selected by an adaptive implementation (in particular, by the least squares implementation considered here). The best hypothesis is defined to be the one with the largest normalized MOE in steady state, provided that the delay it corresponds to is within $T_c/2$ (the delay quantization used) of the true delay. Otherwise, it is defined as the hypothesis that corresponds to a delay closest to the true delay. In the latter instance, the

⁴In this case, if $\|\mathbf{u}_0\|^2/\sigma^2$ is smaller than 10 dB, then the noise level is high enough that no fictitious noise would be needed, so that $\nu = 0$ would suffice.

⁵For the model 7, the error probability for any linear receiver is computed analytically by averaging over the bits $\{b_j[n]\}$ modulating the signal and interference vectors; see [8], for instance.

algorithm would choose an incorrect hypothesis even in steady state, so that acquisition with an adaptive implementation would necessarily be unreliable. In most cases of interest, as the number of least squares iterations increases, the acquisition error probability quickly becomes too small to be estimated directly by simulation. On the other hand, an exact analytical estimate of the acquisition error probability for the least squares implementation appears intractable. We therefore develop a simple approximation as follows.

Let i_{max} denote the index of the best hypothesis. Let $\hat{\zeta}_i$ denote the estimated MOE under hypothesis H_i for a least squares implementation. We will approximate the $\{\hat{\zeta}_i\}$ as jointly Gaussian. Under this approximation, the random variable $\hat{\zeta}_{i_{max}} - \hat{\zeta}_i, i \neq i_{max}$, is Gaussian. Denoting its mean by μ_i and its variance by s_i^2 , we obtain the following approximation for the probability of choosing H_i over the correct hypothesis $H_{i_{max}}$ (this would be exact if the jointly Gaussian assumption were exact):

$$q(i) = P[\hat{\zeta}_{i_{max}} < \hat{\zeta}_i] \approx Q\left(\frac{\mu_i}{s_i}\right). \quad (29)$$

Analytical computation of μ_i and s_i^2 is difficult; hence, we simply estimate these using the empirical statistics of the $\hat{\zeta}_i$ over multiple simulation runs.

Assuming that (29) provides an accurate estimate of the probabilities $q(i)$, we can obtain a union bound on the probability of acquisition error as follows. Let i_{max} and i_{next} index the best hypothesis and the best adjacent hypothesis in steady state. Acquisition error occurs if $H_{i_{max}}$ is not chosen by the adaptive implementation of the algorithm. If $H_{i_{max}}$ is chosen but $H_{i_{next}}$ is not, this is not considered an acquisition error. This is because, if $H_{i_{next}}$ is truly a significant hypothesis, the probability of this event is very small, while if $H_{i_{next}}$ is not significant, then it does not matter if the wrong adjacent hypothesis is chosen, since the combining rule would give a low weight to whichever adjacent hypothesis is chosen. While it is possible, using the jointly Gaussian approximation, to carry out a detailed analysis of the choice of adjacent hypothesis taking into account the combining rule, little additional insight would be gained by doing so.

The event that neither $H_{i_{max}}$ nor $H_{i_{next}}$ is chosen lies in the union of the events $\{\hat{\zeta}_{i_{max}} < \hat{\zeta}_i\}$ for $i \neq i_{max}, i_{next}$. On the other hand, if $H_{i_{next}}$ is chosen, the event that $H_{i_{max}}$ is not the best adjacent hypothesis also lies in the preceding union. Thus, the probability that acquisition error occurs is bounded by

$$q_{e,acq} \leq \sum_{i \neq i_{max}, i_{next}} q(i). \quad (30)$$

V. NUMERICAL RESULTS

We consider a symbol- and chip-asynchronous system with processing gain $N = 15$ and number of transmissions $K = 6$. No attempt is made to optimize the set of signature sequences, so that all numerical results are for a fixed, but random, choice of the set of K signature sequences. Steady state analysis and simulations for other choices of signature sequences yield qualitatively similar results. The delays τ_k of the $K - 1$ interfering transmissions are chosen randomly in $[0, T)$ and

then kept fixed. Two different values of the delay of the first (desired) transmission are considered: $\tau_1 = 3.5T_c$ (which is perfectly matched to hypothesis H_7), and $\tau_1 = 3.25T_c$ (which falls in the “middle” of hypotheses H_6 and H_7 , so that there is mismatch under either of these hypotheses).

The SNR for the equivalent synchronous model (7) is $\|\mathbf{u}_0\|^2/\sigma^2$. Since this depends on the chip delay δ_1 , we define SNR to be that corresponding to a chip-synchronous system ($\delta_1 = 0$), so that $\text{SNR} = \frac{A_1^2 N}{\sigma^2}$. The numerator, $A_1^2 N$, is simply the bit energy E_b , and $\sigma^2 = N_0/2$, so that $\text{SNR} (\text{dB}) = \frac{E_b}{N_0} (\text{dB}) + 3$. We fix the amplitude of the desired transmission, and assume that *each* interfering transmission has power P_I relative to that of the desired transmission. Three different values of P_I are considered: 20 dB (to check for near-far resistance), 0 dB (to examine the performance with perfect power control), and -20 dB (to check that signal cancellation is not excessive, and that the performance is not degraded too much relative to the matched filter receiver). The least squares implementation is run for a value of fictitious noise variance $\nu = \alpha \text{trace}(\hat{\mathbf{R}})$, where $\alpha = .0001$ unless specified otherwise.

A. Steady State Benchmarks

We consider two values of E_b/N_0 , 7 dB (which is moderate for single-user applications, but low for linear multiuser detection, which causes noise enhancement) and 17 dB (which is high for single-user systems, but not uncommon for CDMA systems whose performance is limited by multiple-access interference). These values correspond to (chip-synchronous) SNR's of 10 dB and 20 dB, respectively.

We define $\zeta_{max,good}$ to be the largest MOE among the timing hypotheses within at most $T_c/2$ of the true delay, and $\zeta_{max,bad}$ to be the largest MOE among all other hypotheses. The difference between these two quantities is a measure of how well the algorithm can distinguish between good and bad hypotheses (the bias and variance in estimates of these in a least squares implementation will ultimately determine acquisition performance). These and other relevant steady state quantities are displayed in Tables I and II.

For E_b/N_0 of 17 dB, the delay estimates based on interpolating between good hypotheses are excellent, and the cosine of the angle between the resulting demodulator and the MMSE solution is close to one. The demodulator is near-far resistant, and performs much better than the matched filter even with perfect power control ($P_I = 0$ dB). The separation between $\zeta_{max,good}$ and $\zeta_{max,bad}$ is substantial, so that the scheme is expected to be robust to least squares estimation errors. For a smaller E_b/N_0 of 7 dB, the acquisition scheme can go wrong for high interference levels (see the entry for $P_I = 20\text{dB}$ in Table II) in the presence of mismatch.⁶ In every other case for E_b/N_0 of 7 dB, the acquisition algorithm selects the correct hypotheses and the demodulator and delay estimate based on interpolating the hypotheses is again very close to the MMSE solution. However, in this low SNR regime, the performance gains over the matched filter receiver are not as dramatic. The separation between $\zeta_{max,good}$ and $\zeta_{max,bad}$ is

⁶In this case, increasing E_b/N_0 by a further 2 dB leads to correct acquisition in steady state.

TABLE I
STEADY STATE BENCHMARKS FOR THE PERFORMANCE OF THE BLIND ACQUISITION ALGORITHM (CASE 1: $\tau_1 = 3.5T_c$)

$P_I/E_b/N_0$ (dB)	$\zeta_{max,good}/\zeta_{max,bad}$	$\frac{\hat{\tau}}{T_c}$	κ	P_e	$P_{e,MMSE}$	$P_{e,MF}$
20/17	.53/.12	3.5	.998	1.3×10^{-5}	4.9×10^{-6}	.45
20/7	.75/.67	3.509	.998	.078	.077	.45
0/17	.52/.07	3.501	.997	6.4×10^{-7}	6.1×10^{-7}	.043
0/7	.65/.37	3.505	.999	.035	.035	.091
-20/17	.51/.02	3.500	1.0	1.2×10^{-11}	1.2×10^{-11}	7.6×10^{-11}
-20/7	.60/.13	3.500	1.0	.013	.013	.014

TABLE II
STEADY STATE BENCHMARKS FOR THE PERFORMANCE OF THE BLIND ACQUISITION ALGORITHM (CASE 2: $\tau_1 = 3.25T_c$)

$P_I/E_b/N_0$ (dB)	$\zeta_{max,good}/\zeta_{max,bad}$	$\frac{\hat{\tau}}{T_c}$	κ	P_e	$P_{e,MMSE}$	$P_{e,MF}$
20/17	.31/.11	3.253	.997	1.1×10^{-8}	4.1×10^{-9}	.44
20/7	.65/.71	1.343	-.303	.71	.033	.44
0/17	.15/.07	3.252	.996	7.7×10^{-10}	6.8×10^{-10}	.012
0/7	.51/.38	3.258	.997	.017	.017	.06
-20/17	.10/.02	3.250	.998	2.3×10^{-14}	2.3×10^{-14}	9.2×10^{-14}
-20/7	.45/.13	3.250	1.0	6.6×10^{-3}	6.6×10^{-3}	6.7×10^{-3}

also smaller here, implying that a large number of least squares iterations might be required to provide reliable acquisition. For the remainder of the paper, therefore, we restrict attention to an interference-limited regime more typical of CDMA applications, taking $E_b/N_0 = 17$ dB.

While $\alpha = 0.0001$ is used for the preceding results, it is important to choose α according to signal and interference powers for rapid acquisition using a least squares implementation. See Section V-B, where we illustrate the advantage of choosing larger α when P_I is smaller. Conversely, if we increase P_I to 50 dB while keeping $\alpha = 0.0001$, acquisition error occurs in steady state even for E_b/N_0 of 17 dB. This is because, for fixed α , $\text{trace}(\hat{\mathbf{R}})$, and hence ν , increases with P_I , permitting less interference suppression. Using a smaller $\alpha = 0.00001$ restores reliable steady state performance in this setting.

Finally, it is worth noting that even for very weak interference, the error probability performance for all three demodulators considered is substantially worse than the standard error probability formula $Q(\sqrt{2E_b/N_0})$ for binary phase shift keying (BPSK). This occurs because the receiver is not chip-synchronous with the desired transmission, causing both SNR loss and intersymbol interference.

B. Acquisition Error Probability

In view of the poor performance even in steady state for E_b/N_0 of 7 dB, we restrict attention to E_b/N_0 of 17 dB in order to evaluate the performance of the least squares implementation of the algorithm in an interference-limited regime. Our objective is to explore the dependence of the acquisition error probability on the number M_{LS} of least squares iterations used. We use 10000 simulation runs to

estimate the acquisition probability in each case considered, and compare it with the analytical approximation (29)–(30) obtained using the jointly Gaussian assumption for the output energies $\hat{\zeta}_i$ obtained over the simulation runs (the second-order statistics of the $\{\hat{\zeta}_i\}$ can be well estimated using many fewer runs than required to estimate the acquisition error probability accurately).

The $\{\hat{\zeta}_i\}$ over independent runs will be identically distributed only if the value of ν used for the runs is the same. Since the empirical crosscorrelation matrix $\hat{\mathbf{R}}$ differs over different runs, so does the value of the fictitious noise variance $\nu = \alpha \text{trace}(\hat{\mathbf{R}})$. However, for all the simulation runs considered, $\text{trace}(\hat{\mathbf{R}})$ (which is a rapidly converging estimate of the average received power) is close to the statistical average $\text{trace}(\mathbf{R})$. Thus, from the point of view of analyzing acquisition performance, it suffices to consider a fixed value for the fictitious noise variance, $\nu = \alpha \text{trace}(\mathbf{R})$. For this value of ν , 1000 simulation runs of the least squares implementation of the acquisition algorithm are used to compute the empirical mean and variance of $\hat{\zeta}_{i_{max}} - \hat{\zeta}_i$, $i \neq i_{max}$. This is used to estimate the probability $q(i)$ of choosing a given wrong hypothesis H_i via (29), and then to compute the union bound (30) on the probability $q_{e,acq}$.

In comparing the results of the analysis with direct estimates of the acquisition error probability, we list the following quantities.

- 1) The acquisition error probability $q_{e,acq}$ obtained via 10000 simulation runs and the (simulation-aided) analytical approximation.
- 2) Let $i^* = \text{argmax}\{q(i) : i \neq i_{max}, i_{max} \pm 1\}$ denote the index of the incorrect hypothesis with the largest probability of being chosen over the best hypothesis

TABLE III
ACQUISITION PERFORMANCE AS A FUNCTION OF THE RELATIVE INTERFERENCE POWER P_I , THE NUMBER M_{LS} OF LEAST SQUARES ITERATIONS, AND $\alpha = \nu/\text{trace}(\mathbf{R})$ ($\tau_1 = 3.5T_c$, $E_b/N_0 = 17$ dB)

P_I (dB)/ M_{LS}/α	$q_{e,acq}$ (Sim./Anal.)	$q(i^*)$ (Sim./Anal.)	Bias	Standard Dev.
20/30/.0001	$1.3 \times 10^{-3}/2.5 \times 10^{-2}$	$1.1 \times 10^{-3}/8 \times 10^{-3}$.004	.07
20/40/.0001	$0/2.3 \times 10^{-4}$	$0/1.6 \times 10^{-4}$.003	.02
0/40/.0001	$0/6.3 \times 10^{-2}$	$0/2.6 \times 10^{-3}$	-.01	.1
0/100/.0001	$0/6.8 \times 10^{-18}$	$0/6.7 \times 10^{-18}$.002	.01
0/30/.001	$4 \times 10^{-4}/6.3 \times 10^{-2}$	$4 \times 10^{-4}/6.2 \times 10^{-3}$.003	.05
0/40/.001	$0/6.5 \times 10^{-4}$	$0/3.9 \times 10^{-5}$	-.004	.02
0/40/.01	$0/1.8 \times 10^{-5}$	$0/1.7 \times 10^{-5}$.005	.02
-20/30/.0001	$8.7 \times 10^{-3}/2.76$	$1.8 \times 10^{-3}/.1$.03	.52
-20/100/.0001	$0/4.9 \times 10^{-23}$	$0/4.3 \times 10^{-24}$.0002	.01
-20/30/.001	0/.30	$0/1.1 \times 10^{-2}$	-.0007	.11
-20/30/.01	$0/1.1 \times 10^{-5}$	$0/5.5 \times 10^{-7}$.0006	.02

TABLE IV
ACQUISITION PERFORMANCE AS A FUNCTION OF THE RELATIVE INTERFERENCE POWER P_I , THE NUMBER M_{LS} OF LEAST SQUARES ITERATIONS, AND $\alpha = \nu/\text{trace}(\mathbf{R})$ ($\tau_1 = 3.25T_c$, $E_b/N_0 = 17$ dB)

P_I (dB)/ M_{LS}/α	$q_{e,acq}$ (Sim./Anal.)	$q(i^*)$ (Sim./Anal.)	Bias	Standard Dev.
20/30/.0001	$2.3 \times 10^{-2}/4.3 \times 10^{-2}$	$1.9 \times 10^{-2}/2.6 \times 10^{-2}$.003	.14
20/40/.0001	$8 \times 10^{-4}/2 \times 10^{-3}$	$8 \times 10^{-4}/1.7 \times 10^{-3}$.005	.04
20/60/.0001	$0/3.8 \times 10^{-6}$	$0/3.8 \times 10^{-6}$.004	.02
0/40/.0001	$1.3 \times 10^{-2}/.36$	$2.2 \times 10^{-3}/2 \times 10^{-2}$.06	.76
0/100/.0001	$0/4.3 \times 10^{-6}$	$0/4.4 \times 10^{-7}$.0007	.02
0/40/.001	$2.4 \times 10^{-2}/6.6 \times 10^{-2}$	$2.3 \times 10^{-2}/3.5 \times 10^{-2}$	-.03	.25
0/60/.001	$3.4 \times 10^{-3}/8.1 \times 10^{-3}$	$3.4 \times 10^{-3}/7.2 \times 10^{-3}$	-.003	.11
0/40/.01	$6.1 \times 10^{-3}/7.2 \times 10^{-3}$	$6.1 \times 10^{-3}/7.1 \times 10^{-3}$.005	.08
0/60/.01	$0/7.5 \times 10^{-6}$	$0/3.9 \times 10^{-6}$.03	.02
-20/30/.0001	.21/3.8	$6.5 \times 10^{-2}/.16$.54	2.4
-20/100/.0001	$0/2.4 \times 10^{-6}$	$0/5 \times 10^{-7}$.0002	.02
-20/30/.001	$1.6 \times 10^{-2}/.9$	$6 \times 10^{-3}/4.4 \times 10^{-2}$.0004	.59
-20/60/.001	$0/4.7 \times 10^{-3}$	$0/4.4 \times 10^{-4}$	-.0003	.02
-20/30/.01	$0/3.7 \times 10^{-3}$	$0/3 \times 10^{-4}$	-.0001	.02
-20/60/.01	$0/1.5 \times 10^{-6}$	$0/2.7 \times 10^{-7}$.0001	.01

$H_{i_{max}}$, as estimated by simulations. In order to evaluate the performance of the jointly Gaussian approximation for the $q(i)$ with simulations, we compare $q(i^*)$ obtained using the two methods, rather than listing all the $\{q(i), i \neq i_{max}, i_{max} \pm 1\}$ (if the latter are all zero as estimated from simulations, we list the largest of these values according to our analytical approximation).

- 3) The bias and standard deviation of the delay estimate $\hat{\tau}_1/T_c$.

While all the results from the steady state analysis in Section V-A are for $\alpha = 0.0001$, here we illustrate the effect of varying

α on the performance of the least squares implementation. Tables III and IV list the preceding performance measures as a function of the relative interference power P_I , the number of iterations M_{LS} , and α .

For high SNR, $\alpha = .0001$ is small enough to permit suppression of very strong interference ($P_I = 20$ dB), and gives good performance for $P_I = 0$ dB as well, especially when there is no mismatch under the best hypothesis, as in Table III. Recall that the steady state results in Tables I and II show that $\alpha = .0001$ gives acceptable performance for all values of P_I considered. For the least squares implementation,

TABLE V
ERROR PROBABILITY PERFORMANCE OVER 100 SIMULATION RUNS OF
THE LEAST SQUARES IMPLEMENTATION OF THE BLIND ACQUISITION
ALGORITHM ($E_b/N_0 = 17$ dB, $\tau_1 = 3.5T_c$, $M_{LS} = 40$)

P_I (dB)/ α	P_e range (median)	κ range (mean)
20/.0001	1.2×10^{-5} - 4.7×10^{-5} (1.3×10^{-5})	.906-.998 (.984)
0/.001	7×10^{-7} - 7.9×10^{-2} (1.6×10^{-5})	.203-.996 (.785)
0/.01	1.4×10^{-5} - 2.5×10^{-4} (2.2×10^{-5})	.769-.996 .915
-20/.001	1.1×10^{-11} -.43 (9.3×10^{-5})	.012-1.0 (.541)
-20/.01	1.2×10^{-11} - 2.0×10^{-3} (5.9×10^{-10})	.42-.997 (.856)

however, Tables III and IV show that, for smaller P_I , the acquisition error probability and the delay bias and standard deviation are better for larger values of α , such as .001 and .01. This might be because, for a fixed number of least squares iterations, decreasing P_I makes the empirical cross correlation matrix $\hat{\mathbf{R}}$ more ill conditioned. Further, for fixed α , decreasing P_I amounts to decreasing ν . Thus, the noise enhancement due to inverting $\hat{\mathbf{R}} + \nu\mathbf{I}$ for small M_{LS} appears to be excessive for $\alpha = 0.0001$. If the least squares implementation is run for long enough with $\alpha = .0001$, reliable acquisition is attained for all values of P_I (see the entries for $M_{LS} = 100$), which is consistent with the steady state analysis. However, larger values of α of .001 and .01 produce reliable acquisition much more quickly for P_I of 0 and -20 dB. Of course, these values of α are not universally good either, since they can be shown to cause acquisition errors even in steady state for P_I of 20 dB. If α is chosen appropriately, reliable acquisition is obtained within $M_{LS} = 40$ iterations for $\tau_1 = 3.5T_c$ and within $M_{LS} = 60$ for $\tau_2 = 3.25T_c$.

Comparing Tables XIII and IX in Appendix B with Tables III and IV, it is interesting to note that the automatic choice of α via the algorithm presented in Appendix B does eliminate values of α that provide poor performance for a given value of P_I . However, because of the bias of the algorithm toward smaller values of α (in order to permit more interference suppression), it need not select the value of α that provides the best performance for a given P_I , especially for small or moderate P_I (which requires a large α to optimize performance). Thus, we see from Table IV that $\alpha = .01$ works better for $P_I = 0$ dB, but the results in Table IX show that $\alpha = .001$ is selected much more often in this case.

Comparing Tables III and IV, note that, for the same number of least squares iterations, the fact that there is mismatch even under the best hypothesis for $\tau_1 = 3.25T_c$ leads to a larger acquisition error probability and to a larger bias and standard deviation for the delay estimate, even though the steady state performance for this delay is better (see Tables I and II). The

TABLE VI
ERROR PROBABILITY PERFORMANCE OVER 100 SIMULATION RUNS OF
THE LEAST SQUARES IMPLEMENTATION OF THE BLIND ACQUISITION
ALGORITHM ($E_b/N_0 = 17$ dB, $\tau_1 = 3.25T_c$, $M_{LS} = 40$)

P_I (dB)/ α	P_e range (median)	κ range (mean)
20/.0001	8.2×10^{-9} - 1.2×10^{-5} (1.3×10^{-8})	.807-.998 (.965)
0/.001	7.7×10^{-10} - 2.5×10^{-2} (2.5×10^{-7})	.283-.998 (.809)
0/.01	2×10^{-8} - 1.9×10^{-6} (5.4×10^{-8})	.849-.972 .915
-20/.001	2.5×10^{-14} -.38 (1.8×10^{-5})	.020-.998 (.541)
-20/.01	2.2×10^{-14} - 2.7×10^{-5} (2.4×10^{-12})	.536-.996 (.872)

performance quickly improves as M_{LS} increases, especially for values of α that are well matched to P_I .

The analytical prediction based on the jointly Gaussian approximation is seen to be always larger than the acquisition error probability directly estimated from simulations. The match is better for $\tau_1 = 3.25T_c$. For $\tau_1 = 3.5T_c$, the analytical estimate is sometimes larger by several orders of magnitude. However, even in this case, because the acquisition error probability decreases so rapidly with M_{LS} , the analytical approximation is still a good tool for conservative design; e.g., for $P_I = 20$ dB, $\alpha = .0001$, and a desired $q_{e,acq}$ of 10^{-3} , simulations show that $M_{LS} \approx 30$ should suffice (see Table III), while the analytical approximation would lead to $M_{LS} \approx 35$. Of course, if very low acquisition error probabilities are desired, then the analytical approximation provides the only possible design approach, since direct simulations would be too time consuming.

C. Error Probability Performance

The preceding results show that, for high SNR and an appropriate choice of α , our method quickly provides a good delay estimate. We now evaluate the average error probability P_e of the demodulator \mathbf{c}_{out} produced by the algorithm after M_{LS} least squares iterations. The error probability is evaluated analytically after each simulation run. For each run, we also evaluate the cosine κ of the angle between \mathbf{c}_{out} and \mathbf{c}_{MMSE} . The range and median of P_e (the mean would be weighted too heavily by outliers), and the range and mean of κ are presented in Tables V and VI, which should be compared with the steady state results in Section V-A. As in the previous section, we restrict attention to E_b/N_0 of 17 dB, and vary P_I , keeping $M_{LS} = 40$. For each value of P_I , we choose the values of α determined by results in the previous subsection (and automatically chosen by the algorithm described in Appendix B) to give better acquisition performance. Fewer (100) simulation runs are used here, due to the complexity of the error probability computation.

TABLE VII

ERROR PROBABILITY PERFORMANCE FOR SELECTED SETTINGS. COMPUTED USING 100 SIMULATION RUNS OF THE LEAST SQUARES IMPLEMENTATION OF THE BLIND ACQUISITION ALGORITHM ($E_b/N_0 = 17$ dB, $M_{LS} = 80$)

P_I (dB)/ $\alpha/\tau_1/T_c$	P_e	κ
	range (median)	range (mean)
0/.001/3.5	7×10^{-7} - 5.6×10^{-4} (2.2×10^{-6})	.628-.996 (.910)
0/.001/3.25	7.7×10^{-10} - 1.5×10^{-4} (1.4×10^{-8})	.636-.998 (.909)
-20/.001/3.5	1.1×10^{-11} - 1.7×10^{-2} (3.9×10^{-8})	.307-1.000 (.752)
-20/.001/3.25	2.2×10^{-14} - 7×10^{-3} (1.6×10^{-8})	.347-1.000 (.725)

TABLE VIII

FRACTION OF TIMES EACH VALUE OF α IS CHOSEN BY THE MODIFIED ALGORITHM ($E_b/N_0 = 17$ dB, $\tau_1 = 3.5T_c$, $M_{LS} = 40$)

P_I (dB)	f_1	f_2	f_3	f_4
20	1	0	0	0
0	0	.861	.139	0
-20	0	0	1	0

The results for P_I of 0 dB and -20 dB show that it is important to choose α large enough to prevent excessive signal suppression and noise enhancement in these situations. In particular, the largest bit error probability for $\alpha = .001$ and $P_I = -20$ dB is unacceptable. Even when α is chosen to optimize performance, there is a large variation in bit error probabilities over different runs. Nevertheless, for P_I of at least 0 dB (which is of most interest in a CDMA system, where there are at least a few interferers with strength comparable to that of the desired transmission), the scheme quickly provides a detector with performance far superior to that of the matched filter.

For P_I of 0 dB and -20 dB, since the performance for $\alpha = .001$ is unsatisfactory for $\alpha = 0.001$, we have tried a larger number of least squares iterations, $M_{LS} = 80$, in these cases. The results are shown in Table VII. While the performance for $P_I = 0$ dB improves significantly, the performance for $P_I = -20$ dB remains poor. Running the algorithm for selecting α in this setting, we have found that the likelihood of choosing $\alpha = 0.001$ is small (a few percent) but nonzero. One possibility for rectifying this (if some estimate of received power level relative to background noise were available) might be to bias the algorithm for choosing α toward higher values when the received power level is lower.

VI. CONCLUSIONS

We have presented a blind interference suppression scheme for joint acquisition and demodulation, which requires knowledge only of the spreading sequence of the desired trans-

TABLE IX

FRACTION OF TIMES EACH VALUE OF α IS CHOSEN BY THE MODIFIED ALGORITHM ($E_b/N_0 = 17$ dB, $\tau_1 = 3.25T_c$, $M_{LS} = 40$)

P_I (dB)	f_1	f_2	f_3	f_4
20	1	0	0	0
0	0	.958	.042	0
-20	0	.052	.948	0

mission. The performance of the scheme is illustrated via a steady state analysis and simulations of a least squares implementation. Based on our numerical results, we make the following observations.

- 1) Given that α , and hence the fictitious noise variance ν , is chosen appropriately, the algorithm chooses the correct timing hypotheses with a fairly small number of iterations for a wide range of interference powers. The acquisition algorithm is near-far resistant, so that it can operate in situations in which conventional acquisition methods are useless.
- 2) If the correct (coarsely quantized) delay hypotheses are chosen, the combining rule for interpolating between these hypotheses produces an excellent delay estimate, which eliminates the need for "pull-in" using a code tracking loop. The interpolation also produces a near-far resistant demodulator which is close to the MMSE solution, and outperforms the conventional matched filter (which assumes perfect knowledge of timing) when the interference levels are significant, including the situation of perfect power control.
- 3) A Gaussian approximation used to estimate the acquisition error probability is found to be consistently conservative in regimes where acquisition error probabilities are large enough to be directly estimated by simulation. Application of this approximation shows that the acquisition error probability drops rapidly with the number of least squares iterations when the SNR is high enough. For low SNR's and high relative interference powers, noise enhancement due to linear interference suppression causes acquisition errors regardless of the number of iterations. These errors can be predicted using the steady state analysis.
- 4) The biggest disadvantage of the algorithm presented here is its sensitivity to the choice of ν . One possible method for automatically choosing ν is given, but it is not completely satisfactory because of its high complexity. An analogous training based method for joint acquisition and demodulation [6], [7] does not suffer from such sensitivity to the choice of algorithm parameters.

In future work, it is of interest to seek lower complexity methods for automating the choice of ν . Extending the range of operation of the algorithm to lower SNR's is another problem that needs to be addressed. Finally, while there are now a number of low-complexity methods for interference suppression in CDMA systems, extensive performance evaluation of these schemes in a typical time-varying wireless environment

is required to quantify the trade-offs involved in designing a functioning system based on these ideas.

APPENDIX A

COMBINING RULE FOR INTERPOLATING BETWEEN HYPOTHESES

We supply here the details of obtaining the best interpolation between the nominals for the best two hypotheses. Simplifying the notation, consider two nominals $\hat{\mathbf{u}}_0 = \mathbf{x}_1$ and $\hat{\mathbf{u}}_1 = \mathbf{x}_2$, and consider an interpolated nominal $\mathbf{z} = \mathbf{z}(\lambda) = \lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$, where $\lambda \in [0, 1]$. The normalized MOE for this nominal is given by

$$\xi(\lambda) = \frac{\mathbf{z}^T(\mathbf{R} + \nu\mathbf{I})^{-1}\mathbf{R}(\mathbf{R} + \nu\mathbf{I})^{-1}\mathbf{z}}{\mathbf{z}^T\mathbf{z}}$$

where \mathbf{R} denotes the statistical correlation matrix, and is replaced by $\hat{\mathbf{R}}$ for a least squares implementation. We want to maximize $\xi(\lambda)$ over $\lambda \in [0, 1]$.

The numerator and denominator of $\xi(\lambda)$ are quadratics in λ , whose coefficients already need to be evaluated when running the acquisition algorithm. Differentiating ξ with respect to λ and simplifying, we find that the unconstrained extrema of $\xi(\lambda)$ solve a quadratic equation. Denoting by λ_1 and λ_2 the solutions to the quadratic equation, choose λ_{max} to be the value that maximizes $\xi(\lambda)$ over $\lambda = 0, 1$ and either of λ_1, λ_2 that fall in $[0, 1]$. Thus, evaluating λ_{max} involves a comparison of at most four real numbers. The demodulator \mathbf{c}_{out} can then be obtained using (25).

Obtaining the delay estimate requires expressing the nominals \mathbf{x}_1 and \mathbf{x}_2 as linear combinations of shifts of the spreading sequence. Suppose, without loss of generality, that $\mathbf{x}_1 = \frac{T_R^n \mathbf{a}_1}{\|T_R^n \mathbf{a}_1\|}$ and $\mathbf{x}_2 = \frac{T_R^n \mathbf{a}_1 + T_R^{n\pm 1} \mathbf{a}_1}{\|T_R^n \mathbf{a}_1 + T_R^{n\pm 1} \mathbf{a}_1\|}$. Then the interpolated nominal is written as $\mathbf{z}(\lambda_{max}) = aT_R^n \mathbf{a}_1 + bT_R^{n\pm 1} \mathbf{a}_1$, where

$$a = \frac{\lambda_{max}}{\|T_R^n \mathbf{a}_1\|} + \frac{1 - \lambda_{max}}{\|T_R^n \mathbf{a}_1 + T_R^{n\pm 1} \mathbf{a}_1\|}, b = \frac{1 - \lambda_{max}}{\|T_R^n \mathbf{a}_1 + T_R^{n\pm 1} \mathbf{a}_1\|}$$

and the delay estimate is given by

$$\hat{\tau}_1 = n \pm \frac{b}{a+b}$$

APPENDIX B

CHOOSING α AUTOMATICALLY

We describe here a modified algorithm in which the MOE detectors under each hypothesis are computed as in (21) for m different values of α , so that there are m parallel algorithms using values of ν given by $\nu_p = \alpha_p \text{trace}(\hat{\mathbf{R}})$, $1 \leq i \leq m$. This is admittedly complex, since it requires m matrix inversions (or m RLS algorithms running in parallel). If m is large, it might even be easier to compute the m inverses based on an eigendecomposition of $\hat{\mathbf{R}}$, which would make the complexity comparable to that of the MUSIC method in [2], [15].

The algorithm chooses between the values of ν based on the following reasoning. In general, it is preferable to use the smallest possible value of ν , since this yields better interference suppression and a better approximation to the MMSE detector. However, small ν permits a large detector energy and more noise enhancement, which becomes especially

significant when the interference is weak and the detector energy is expended mainly in suppressing the desired signal. Since the detector energy required to suppress the interference should be of the order of $\beta_I = 1/\eta_I$ (see Section II-C), near-far resistance should still be obtained if the maximum allowable value of β is of the order of β_I . This leads to the following modification to the algorithm in Section III-B, based on constraining the maximum detector energy to be at most β_{max} , where β_{max} is chosen large enough to allow adequate interference suppression.

For each value of $\alpha = \alpha_p$, $1 \leq p \leq m$, compute the correlators $\mathbf{c}^{(i)}$ using (21). For $\alpha = \alpha_p$, define $\beta_p^{(max)} = \max_i \frac{\|\mathbf{c}^{(i)}\|^2}{\langle \mathbf{c}^{(i)}, \hat{\mathbf{u}}_0^{(i)} \rangle^2}$ as the largest detector energy among all hypotheses. Choose $\alpha = \alpha^*$ to be the *smallest* α_p such that $\beta_p^{(max)} < \beta_{max}$ (set $\alpha^* = \max_p \alpha_p$ if the latter condition does not hold for any p). Now follow Steps 1 through 4 of the algorithm using the correlators $\mathbf{c}^{(i)}$ obtained for $\alpha = \alpha^*$.

In an RLS implementation, it might be possible to use a single value of ν instead of running m parallel RLS algorithms, and to adapt this value based on the detector energies produced by the algorithm. We leave the study of such modifications for future work.

While we did not perform extensive simulations with this modified algorithm due to its complexity, its effectiveness is illustrated by the following example. Consider $m = 4$, with $\alpha_1 = 0.0001$, $\alpha_2 = 0.001$, $\alpha_3 = 0.01$, and $\alpha_4 = .1$. For $M_{LS} = 40$ and $\beta_{max} = 10$ (corresponding to 10 dB noise enhancement), we present in Tables VIII and IX the fraction f_p of 1000 simulation runs in which α_p is chosen, as a function of P_I . Note that the values of α chosen decrease with increasing P_I , as they should. Referring back to Tables V and VI, we see that the values of α chosen for a given value of P_I are the ones that give good performance for that interference power level. When the algorithm for choosing α is applied in steady state, however, the choice is $\alpha = \alpha_1 = .0001$ for all values of P_I . It appears, therefore, that least squares estimation errors cause a larger value of detector energy β than would be obtained in steady state, causing larger values of α to be chosen by the algorithm.

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