Blind Correction of Gain and Timing Mismatches for a Two-Channel Time-Interleaved Analog-to-Digital Converter

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***Abstract*—A time-interleaved analog-to-digital converter (TIADC) structurally provides faster data conversion, but its spectral performance is limited by gain and timing mismatches between sub-converters. In this paper, we propose a novel blind method for TIADC mismatch correction. Assuming a wide-sense stationary (WSS) input, we adjust the reconstruction system until the entire system restores shift-invariance, yielding a WSS output. We also prove that this input-output WSS condition is sufficient as well as necessary for a two-channel TIADC mismatch correction. Supportive simulation results are included. The proposed method can be used for online calibration of a TIADC without interruption or any dedicated signal.**

I. Introduction

A time-interleaved analog-to-digital converter (TIADC) has a parallel structure where the input signal is cyclically sampled by a number of analog-to-digital converter (ADC), and the digital output is similarly taken to re-align the signal stream. The overall sampling rate is therefore multiplied by the number of ADC's. A TIADC naturally finds its application in wide-band electronic systems such as radar, direct digital receivers, base-station receivers, and high-speed instrumentation.

One of the unique features of time-interleaving data conversion is its low sensitivity to process technologies (except sample-and-hold circuitry). Its performance is, however, highly sensitive to mismatches in electrical characteristics between sub-ADCs (e.g., dc offset, gain, sampling time, etc) [1]-[5], [7], [8]. Such mismatches periodically modulate the input signal, and creates spurious signal, corrupting signal-to-noise ratio. Tunable analog circuitry can correct mismatches, although this is generally subject to drift and process variation. Digital signal processing, on the other hand, provides a reliable and flexible way of mismatch correction. It is also becoming increasingly inexpensive due to the continued scaling of process technologies.

The authors demonstrated an offline method of digital mismatch correction [3]. Offline calibration provides superior accuracy due to the dedicated characterization setup. However, it is not a suitable correction method when mismatches are time varying, or when the system interruption is not allowed, for example. Under these special circumstances, we have to find another way of mismatch correction with the system continuously working on data conversion. A "blind" method serves this need in that no special calibration signal or system stoppage is required [4]-[5], [7]-[8], and therefore complements the offline calibration methods. The present paper proposes a novel correction method which is blind as long as the input signal is wide-sense stationary (WSS). The proposed method is more comprehensive than previous efforts since we do not rely on special distribution or bandwidth restriction of the input signal, other than WSS property and Nyquist sampling criterion. It is also novel that gain and sampling time mismatches are incorporated within a common framework of parameterized filter banks. Based on the cyclic spectral density representation of wide-sense cyclostationary (WSCS) signals, we give a proof that the proposed algorithm always achieves mismatch correction for a two-channel TIADC, which is the first in the literature to the authors' knowledge.

II. Background and Problem Description

Fig.1 (a) shows a two-channel TIADC system. Individual A/D converters have $2T_S$ of conversion time so that the aggregate sampling rate is $f_S\left(=1/T_S\right)$. Each channel has respective gain (G_i) and timing error (Δt_i). In practice, sub-converters have different dc offsets, but we assume they are independently compensated. The input $x(n)$ is assumed bandlimited from dc to $f_S/2$. Fig.1 (b) is an equivalent system where quantization effects are ignored. A simple transform yields a normalized system in Fig.1 (c). If we regard $x'(n)$ as the TIADC input, then the upper channel becomes error-free, and it is immediately seen only relative errors ($G \equiv G_1/G_0$, $\Delta t \equiv \Delta t_1 - \Delta t_0$) are relevant. This normalization is justified when raw mismatches are relatively small, or when we are not interested in the change of the output signal in absolute timing and magnitude. Whenever $G \neq 1$ or $\Delta t \neq 0$,

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the TIADC effectively modulates the input. Modulation sidebands (i.e., "aliasing spurs") are then produced at the output, limiting the maximum signal-to-noise plus distortion ratio (SNDR) and spurious-free dynamic range (SFDR) achievable.

Once mismatches are estimated or measured, a reconstruction system can be cascaded [Fig.1 (d)] for mismatch correction so that the aliasing spurs no longer limit the spectral performance. In standard offline calibration methods [3], we apply known signals, and observing the output enables us to characterize channel mismatches. Our objective is, however, to estimate and correct the mismatches without explicit (in a deterministic sense) knowledge of the input $x(n)$. We first note the key fact that a TIADC system is linear time-invariant (LTI) if and only if it is aliasing-free, and linear periodically time-varying (LPTV) otherwise [9]. Thus, an equivalent statement of our goal is, to design the reconstruction system so that the overall TIADC system regains LTI property, without explicitly knowing the input. It is also known that, for every WSS input, an LTI and LPTV system yields a WSS and WSCS output, respectively. This is basic motivation for the proposed blind correction method which can be stated as follows: *assuming $x(n)$ is a zero-mean WSS random process, design reconstruction filters so that the output $y(n)$ also becomes WSS*. We emphasize that the sufficiency of this input-output pairwise WSS condition remains to be proven. Equivalently, *we need to check if this pairwise WSS condition guarantees that the entire TIADC system is LTI*. This check is practically important due to the undesirable possibility of *false correction*, i.e., both input and output are WSS, but with nonzero residual mismatches.

We discuss about the parameterized filter bank as a reconstruction system in Section III. Characterization of WSCS processes is discussed in Section IV. The proposed algorithm is described along with the sufficiency check in Section V.

III. PARAMETERIZED FILTER BANK

There can be several realizations of the reconstruction system in Fig.1 (d) [4]-[8]. Although their signal processing is all equivalent, i.e., mismatch correction, it is noted that gain and timing mismatches traditionally have been individually equalized. In this paper, we employ a parameterized filter bank for unified treatment of gain and timing errors. The filter bank representation also provides convenient framework for the sufficiency check of the pairwise WSS condition, as will be seen in Section V.

A two-channel TIADC is equivalent to a filter bank in Fig.2 with the choice of an analysis bank, $H_0(\omega)=1$, and $H_1(\omega)=Ge^{-j\omega(\Delta t-T_s)}$, for $|\omega|<\omega_s/2=\pi/T_s$ (Undersampling switches in Fig.1 (d) is equivalent to the cascade of two-fold decimator and interpolator in Fig.2). The alias component (AC) matrix for the analysis ($H_i(\omega)$'s) and synthesis ($F_i(\omega)$'s) bank is then defined by [9]

$$\mathbf{H_{AC}}(\omega)=\begin{pmatrix} H_0(\omega) & H_1(\omega) \\ H_0(\omega-\omega_s/2) & H_1(\omega-\omega_s/2)\end{pmatrix}=\begin{pmatrix} 1 & Ge^{-j\omega(\Delta t-T_s)} \\ 1 & Ge^{-j(\omega-\omega_s/2)(\Delta t-T_s)}\end{pmatrix}, \quad (0\le\omega<\omega_s/2) \qquad (1)$$

$$\mathbf{F_{AC}}(\omega)=\begin{pmatrix} F_0(\omega) & F_1(\omega) \\ F_0(\omega-\omega_s/2) & F_1(\omega-\omega_s/2)\end{pmatrix}.$$

It can be shown that the perfect reconstruction condition for this two-channel filter bank [9], in terms of AC matrices, is given by

$$\mathbf{H_{AC}}(\omega)\mathbf{F^T_{AC}}(\omega)=2\mathbf{I}.$$

It follows that

$$\mathbf{F_{AC}}(\omega,G,\Delta t)=2\mathbf{H^{-T}_{AC}}(\omega,G,\Delta t). \qquad (2)$$

where $\mathbf{I}$ is a 2×2 identity matrix. The dependence on mismatch parameters is explicitly shown for clarity. The first row of $\mathbf{F_{AC}}$, $(F_0(\omega)\ F_1(\omega))$, completely specifies the reconstruction system required for gain and timing mismatch correction. The AC matrix form in (2) is however needed for the statistical characterization of the filter bank output as will be seen in the next section. In practice, the actual mismatch parameters $(G,\Delta t)$ are unknown, and we instead rely on the estimation $(\widetilde{G},\Delta\widetilde{t})$ to calculate the reconstruction filters by (2).

IV. CHARACTERIZATION OF CYCLOSTATIONARY PROCESSES

The characterization of WSCS processes is central to the present paper, and is briefly reviewed in this section following the convention in the literature [10]-[11]. The autocorrelation function of a real-valued zero-mean random process $x(n)$ is given by $R_x(n,n')=E[x(n)x(n')]$. If $x(n)$ is WSS, the autocorrelation function, by definition, only depends on a time lag, such that

$$\text{WSS}: R_x(u)=R_x(n+u,n) \quad \text{for all } n. \qquad (3)$$

We note again that the output of an LTI system (e.g., mismatch corrected TIADC), with a WSS input, is always WSS. On the other hand, the output autocorrelation out of an LPTV system (e.g., mismatch uncorrected TIADC) features periodic shift-dependence, such that

$$\text{WSCS}: R_x(n+M,n'+M)=R_x(n,n') \quad \text{for all } n. \qquad (4)$$

Equation (4) defines WSCS random processes with period *M*.

Since $R_x(n+u,n)$ is periodic with respect to the n, its Fourier series coefficient can be obtained by

$$\text{WSCS}: R_x^{\alpha}(u) = \frac{1}{M}\sum_{k=0}^{M-1} R_x(k+u,k)\, e^{-j2\pi\alpha k} \tag{5}$$

where $\alpha \in \{0, 1/M, \ldots, (M-1)/M\}$ has a physical interpretation of frequency. Each coefficient is a function of the time lag, u, which suggests we can define a spectral density for each $R_x^{\alpha}(u)$ as follows.

$$\text{WSCS}: S_x^{\alpha}(\omega) = \sum_{u=-\infty}^{+\infty} R_x^{\alpha}(u)\, e^{-j\omega u} \tag{6}$$

$R_x^{\alpha}(u)$ and $S_x^{\alpha}(\omega)$ are called the cyclic correlation function and cyclic spectral density of $x(n)$, respectively, and either of one for all α completely characterizes a WSCS process. In the special case when $x(n)$ is WSS, only $R_x^0(u)$ and $S_x^0(\omega)$ are nonzero, and they reduce to the conventional autocorrelation function and spectral density for a WSS process.

For the M=2 filter bank in Fig.2, cyclic spectral density matrices for the input($x(n)$) and output ($y(n)$) is defined as follows [11], where $x(n)$ is assumed WSS.

$$\mathbf{S_x}(\omega) = \begin{pmatrix} S_x^0(\omega) & 0 \\ 0 & S_x^0(\omega-\omega_s/2) \end{pmatrix}, \quad \mathbf{S_y}(\omega) = \begin{pmatrix} S_y^0(\omega) & S_y^{1/2}(\omega) \\ S_y^{-1/2}(\omega-\omega_s/2) & S_y^0(\omega-\omega_s/2) \end{pmatrix}$$

It is noted these matrices has a diagonal form for a WSS signal. $\mathbf{S_y}(\omega)$ can be written by $\mathbf{S_x}(\omega)$ and AC matrices [11] as,

$$\mathbf{S_y}(\omega) = \frac{1}{4}\left(\mathbf{F_{AC}}\mathbf{H_{AC}^T}\right)\mathbf{S_x}(\omega)\left(\mathbf{F_{AC}}\mathbf{H_{AC}^T}\right)^{\mathbf{H}} \tag{7}$$

where the frequency and mismatch parameter dependency of AC matrices is suppressed for simplicity. $(\cdot)^*$ and $(\cdot)^{\mathbf{H}}$ denote complex conjugate, and complex conjugate transpose, respectively.

Cyclic correlation functions or cyclic spectral densities provide a convenient measure of how a given signal is close to being WSS or WSCS, which is exploited by the proposed algorithm described next.

V. Algorithm Description and Sufficiency Check

Referring to Fig.1 (d), let $x(n)$ be the TIADC input which is WSS, and $y(n)$ be the output of the reconstruction system designed by (2) with estimated parameters $(\tilde{G}, \Delta\tilde{t})$. Then, the best estimation $(\tilde{G}_{opt}, \Delta\tilde{t}_{opt})$ can be obtained by minimizing the following error measure.

$$J(\tilde{G}, \Delta\tilde{t}) \equiv \sum_{\text{for all } u}\ \sum_{\text{for all } \alpha\neq 0} \left(R_y^{\alpha}(u)\right)^2. \tag{8}$$

When $(\tilde{G}_{opt}, \Delta\tilde{t}_{opt})$ is equal to $(G, \Delta t)$, which is the desired case, $y(n)$ becomes WSS, and J is identically zero. We now want to answer this question: Is there any other $(\tilde{G}, \Delta\tilde{t}) \neq (G, \Delta t)$ which will also yield zero error measure? We first assume J is zero, i.e., pairwise WSS condition, with both $\mathbf{S_x}(\omega)$ and $\mathbf{S_y}(\omega)$ being a diagonal matrix. For simplicity, let $\mathbf{H_{AC}} = \mathbf{H_{AC}}(\omega, G, \Delta t)$, and $\tilde{\mathbf{H}}_{\mathbf{AC}} = \mathbf{H_{AC}}(\omega, \tilde{G}, \Delta\tilde{t})$. With the parameterized reconstruction filter bank $\mathbf{F_{AC}} = 2\tilde{\mathbf{H}}_{\mathbf{AC}}^{-\mathbf{T}}$, (7) can be rewritten as $\tilde{\mathbf{H}}_{\mathbf{AC}}^{\mathbf{T}}\mathbf{S_y}(\omega)\tilde{\mathbf{H}}_{\mathbf{AC}}^{*} = \mathbf{H_{AC}^T}\mathbf{S_x}(\omega)\mathbf{H_{AC}^*}$ for $|\omega| < \omega_s/2$. This, in turn, can be cast into the form,

$$\mathbf{C}(\omega)\mathbf{x}(\omega) = \mathbf{0}, \tag{9}$$

where $\mathbf{C}(\omega)$ is a 4×4 coefficient matrix, and $\mathbf{x}(\omega) = \begin{pmatrix} S_x^0(\omega) & S_x^0(\omega-\omega_s/2) & S_y^0(\omega) & S_y^0(\omega-\omega_s/2)\end{pmatrix}^{\mathbf{T}}$. Note the implicit constraint from physical reasoning: elements of $\mathbf{x}(\omega)$ are nonnegative, and at least one of them is nonzero at some frequency. Any combination of $(\tilde{G}, \Delta\tilde{t})$ and $(G, \Delta t)$ will result in the pairwise WSS condition, as long as it supports a nontrivial null space vector of $\mathbf{C}(\omega)$ with the above constraint. It can further be shown that, the only possible combination is $\tilde{G} = \pm G$ and $\Delta\tilde{t} = \Delta t$, provided $|\Delta t|, |\Delta\tilde{t}| < T_s$. Under the small-mismatch regime, which is usually met in practice, sign ambiguity in gain is easily resolved, and timing mismatches are also smaller then the sampling interval. This proves that the accomplishment of pairwise WSS condition is indeed sufficient for mismatch correction, and therefore we are assured that there is no *false correction* for a two-channel TIADC.

The rank of $\mathbf{C}(\omega)$ in (9) is two, and its null space is spanned by any $\mathbf{x}(\omega)$ satisfying $S_y^0(\omega) = S_x^0(\omega)$, and $S_y^0(\omega-\omega_s/2) = S_x^0(\omega-\omega_s/2)$. This is a direct result of the perfect reconstruction property by a parameterized filter bank.

VI. Simulation Results

This section presents simulation results for a M=2 TIADC. Two equal-power signals are generated as a representative narrowband and wideband input.

(1) 'SINE' : Single sinusoid at frequency $0.15\omega_s$.

(2) 'WIDE' : Uniformly distributed samples filtered by

$h = [0.925 \quad -0.277 \quad -0.185 \quad 0.185]$, and further bandlimited to $0.45\omega_s$ by 5th-order Butterworth filter.

The impulse response of a raised-cosine filter with 10% excessive bandwidth, after shifted by Δt, multiplied by G, and sampled at every T_s, serves as a lower channel for M=2 TIADC in Fig.1 (d). Then, 10-bit quantization is applied throughout the simulation. The reconstruction system consists of two 51-tap finite impulse-response (FIR) filters designed by a conventional frequency-sampling method, and parameterized by $(\tilde{G}, \Delta\tilde{t})$.

Empirical autocorrelation function for a raw (uncorrected) TIADC output is first estimated by averaging over samples. Then, the autocorrelation function, after reconstruction filters, is obtained by double-sided convolution, from which $R_y^{\alpha}(u)$ follows. For the calculation of the error measure in (8), time lag from 0 to 10 is considered for all cases. The number of samples, N, for correlation estimation, is a key simulation parameter along with $(G, \Delta t)$.

First, the error measure (8) is examined on a $(\tilde{G}, \Delta\tilde{t})$ space as shown in Fig.3 (a) and (b). The actual shape depends on the input signal, but a well-defined global minimum at $G = 0.917$ and $\Delta t / T_s = 0.041$ is clearly seen in both cases.

Next, Monte Carlo run is performed for each input signal varying the sample size N from 10^2 to 10^6 (30 simulations for each sample size). Mismatch parameters are uniformly distributed within ±10% and ±5% for G and $\Delta t / T_s$, respectively. After the blind mismatch correction, the standard deviation of residual mismatches, $G / \tilde{G}$ and $(\Delta t - \Delta\tilde{t}) / T_s$ are recorded. Fig.4 (a) and (b) shows the standard deviation of gain and sampling time mismatches, respectively. Correction accuracy is seen to improve as we observe more samples. The estimation accuracy for SINE input is mainly limited by the reconstruction filter for this simulation, which explains the flattening of the deviation curve around N=10^5. The residual error, however, can be further reduced by using longer FIR filters (>51 taps). In contrast, for WIDE input, the estimation error of sample autocorrelation function is seen to dominate residual mismatches. Richer spectrum of WIDE input than SINE renders its performance more sensitive to observation sample size.

At N=10^5, residual mismatches are suppressed by ~20dB and ~50dB with WIDE and SINE input, respectively. This directly translates to the increase in SFDR or SNDR by the same amount under mismatch dominant regime.

VII. Conclusion

Blind correction methods complement offline calibration methods by providing 100% availability of data conversion and the ability to track time-varying mismatches.

This paper presented a novel blind method of mismatch correction for an M=2 TIADC, based on a parameterized filter bank and WSCS characterization of relevant signals. Assuming WSS input, the proposed algorithm corrects gain and timing errors by restoring the shift-invariance of the output autocorrelation function.

Experimental verification and detailed performance analysis is currently under way. Extension of the proposed algorithm to M>2 cases should be straightforward, but its corresponding sufficiency check seems to be an open question to the authors due to the complexity of the problem.

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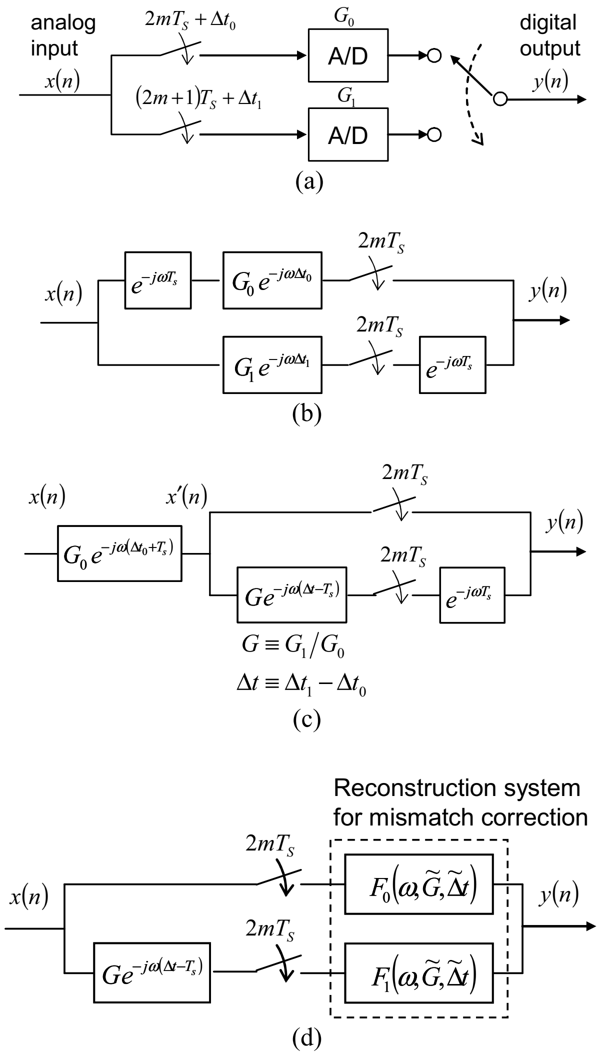

Fig.1. Two-channel TIADC system: (a) actual, (b) equivalent, and (c) normalized system. (d) Cascaded with a reconstruction system (The output delay element in (c) is absorbed in $F_1(\omega)$).

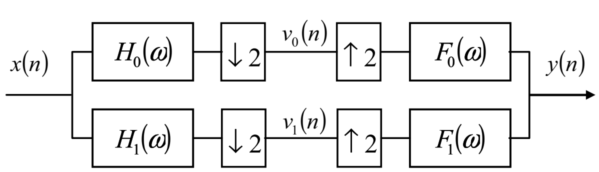

Fig.2 Two-channel filter bank (or equivalently, two-channel TIADC with a reconstruction system in Fig.1 (d)).

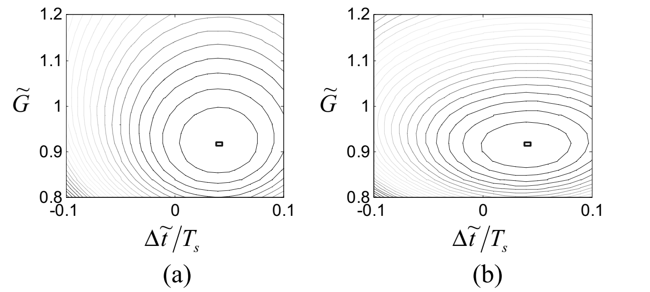

Fig.3 Error surface defined by (8) for the (a) SINE, and (b) WIDE signal (N=10^4)

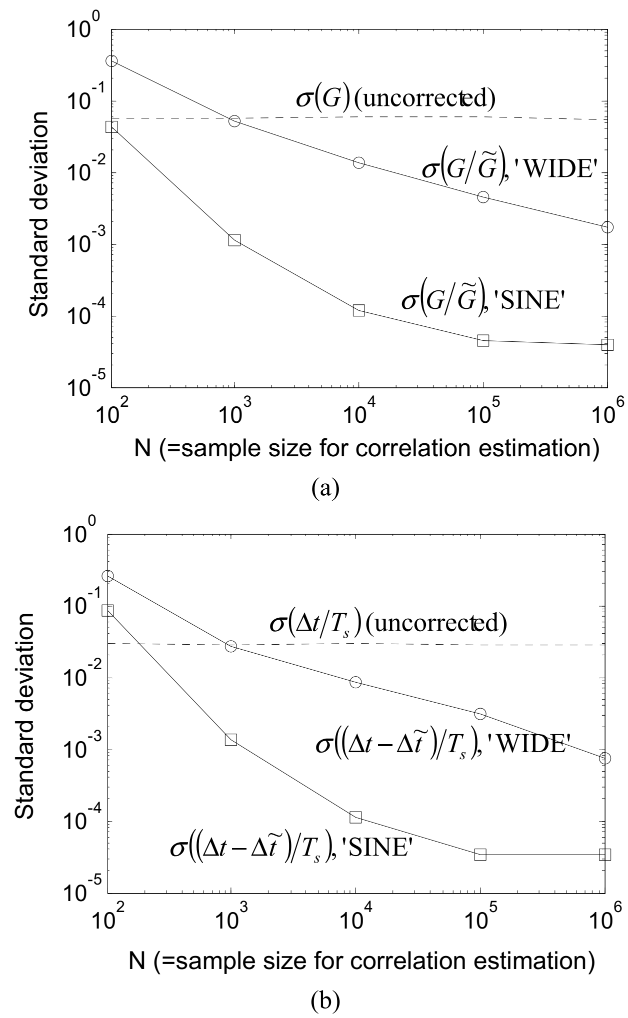

Fig.4 Monte Carlo simulation result for (a) gain, and (b) sampling time mismatch correction. Dotted and solid lines denote mismatch distribution before and after blind correction, respectively. Residual mismatches after correction are marked with a circle and square for WIDE and SINE input, respectively.