Blind Adaptive Interference Suppression for Direct-Sequence CDMA

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Invited Paper

Direct sequence (DS) code division multiple access (CDMA) is a promising technology for wireless environments with multiple simultaneous transmissions because of several features: asynchronous multiple access, robustness to frequency selective fading, and multipath combining. The capacity of DS-CDMA systems is interference-limited and can therefore be increased by techniques that suppress interference. In this paper, we present recent developments in interference suppression using blind adaptive receivers that do not require knowledge of the signal waveforms and propagation channels of the interference, and that require a minimal amount of information about the desired signal. The framework considered generalizes naturally to include additional capabilities such as receive antenna diversity. The most powerful application of the methods described here is for linearly modulated CDMA systems with short spreading waveforms (i.e., spreading waveforms with period equal to the symbol interval), for which they provide substantial performance gains over conventional reception. Implications for future system design due to the restriction of short spreading waveforms and directions for further investigation are discussed.

Keywords— *Adaptive equalizers, blind equalization, code division multiple access, direct sequence, interference suppression, multiuser detection, spread spectrum, timing acquisition.*

I. Introduction

In this introductory section, we motivate the problem considered, provide a perspective on the state of the art in this area, and discuss the new thinking in terms of system design that would be needed to exploit the relatively new techniques discussed in this paper. We end with an outline of the remainder of the paper. Our objective in this paper is not to survey the rapidly changing state of the art in this field, but to convey a basic understanding of some key concepts.

A. Motivation

We are interested in the following problem: *given multiple digitally modulated signals being heard simultaneously by a receiver, how does the receiver reliably demodulate a particular user of interest?* While this problem is fundamental to communication theory, it is currently receiving particularly intense scrutiny because of the key practical role it plays in the emerging vision of "anywhere, anytime" communications promised by systems such as personal communications, digital cellular telephony, and mobile computing. In essence, since mobile or untethered communications must be wireless, and wireless is a broadcast medium, the multiple-access interference (MAI) due to many simultaneous users is what ultimately limits performance. In this paper, we consider the problem of receiver design for a specific multiple-access technology, direct-sequence (DS) code division multiple access (CDMA). Other multiple-access techniques that are applicable to a cellular setting include combinations of time division multiple access (TDMA), frequency division multiple access (FDMA), or frequency-hop (FH) CDMA [56].

The information-bearing signal for each user in a DS-CDMA system is spread over a wider bandwidth by means of a spreading waveform unique to that user. Fig. 1 shows an example of the information signal, the spreading waveform, and the transmitted signal for a single DS transmitter. Note from the figure that the spreading waveform is determined by a spreading code or spreading sequence (hence the term CDMA); from Fig. 1(b), this is seen to be the sequence $\{1, -1, -1, 1, -1\}$.

If the spreading waveforms for different receivers as seen at the receiver were orthogonal, MAI could be eliminated. However, in practice, signals from different users arrive at the receiver at different delays, and it is not possible to make the waveforms orthogonal at all possible relative delays. Hence, the effort is to design waveforms with small expected cross correlations averaged over relative delays. Conventional system designs exploit this property of the waveforms and ignore MAI in receiver design. For instance, current implementations of the IS-95 U.S. digital cellular standard [23] use matched filter reception (which is optimal only if there is no MAI).

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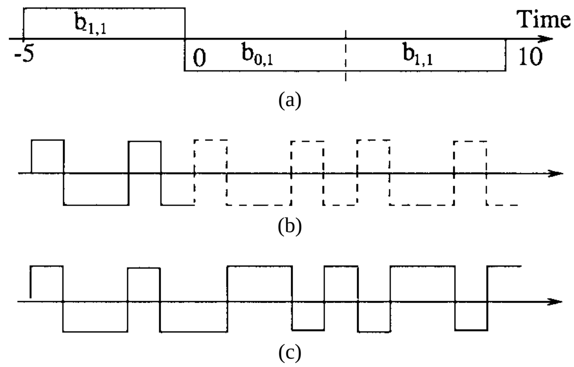

Fig. 1. Construction of a typical DS signal. (a) Information signal consisting of a symbol sequence at rate 1/5. (b) Spreading waveform (two shifted copies of the waveform are shown with dashed lines). (c) Transmitted waveform obtained by modulating spreading waveform in (b) by the symbol sequence in (a).

If the signals due to all users arrive at the receiver at roughly equal powers, the near-orthogonality of the spreading waveforms implies that conventional reception would work fairly well.[1] However, the motivation for the work reported in this paper is supplied by the following drawbacks of conventional reception.

1) *Interference floor*: Since the interfering signals are not truly orthogonal to the desired signal, the output of the conventional matched filter for the desired user contains contributions from the MAI. Thus, even if the receiver thermal noise level goes to zero, the error probability of the conventional receiver exhibits a nonzero floor because of the MAI. This makes it difficult to attain the low bit error rates required by emerging applications such as data and video using conventional reception without an excessive reduction in system capacity (i.e., in the number of simultaneous users permitted).
2) *Near–far problem*: If an interfering signal is much stronger than the desired signal, its contribution to the matched filter output for the desired signal can become large enough to make reliable reception impossible. In order to avoid this, the IS-95 system uses stringent closed loop power control, which has several disadvantages. The overhead associated with feedback-based power control may turn out to be excessive in future packet CDMA systems (IS-95 is a circuit-based system with relatively long lived connections). Further, closed loop power control requires coordination between transmitters and receivers at a level which might be difficult, for instance, in *ad hoc* wireless networks with arbitrary and/or rapidly time-varying topologies.[2]

[1] For analysis of the performance of conventional reception under such circumstances, see [41], [48], and [72] (see also [36] for fundamental limits on the performance of conventional timing acquisition methods).

[2] The term *ad hoc* networks has been recently coined for (typically multihop) wireless networks which do not conform to the cellular paradigm of a user accessing a wireline backbone via a single wireless hop to a controlling base station.

The interference floor and the near–far problem encountered by conventional reception can be alleviated or eliminated by the use of multiuser detection, a term we use generically for any receiver that, unlike conventional receivers, attempts to exploit the structure of the MAI. Thus, the term includes receivers that are only interested in reliable demodulation of a single user. See [71] for a comprehensive treatment of multiuser detection. The power of multiuser detection was first rigorously demonstrated by Verdu in [68], where it was shown, under a mild condition, that the near–far problem does not occur if optimal maximum likelihood (ML) detection is used. The complexity of implementing ML detection is exponential in the number of active users, which has motivated the invention of a number of suboptimal multiuser detectors with lower complexity, typically linear in the number of active users [11], [12], [30], [31], [66], [67], [77] (see [70] and [71] for a survey). The receiver front end assumed by all the aforementioned detectors is a bank of filters matched to the transmitted waveforms and channels of the active users, where each filter is sampled at the symbol rate based on the timing of the corresponding user. Thus, while each filter in the bank is a conventional receiver for a given user, collectively the matched filter outputs form sufficient statistics (i.e., contain all the relevant information) for making joint symbol decisions for all users [68]. A multiuser detection scheme based on this centralized front end is, therefore, simply a means of jointly processing the matched filter outputs to obtain the bit estimates for one or more users.

Implementation of the preceding front end could be cumbersome since it requires knowledge of the spreading waveforms and the propagation channels of all users, even if only one particular user is of interest. In particular, it may be difficult to obtain reliable estimates of the users' propagation channels in a typical wireless environment with impairments such as fading and MAI. Furthermore, the complexity of both the implementation of the front end and of the processing of the outputs grows with the number of users. As we shall see in Section I-B, for a certain class of DS-CDMA systems these difficulties can be overcome by using adaptive implementations of multiuser detection based on an alternative front end.

B. Adaptive Interference Suppression

In this paper, we concentrate on the class of DS-CDMA systems based on short spreading codes; the spreading codes (and hence spreading waveforms) for each user are periodic with period equal to the symbol interval. The example shown in Fig. 1 falls within this class. In this setting, the MAI seen by a given symbol of the desired user is statistically identical to that seen by the next symbol of that user, provided that the propagation channels for the users vary relatively slowly. This observation (which is made precise later in the paper) simplifies the task of multiuser detection because the receiver can now adaptively "learn" enough about the structure of the MAI to be able to suppress it. Since such adaptation is implemented using digital signal processing, the front end needed for such methods consists of a wideband filter sampled at (a multiple of) the chip rate, thus preserving the information in the continuous-time received signal. This front end is independent of the spreading waveforms or propagation channels of the interfering users. It now becomes possible to do adaptive interference suppression, which is the term we will use to describe multiuser detection schemes that do not require explicit knowledge of the parameters of the MAI.

Adaptive interference suppression is analogous to adaptive equalization of a time invariant (or slowly time-varying) channel by virtue of the analogy between MAI and intersymbol interference (ISI). Application of these methods to DS-CDMA is a relatively recent concept proposed by a number of different authors at approximately the same time, e.g., Abdulrahman *et al.* [2], Madhow and Honig [35], Miller [38], and Rapajic and Vucetic [49]. All of these authors proposed adaptive receivers based on the linear minimum mean squared error (MMSE) criterion. These receivers require only a training sequence of symbols transmitted by the desired user and a coarse knowledge of the timing of the desired user, and can be implemented adaptively using standard algorithms such as least mean squares (LMS) or recursive least squares (RLS) [16]. After the training phase, the receivers can continue to adapt in decision-directed mode, in which symbol decisions made by the receiver are fed back for further adaptation. Some basic properties of linear MMSE receivers (including their immunity to the near–far problem), as well as implementations of varying complexity, can be found in [35], while simulations of adaptive implementations over time invariant channels appear in [2], [38], and [49]. Assuming convergence of the adaptive receiver (which is easy to achieve in a time invariant setting), linear MMSE reception has been shown to provide large performance gains over conventional reception: not only does it suppress interference, but it also provides automatic multipath combining for the desired user.

The preceding schemes all assume the availability of a known sequence of symbols for the desired user in the training phase, to be followed by a decision-directed phase in which the decisions from the adaptive receiver are used for continuing adaptation. They also assume that some coarse knowledge of the timing of the desired user is available, but this assumption can be removed by using the training sequence for initial timing acquisition as well [4], [34], [53], [79]. A major hurdle that remains is to make these adaptive algorithms robust to the severe time variations typical of a wireless channel. In particular, conventional LMS or RLS adaptation does not work over rapidly fading channels either in training or in decision-directed modes [80]. However, promising results have been obtained using recently proposed modifications to LMS and RLS that exploit differential modulation to relieve the adaptive algorithm of the burden of channel tracking [20], [80].[3] Another approach to fading channels [74] is to track fading gains explicitly using periodically transmitted pilot symbols. Further modifications (typically involving decorrelation of independently faded multipath components of the desired user by using timing information, as in [22], [74], and [82]) are needed to deal with channels in which the multipath components undergo rapid independent fades, since the automatic multipath combining capability of the MMSE receiver is impaired in this setting. Finally, the performance of adaptive algorithms, taking into account transients in the interference pattern due to the arrival and departure of interfering users, has been studied in [21].

In view of the work cited in the previous paragraph, we anticipate that robust adaptive algorithms for both the training and decision-directed phases that work in the presence of severe channel time variations will soon become available. If so, such algorithms may well constitute the most practical approach to receiver design future high-capacity systems based on DS-CDMA. Decision-directed adaptation, however, is still vulnerable to sudden channel variations. For example, it could be derailed by the appearance of an extremely strong incoming interfering signal to which the receiver has not already adapted (e.g., due to a new interfering user, or due to a new multipath component appearing for an existing interfering user). Similarly, appearance and disappearance of multipath components for the desired user could affect decision-directed adaptation adversely. In order to recover from failure of decision-directed adaptation without requiring the transmitter to send a fresh training sequence, it is necessary to develop blind adaptive mechanisms that do not require knowledge (or reliable explicit estimates) of the symbol sequence of the desired user. For example, the blind receiver in [18] is used to recover from failure of decision-directed adaptation due to fading or interference transients in [20] and [21]. Blind reception is also of interest in its own right in broadcast or multicast settings, since it enables receivers to asynchronously tune in to a transmission of interest at any time. Achieving this using a training-based mechanism would require significant overhead in the form of transmission of training sequences at regular intervals.

The preceding arguments motivate the subject of this paper, namely, blind adaptive interference suppression, which at its minimum means that the receiver does not require a training sequence for the desired user (in addition to not requiring knowledge of the interference parameters).

C. Classification of Blind Receivers

We classify blind schemes into three categories according to the knowledge that the receiver assumes.

[3] In very recent work, we have identified a constrained optimization problem for differentially modulated systems whose solution is the linear MMSE detector [81]. The *ad hoc* algorithm in [80], which can be shown to provide an approximate solution to the preceding optimization problem, is just one of several adaptive algorithms that can be derived based on the preceding optimization problem.

C1) The receiver knows the timing (or more generally, the propagation channel) and spreading waveform of the desired user.
C2) The receiver knows only the spreading waveform of the desired user.
C3) The receiver does not know any information about the desired user, other than the fact that the desired signal is digitally modulated at a given symbol rate.

A receiver in category C1) was obtained by Honig *et al.* in [17] and [18], where an adaptive implementation of the linear MMSE receiver based on a constrained minimum output energy (CMOE) criterion was presented. A related approach appeared in [13] at the same time as [17], but the approach in [13] has the drawback of requiring knowledge of the spreading waveforms and propagation channels of all users. The idea behind the receiver in [18] is similar to minimum variance beamforming for adaptive antenna arrays [24], where, assuming that the direction of arrival for the desired signal is known, interference can be nulled by adapting the array to minimize the output variance, subject to the constraint of not putting a null in the direction of the desired signal. Knowing the spreading waveform and propagation channel of the desired user is analogous to knowledge of the desired user's array response in beamforming. Further discussion of the CMOE-based category C1) receiver in [18] and its extensions is provided in Section III-B1.

Our primary focus in this paper is on category C2); we consider interference suppression methods for timing acquisition and demodulation of a desired user whose spreading waveform is known. Category C3) would be classified as blind equalization for a system with one user (see [14], [25], [60], and [63]), and blind source separation for a multiuser system (see [7]). Receivers in this category are traditionally applied to narrow-band systems and array processing. In theory, such receivers apply to CDMA as well, as pointed out in Section IV. However, if additional information is available regarding the spreading waveform of the desired user, then it can be exploited to simplify implementation as well as to obtain better performance. Thus, the practical utility of category C3) receivers for CDMA applications beyond noncooperative applications such as eavesdropping is unclear.

Remark I.1: We focus in this paper on the linear MMSE receiver, since blind methods can be devised for its computation. It is worth noting, however, that a number of training-based adaptive schemes based on criteria other than the linear MMSE criterion have been considered in the literature. As shown in [9] and [40], it is also possible to obtain recursive implementations of the decorrelating detector [30], [31]. However, the implementations in [9] and [40] require (differing degrees of) knowledge of the delays and spreading waveforms of all users. It is also worth mentioning attempts to adapt (using training sequences) neural networks for multiuser detection [1], [39]. However, the neural network approach outperforms linear interference suppression only if training sequences are available for all users [1]. When a training sequence is available only for the desired user, the convergence and steady-state performance of a neural network is typically a little worse than that of a linear MMSE detector adapted using the LMS algorithm [39]. Thus, linear MMSE receivers implemented using faster LS type algorithms would be expected to perform better than a neural network implementation when training sequences for interfering users are not available.

D. System Design Considerations

Most current CDMA systems (including IS-95, as well as military DS communication systems) are based on long spreading waveforms (i.e., the period of the spreading waveform is much longer than the symbol interval). For such systems, the structure of the MAI changes from symbol to symbol. This is an advantage for systems based on conventional reception, since it prevents systematic recurrence of bad realizations of the MAI at the output of the conventional receiver. However, it is a disadvantage from the point of view of implementing multiuser detection, since the detector must be time-varying and explicit knowledge of interference parameters is required. Thus, for future high-capacity systems designed around the notion of multiuser detection, short spreading waveforms would appear to be the logical choice. Concerns regarding privacy could be overcome with suitable encryption schemes, or with periodic changes in the spreading codes used (e.g., on a packet-by-packet basis). Another concern is that, even with interference suppression, a set of relative delays and spreading waveforms that leads to poor performance could persist for a long period of time for short spreading waveforms. Mechanisms for overcoming this might include power control,[4] assignment of a new spreading waveform or new delay, or injection of slow drifts in the transmitter and receiver clocks to produce drifts in the relative delays. In short, we believe that research results in adaptive interference suppression have reached the point that a complete system design based on it is within reach (assuming application of sufficient engineering effort and ingenuity), and that such a system would provide large performance gains even over severely time-varying channels, compared to existing DS-CDMA systems based on long spreading waveforms and conventional reception.

For CDMA receivers equipped with an antenna array, further interference suppression in the spatial domain is possible. Due to lack of space, we do not discuss such techniques in any detail. However, the following comments may form a useful starting point for further exploration of this topic. The array response corresponding to a given user, which depends on its direction of arrival (DoA), can be thought of as its spatial spreading waveform. In analogy with our notion of short and long spreading sequences, the spatial spreading waveforms are "short" in the sense that variations in the array response for (a given multipath

[4] Since adaptive interference suppression schemes are near–far resistant, allowing users whose performance is poorer to send at higher powers should have only a minor effect on the performance of users who are currently enjoying good performance.

component of) a given user are typically slow relative to the symbol rate. Since different users have different DoA's (and hence different array responses), adaptive interference suppression in the spatial domain can be applied.[5]

Due to the slow variation in the array responses, adaptation in the spatial domain is possible for DS-CDMA systems with either short or long spreading sequences. For short spreading sequences, since both the array responses and the spreading waveforms are time invariant (or slowly varying) from symbol to symbol, blind or training-based adaptive interference suppression is directly applicable to the spatiotemporal received signal corresponding to the outputs of the antenna array elements over a given time interval. The spatiotemporal filters resulting from such a direct extension have a number of taps equal to the product of the number of array elements and the number of temporal taps, which leads to increased complexity and slower convergence speed. While the performance advantage of this approach is significant [35], [57], the problem of low-complexity rapid spatiotemporal adaptation is far from solved.

For DS-CDMA systems with long-spreading sequences, the fact that the array responses are time invariant (or slowly varying) implies that adaptive interference suppression can still be performed in the spatial domain, while using either conventional reception or multiuser detection (using explicit knowledge about the interference) to exploit the discrimination between users provided by the DS spreading in the time domain. From a practical point of view, spatial adaptation coupled with conventional reception appears to be the most feasible. A number of algorithms using the latter approach have been proposed recently [29], [32], [42], [76]. In particular, as shown in [32], knowledge of the desired user's spreading sequence can be used as a "training sequence" for antenna array adaptation even when the desired user's symbol sequence is unknown..

As with temporal interference suppression, the goal of most algorithms using antenna arrays for interference suppression is computation of approximations to the linear MMSE receiver.

E. Outline

Section II contains a description of the system model, showing how a continuous-time asynchronous CDMA system can be reduced to an equivalent discrete-time synchronous CDMA model for the purpose of digital signal processing. This is an important observation, thus we illustrate it in some detail via several examples. Section III describes blind interference suppression schemes which use only the second-order statistics (SOS) of the received signal. We focus mainly on category C2) receivers, since these form the main theme of the paper. We describe the geometry behind the algorithms instead of giving a detailed description of the computations involved. The algorithms described here are minor extensions of algorithms that have appeared recently in the literature [5], [33], [55], [64]. A brief discussion of SOS-based category C3) receivers is provided at the end of Section III. Section IV discusses the possible applicability of higher order statistics (HOS) for devising receivers in category C3) for CDMA applications. This includes comments on the fundamental limits of HOS methods as well as on the local minima of the popular constant modulus algorithm (CMA) [15], [62] when applied in the present context. Section V contains some concluding remarks.

[5]If the multipath components for the user have a significant angular spread, then the array response for different multipath components may be different. However, each such response would still vary slowly with time, hence making adaptation feasible.

II. System Model

We begin by illustrating key features of the system model for DS-CDMA by means of examples, followed by a succinct statement of the general system model used in the remainder of the paper.

Notation: Boldface small letters denote column vectors, while boldface capital letters denote matrices. Since we work with complex baseband system models, vectors and matrices with complex components must be considered. The row vectors $\mathbf{x}^T$ and $\mathbf{x}^H$ are the transpose and the complex conjugate transposed, respectively, of $\mathbf{x}$. The inner product, or correlation, of two vectors $\mathbf{x} = (x_1, \cdots, x_L)^T$ and $\mathbf{y} = (y_1, \cdots, y_L)^T$ of length L is defined as

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^H \mathbf{y} = \sum_{i=1}^{L} x_i^* y_i$$

where z^* denotes the complex conjugate of a scalar z. For real vectors, the preceding expression specializes to the conventional inner product $\mathbf{x}^T \mathbf{y}$. Finally, the norm $\|\mathbf{x}\|$ of a vector $\mathbf{x}$ is defined as $\|\mathbf{x}\| = \sqrt{\langle \mathbf{x}, \mathbf{x} \rangle}$.

A. Examples

We have already seen a DS user (labeled 1 in subsequent discussion) in Fig. 1. The bit rate is denoted by $1/T$, where the symbol interval $T = 5$ in the example. Fig. 1(a) shows the bit transitions: letting $b_{n,1}$ denote the bit (taking values ± 1) sent in the time $nT \leq t < (n+1)T$, we have $b_{-1,1} = +1$, $b_{0,1} = -1$, and $b_{1,1} = -1$. The DS spreading waveform $s_1(t)$ is shown in Fig. 1(b). Copies of the spreading waveform shifted by the bit interval are shown with dashed lines. The spreading waveform is constructed by modulating a chip waveform $\psi(t)$ using a spreading code, or spreading sequence. In the example, $\psi(t)$ is a rectangular pulse of duration $T_c = 1$, where T_c is the chip interval, and the spreading code $\mathbf{s}_1$ is given by

$$\mathbf{s}_1 = (1, -1, -1, 1, -1)^T.$$

Fig. 1(c) shows the modulated baseband DS signal corresponding to the multiplication of the given bit sequence and the spreading waveform. Clearly, the bandwidth of the signal is of the order of $1/T_c$, which is a factor $N = T/T_c$ higher than the information rate. The factor N is called the processing gain ($N = 5$ in the example). Typically, it is necessary for the processing gain to scale linearly with the number of active users in a CDMA system.

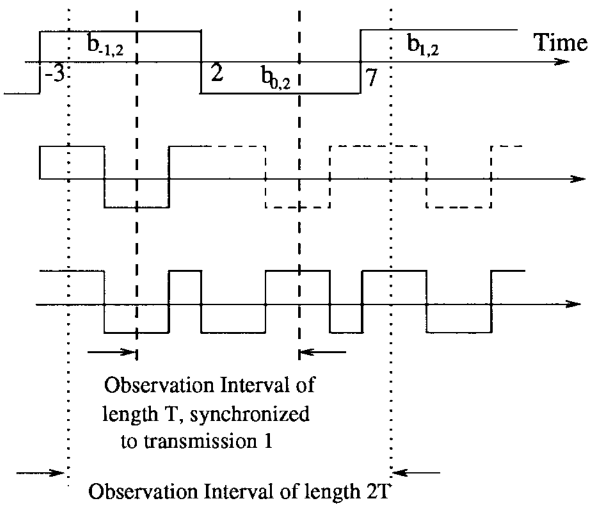

Fig. 2. The information signal, spreading sequence, and transmitted waveform for an interfering user. The interference vectors generated by the user depend on the observation interval used to demodulate the desired user.

Consider first a receiver synchronized to user 1, which obtains discrete time samples by integrating the received signal chip by chip. This corresponds to sampling the output of a filter matched to the chip pulse $\psi(t)$ at the chip rate. Our objective in this section is to model these discrete-time samples in terms of the signal and interference parameters. Such a model is the first step to designing interference suppression schemes. Consider the discrete-time samples in the interval $[0, T]$ $(T=5)$; these form a vector $\mathbf{r}$ of length five. From Fig. 1(c), the contribution to $\mathbf{r}$ due to user 1 is $(-1, 1, 1, -1, 1)^T$. While this corresponds to $b_{0,1}=-1$, in general, the contribution due to user 1 in $[0, T]$ is given by $b_{0,1}\mathbf{s}_1$. Thus, in order to implement matched filter reception for user 1, the receiver would simply correlate the vector $\mathbf{r}$ with the spreading sequence $\mathbf{s}_1$ and decide on the bit $b_{0,1}$ based on the sign of the output.

Consider now the effect of an interfering user (user 2) whose contribution to the received signal is offset from that of user 1 by $2T_c$. Fig. 2 shows the bit sequence, spreading waveform, and modulated signal due to user 2.

The spreading sequence for user 2 is $\mathbf{s}_2 = (1, 1, -1, -1, 1)^T$. From Fig. 2(c), the contribution of user 2 to the received vector over $[0, T]$ is given by $(-1, 1, -1, -1, 1)^T$. Letting $b_{n,2}$ denote the bit of user 2 that spans the interval $nT + 2T_c \le t < (n+1)T + 2T_c$, this contribution can be written as

$$b_{-1,2}\mathbf{v}_{-1,2} + b_{0,2}\mathbf{v}_{0,2}$$

where $b_{-1,2}=+1$ and $b_{0,2}=-1$ are the bits sent by user 2 that overlap with the interval $[0, T]$, and

$$\mathbf{v}_{-1,2} = (-1, 1, 0, 0, 0)^T \qquad \mathbf{v}_{0,2} = (0, 0, 1, 1, -1)^T.$$

The vector $\mathbf{v}_{-1,2}$ corresponds to the part of the spreading waveform modulating bit $b_{-1,2}$ that falls in the interval $[0, T]$. The vector $\mathbf{v}_{0,2}$ corresponds to the part of the spreading waveform modulating bit $b_{0,2}$ that falls in the same interval. These can be expressed as left and right shifts, respectively, of the spreading sequence $\mathbf{s}_2$ for user 2 as follows.

Let $\mathcal{T}$ denote the acyclic right shift operator, and let $\tilde{\mathcal{T}}$ denote the acyclic left shift operator, both operating on vectors of length n. Thus, for a vector $\mathbf{x} = (x_1, \cdots, x_L)^T$ we have $\mathcal{T}\mathbf{x} = (0, x_1, \cdots, x_L)^T$ and $\tilde{\mathcal{T}}\mathbf{x} = (x_2, \cdots, x_L, 0)^T$. We can now write $\mathbf{v}_{-1,2} = \tilde{\mathcal{T}}^3\mathbf{s}_2$ (spreading sequence for user 2, shifted left by three) and $\mathbf{v}_{0,2} = \mathcal{T}^2\mathbf{s}_2$ (spreading sequence for user 2, shifted right by two).

Overall, the vector received over the interval $[0, T]$ can be written as

$$\mathbf{r}_0 = b_{0,1}\mathbf{s}_1 + b_{-1,2}\mathbf{v}_{-1,2} + b_{0,2}\mathbf{v}_{0,2} + \mathbf{w}_0 \qquad (1)$$

where we introduce the notation $\mathbf{r}_n$ to denote the vector of samples obtained over the interval $[nT, (n+1)T]$, and where $\mathbf{w}_n$ is the contribution due to noise in the interval. The received vector $\mathbf{r}_0$ is therefore modeled as the sum of signal vectors modulated by bits, plus noise. The vector $\mathbf{s}_1$ is the desired vector modulated by the desired bit $b_{0,1}$. The vectors $\mathbf{v}_{-1,2}$ and $\mathbf{v}_{0,2}$ are interference vectors modulated by interfering bits.

1) Linear Receivers: We restrict attention to linear receivers in this paper. In the context of (1), this corresponds to generation of a decision statistic $Z_0 = \langle \mathbf{c}, \mathbf{r}_0 \rangle$ by linear correlation of the received vector. The statistic could be used for either hard or soft decisions, with the former being given by

$$\hat{b}_{0,1} = \text{sgn}(Z_0)$$

for binary signaling and real valued signals. Writing out the decision statistic using (1), we obtain that

$$\begin{aligned} Z_0 &= \langle \mathbf{c}, \mathbf{r}_0 \rangle \\ &= b_{0,1}\langle \mathbf{c}, \mathbf{s}_1 \rangle + b_{-1,2}\langle \mathbf{c}, \mathbf{v}_{-1,2} \rangle \\ &\quad + b_{0,2}\langle \mathbf{c}, \mathbf{v}_{0,2} \rangle + \langle \mathbf{c}, \mathbf{w}_0 \rangle. \end{aligned} \qquad (2)$$

For interference suppression, the contribution of the interference vectors and the noise to the correlator output [i.e., the last three terms in the right-hand side of (2)] should be small compared to that of the desired vector [the first term in the right-hand side of (2)]. This motivates the definition of the signal-to-(noise+)interference ratio (SIR) as the ratio of the energy of the signal contribution relative to that of the noise plus interference [35]. In the preceding example, the SIR is given by

$$\text{SIR} = \frac{(|\langle \mathbf{c}, \mathbf{s}_1 \rangle|^2}{|\langle \mathbf{c}, \mathbf{v}_{-1,2} \rangle|^2 + |\langle \mathbf{c}, \mathbf{v}_{0,2} \rangle|^2 + |\mathbf{c}^H \Sigma_w \mathbf{c}|^2}$$

where Σ_w is the noise covariance matrix. While the ultimate performance measure is the error probability, the SIR is easier to compute and serves as a convenient measure for comparing linear receivers. Moreover, in certain contexts, the SIR has been found to be an excellent predictor of the error probability [44].

One realization of a receiver in category C1) (see Section I) is an adaptive correlator $\mathbf{c}$ that maximizes the SIR without requiring prior knowledge of the interference vectors. To see why this could be possible, consider now the vector $\mathbf{r}_1$ of samples corresponding to the interval $[T, 2T]$. Since the delays and spreading sequences remain the same, the desired and interference vectors remain the same, but the desired and interfering bits change. We therefore obtain

$$\mathbf{r}_1 = b_{1,1}\mathbf{s}_1 + b_{0,2}\mathbf{v}_{-1,2} + b_{1,2}\mathbf{v}_{0,2} + \mathbf{w}_1. \quad (3)$$

Comparing (1) and (3), we realize that the structure of the received vectors corresponding to observation intervals offset by T is the same. In particular, a correlator that maximizes the SIR for $\mathbf{r}_0$ also maximizes that for $\mathbf{r}_1$. Indeed, this correlator maximizes the SIR in any observation interval so that, by observing the sequence of received vectors $\{\mathbf{r}_n\}$, it is plausible that we could be able to learn the best choice of correlator. One mechanism for achieving this is adaptive implementation of the linear MMSE receiver mentioned earlier.

The discussion in the previous paragraph may be made precise as follows: the continuous-time received signal is cyclostationary [47, pp. 75–76], in that its statistics do not change under time shifts by the symbol interval T. Hence, received vectors obtained from observation intervals offset by multiples of T have identical statistics, i.e., the sequence $\{\mathbf{r}_n\}$ is stationary. This stationarity is what makes adaptive reception feasible.

In the preceding example, the propagation channel for the desired user is assumed to be an ideal channel which only scales and delays the transmitted signal. The receiver is assumed to know the delay of the desired user, so that the received vector $\mathbf{r}_n$ corresponds to the nth bit interval of the desired user. In addition, adaptive implementation of the linear MMSE receiver requires a sequence of training symbols $\{b_{n,1}\}$ for the desired user, followed by decision-directed operation using the estimates $\{\hat{b}_{n,1}\}$. While the latter does provide interference suppression without explicit knowledge of interference parameters, recall that our objective is to also dispense with the need for prior information regarding the desired signal. Furthermore, we are interested in multipath channels, for which the received signal due to a given user is a sum of several weighted and delayed replicas of the transmitted signal.

The first step to dispensing with knowledge of the channel seen by the desired user is to enlarge the observation interval used for each symbol decision. In particular, consider samples in the observation interval [−2, 8] shown in Fig. 2. For each of the users shown in Figs. 1 and 2, one complete bit falls in this interval. This property holds for an observation interval of length $2T$ regardless of delays, and is key to devising blind detection schemes.[6] As before, we wish to use the received vector corresponding to this interval to decide on the bit $b_{0,1}$. Since bit transitions are spaced by T, this observation interval must be advanced by $T = 5$ in order to decide on the next bit $b_{1,1}$ of the desired user. Generalizing the notation introduced earlier, let $\mathbf{r}_n$ denote the vector of samples corresponding to the nth observation interval. For the example at hand, $\mathbf{r}_n$ denotes samples in the interval $[nT-2, nT+8]$. Proceeding as before, we can write each received vector as a sum of vectors modulated by bits, plus noise. For example, three bits of each user contribute to the zeroth observation interval [−2, 8]. Mimicking the notation in (1), we have a received vector of length ten that can be written as

$$\mathbf{r}_0 = \sum_{i=-1}^{1} b_{i,1}\mathbf{v}_{i,1} + \sum_{i=-1}^{1} b_{i,2}\mathbf{v}_{i,2} + \mathbf{w}_0 \quad (4)$$

where $\mathbf{w}_0$ now denotes a noise vector of length ten. From Fig. 1, the desired vector $\mathbf{v}_{0,1}$ (modulating the bit $b_{0,1}$ of the desired user) is given by

$$\mathbf{v}_{0,1} = (0, 0, 1, -1, -1, 1, -1, 0, 0, 0)^T = \mathcal{T}^2\mathbf{s}_1. \quad (5)$$

The interfering vectors modulating the MAI bits (due to user 2) can be read off from Fig. 2(b) as follows:

$$\begin{aligned} \mathbf{v}_{-1,2} &= (1, -1, -1, 1, 0, 0, 0, 0, 0, 0)^T = \tilde{\mathcal{T}}^1\mathbf{s}_2 \\ \mathbf{v}_{0,2} &= (0, 0, 0, 0, 1, 1, -1, -1, 1, 0)^T = \mathcal{T}^4\mathbf{s}_2 \\ \mathbf{v}_{1,2} &= (0, 0, 0, 0, 0, 0, 0, 0, 0, 1)^T = \mathcal{T}^9\mathbf{s}_2. \end{aligned} \quad (6)$$

The interference vectors $\mathbf{v}_{-1,1}$ and $\mathbf{v}_{1,1}$ modulating the ISI bits (due to user 1) are similarly obtained from Fig. 1.

For the nth observation interval, we obtain

$$\mathbf{r}_n = \sum_{i=-1}^{1} b_{i+n,1}\mathbf{v}_{i,1} + \sum_{i=-1}^{1} b_{i+n,2}\mathbf{v}_{i,2} + \mathbf{w}_n. \quad (7)$$

Thus, the signal vectors are identical to those in (4), but the indexes of the bits involved are incremented by n, so that the sequence $\{\mathbf{r}_n\}$ is stationary, as before.

2) Effect of Chip Asynchronism: In the examples thus far, we have restricted attention to a chip-synchronous system, in that the chip intervals of the two users are aligned, and the receiver knows the chip timing when generating discrete time samples. In practice, users are chip- as well as symbol-asynchronous, and the receiver does not know the chip timing for (different multipath components of) the desired user *a priori*. To model this effect, consider the observation interval [−1.5, 8.5] in which chip-spaced samples are generated by integrating over [−1.5, −0.5], [−0.5, 0.5], etc. Clearly, the form of the model (7) still applies to the received vector thus obtained, but the signal vectors change. In particular, it is easy to see from Fig. 1(b) that the vector modulating the desired bit $b_{0,1}$ is now given by

$$\mathbf{v}_{0,1} = (0, .5, 0, -1, 0, 0, -.5, 0, 0, 0)^T. \quad (8)$$

With a little thought, the preceding equation can be rewritten as

$$\mathbf{v}_{0,1} = 0.5\mathcal{T}^1\mathbf{s}_1 + 0.5\mathcal{T}^2\mathbf{s}_1. \quad (9)$$

Thus, chip asynchronism leads to a signal vector, which is a linear combination of shifts of the spreading sequence, where the coefficients of the linear combination depend on the fractional chip offset between the receiver's integration intervals and the chip timing for the corresponding user. Expressions similar to (9) can be obtained for the other signal vectors involved in (7).

[6] For a multipath channel, one complete symbol falls into each observation interval if the length of the observation interval is at least $2T$ plus the channel delay spread.

3) Sampling Faster Than the Chip Rate: Chip-spaced samples in a chip-asynchronous system can lead to significant loss in SNR, as can be seen by comparing the energies (sum of the squares of the components) of the desired vector $\mathbf{v}_{0,1}$ in (5) and (8). To roughly quantify the loss, note that (9) can be generalized to obtain a desired vector of the form

$$\mathbf{v}_{0,1} = (1-\delta)\mathcal{T}^n \mathbf{s}_1 + \delta \mathcal{T}^{n+1}\mathbf{s}_1 \tag{10}$$

when the delay of the desired bit from the left edge of the observation interval is $(n+\delta)T_c$, where n is an integer and $0 \le \delta < 1$. Assuming that the two adjacent shifts of the spreading sequence are roughly orthogonal, the relative energy of the vector compared to $||\mathbf{s}_1||^2$, the energy in the chip-synchronous case is approximately $\delta^2 + (1-\delta)^2$. When minimized over $0 \le \delta < 1$, we obtain a relative energy of $\frac{1}{2}$ (attained at $\delta = \frac{1}{2}$). Thus, for chip-rate sampling, there is a worst-case SNR loss of approximately 3 dB, corresponding to a half-chip offset between the chip timing for the received signal due to the desired user, and the receiver sampling times.

The SNR loss due to chip asynchronism can be alleviated by obtaining more samples, e.g., for an observation interval [−1.5, 8.5], use overlapping integration intervals [−1.5, −0.5], [−1, 0], [−0.5, 0.5], etc. This is equivalent to sampling the chip matched filter at twice the chip rate. An estimate of the SNR loss in this case can be obtained by considering (10) again. Sampling at twice the chip rate is equivalent to obtaining two interleaved chip-rate subsequences. While the noise samples for the overall sample sequence are correlated, the noise samples for each subsequence are uncorrelated. Since the SNR (accounting for the noise coloring) based on the entire sample sequence is at least as large as that for any of the subsequences, we consider the chip-rate subsequence that is closest to the chip timing of the desired user to obtain a worst-case value for the SNR loss. For this subsequence, we must have $0 \le \delta \le 1/4$ or $3/4 \ge \delta \le 1$ in (10), so that the minimum value of the relative energy $\delta^2 + (1-\delta)^2$ occurs at $\delta = 1/4, 3/4$, and is given by 5/8. This corresponds to a 1 dB loss relative to a chip-synchronous system. In general, for a rectangular chip waveform, sampling at m/T_c for an asynchronous system would lead to an SNR relative to a chip-synchronous system of 10 $\log_{10}[(1/2m)^2 + (1-1/2m)^2]$ dB, a loss that quickly becomes negligible as m increases.

For the same observation interval, sampling at rate $2/T_c$ results in a received vector of twice the length. A linear receiver $\mathbf{c}$ operating on this vector would therefore also need to have twice as many taps, which can slow down the convergence of adaptive implementations. One approach to this problem is to consider separately each of the m interleaved streams of chip-rate samples corresponding to sampling at rate m/T_c, to apply algorithms designed for chip rate samples to each stream, and to then combine the outputs of the m algorithms in some manner. This is the approach implicitly adopted in the presentation of the algorithms in Section III, which assumes chip-rate samples. Note, however, that simple modifications of these algorithms could also be applied directly to the high-rate stream, since the required stationarity of the $\{\mathbf{r}_n\}$ still holds.

In our example, sampling at $2/T_c$ yields two interleaved versions of the model (7): the desired signal vector is given by (5) in one version, and by (8) in the other. If these two chip-rate models are interleaved to form a single stream at rate $2/T_c$, we still obtain a model of the form (7) when using an observation interval of length $2T$, except that the received vector is now of length 20 rather than ten.

In Section II-B, we note that the modeling done via examples in this subsection [in particular, (7)] extends to a general setting. This sets the stage for discussion of interference suppression schemes in Section III.

B. General Model

The system model from the point of view of user k is depicted in Fig. 3. Throughout this paper, we will work with the low-pass equivalent, or complex baseband, representation of passband signals. The symbol sequence changes at rate $1/T$ and is multiplied by the spreading sequence which changes (N times faster) at rate $1/T_c$, resulting in a sequence at rate $1/T_c$ being input to a wideband chip filter with impulse response $\psi(t)$. This input sequence may be complex, since the elements of the symbol or chip sequences may be chosen from a two-dimensional constellation. For example, in the IS-95 downlink, the spreading sequence is based on a complex quadrature phase shift keyed (QPSK) constellation, but the symbol sequence is from a real binary PSK (BPSK) constellation. Letting $\Psi(f)$ denote the frequency response of the chip filter $\psi(t)$, the chip filter is typically chosen such that $|\Psi(f)|^2$, which is the frequency response of the cascade of the chip filter and chip matched filter, is a Nyquist pulse at rate $1/T_c$. Provided chip timing is available, this avoids interchip interference for a given user. The rectangular $\psi(t)$ in Section II-A is an example of such a pulse, but in practice one might use pulses with better spectral properties, e.g., $\Psi(f)$ is chosen to be the square root of a raised cosine in the IS-95 system.

Let $s_k(t)$ denote the spreading waveform for user k. Let $\tilde{\mathbf{s}}_k(t)$ denote the effective spreading waveform for the kth user after going through the channel (which may be different for different users). The received signal due to the k user is then given by

$$r_k(t) = \sum_{n=-\infty}^{\infty} b_{n,k}\tilde{\mathbf{s}}_k(t-kT)$$

where $1/T$ is the symbol rate and $b_{n,k}$ is the nth symbol transmitted by the kth user. The net received signal is given by the sum of the received signals due to K active users, plus AWGN

$$r(t) = \sum_{k=1}^{K} r_k(t) + n(t). \tag{11}$$

As shown in Fig. 3, the received signal is converted to discrete time after chip matched filtering and sampling. As in the examples, we consider use of a finite observation

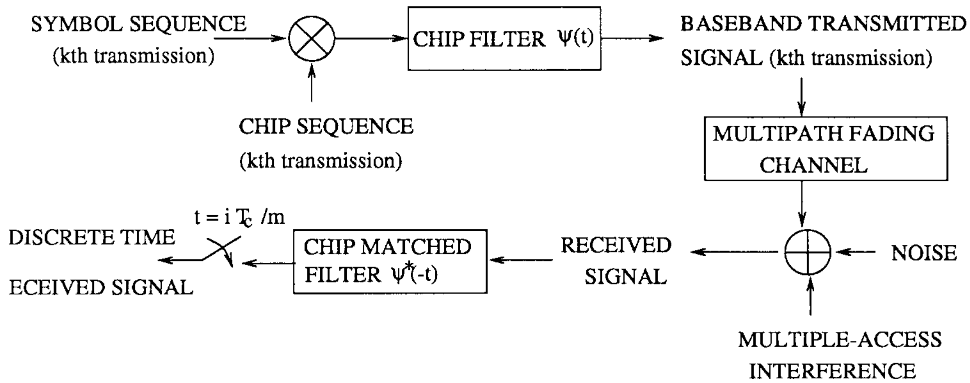

Fig. 3. System model from the viewpoint of user k.

interval for each symbol decision. Letting $\{\mathbf{r}_n\}$ denote the vector of samples in the nth observation interval, as illustrated in Section II-A, we may write $\mathbf{r}_n$ as the sum of signal vectors modulated by symbols, together with Gaussian noise. The signal vectors are simply the effective spreading waveforms $\tilde{\mathbf{s}}_k(t-nT)$ after filtering, sampling, and windowing in a given observation interval. The number of such vectors within a given observation interval depends on the length of the interval, the duration of the chip waveform, and the delay spread of the channel. In particular, we can work with the following generic equivalent synchronous model.

1) Equivalent Synchronous Model: The received vector in the nth observation interval is an L-dimensional complex vector given by

$$\mathbf{r}_n = b_0[n]\mathbf{u}_0 + \sum_{j=1}^{J} b_j[n]\mathbf{u}_j + \mathbf{w}_n \tag{12}$$

where $b_0[n]$ is the desired symbol for the nth observation interval and $\mathbf{u}_0$ is the desired signal vector. The symbols $\{b_j[n],\, 1 \le j \le J\}$ are interfering symbols, with corresponding interference vectors $\{\mathbf{u}_j,\, 1 \le j \le J\}$. The vector $\mathbf{w}_n$ is discrete-time Gaussian noise, which is colored if the sample spacing is closer than the chip interval T_c.

Remark II.1: The notation in (12) is deliberately chosen to be different from that in the examples in Section II-A in order to emphasize the generic nature of the model. For the example in Section II-A corresponding to an observation interval of length $2T$, a comparison of (7) and (12) yields that the desired bit $b_0[n] = b_{n,1}$, and that the interfering bits $b_j[n]$ and $1 \le j \le J$ consist of the ISI bits $b_{n-1,1}$ and $b_{n+1,1}$ from the desired user 1 and the MAI bits $b_{n-1,1}$, $b_{n,1}$, and $b_{n+1,1}$ from the interfering user 2, so that $J = 5$ is the number of interference vectors. For chip rate samples, the dimension $L = 2N$. On the other hand, for an observation interval of length T synchronized to the desired user, $J = 2$ (no ISI, and two MAI bits due to user 2) and $L = N$.

Remark II.2: Different symbols of the same user, as well as symbols from different users, are assumed to be uncorrelated, which implies uncorrelatedness for the symbols $b_0[n]$, $b_1[n], \cdots, b_J[n]$ in the equivalent synchronous model. Thus, $E[b_j[n]b_k^*[n]] = 0$ for $j \ne k$. This property is crucial for distinguishing the desired vector from the interfering users.

Remark II.3: The model (12) is applicable to any linearly modulated system in which the symbol waveform and the channel are time invariant. For instance, it also applies for modeling intersymbol interference in narrow-band systems, and therefore provides a simple framework for developing a geometric understanding of blind equalization, as discussed in Section IV. It also brings out the close analogy between ISI and MAI: each phenomenon simply contributes interference vectors, modulated by interfering symbols, to the received vector. The only difference is that ISI scales with the desired user's power, and therefore cannot cause a near–far problem. Finally, note that the model applies to either finite or infinite observation intervals (the number of interference vectors J and the dimension L of the received vectors are infinite in the latter case).

Remark II.4: The adaptive interference suppression algorithms considered here do not require explicit knowledge of the interference vectors appearing in (12). It is not necessary, therefore, to keep track of the spreading sequences and propagation channels for interfering users. The only relevant property of the model is that the signal vectors $\{\mathbf{u}_j\}$ are the same for (or slowly varying over) different observation intervals, due to our assumption of short spreading sequences and time invariant (or slowly varying) channels.

Remark II.5: Most receiver algorithms in category C3 (i.e., algorithms which do not utilize any information regarding the desired user) rely only on the time invariance of the $\{\mathbf{u}_j\}$ and the independence of the $\{b_j[n]\}$ for different j and n.[7] However, for receivers in category C2) (which do exploit information regarding the spreading sequence of the desired user), we require, in addition, a model for the desired signal vector $\mathbf{u}_0$ in terms of the spreading sequence and the unknown channel. If this information can be economically parametrized, blind adaptive interference suppression can be achieved by fitting the parametric model to the observed received vectors $\{\mathbf{r}_n\}$ so as to obtain a

[7] There are blind equalizers in category C3) that also exploit the memory in the sequence $\{\mathbf{r}_n\}$ [59], but we do not consider these here, since their generalization to multiuser applications appears to be difficult.

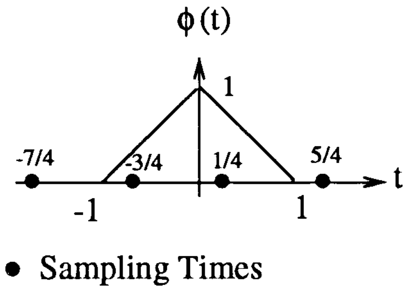

Fig. 4. Net chip response $\phi(t)$ due to a rectangular chip waveform.

receiver that preserves the energy due to the desired user while suppressing MAI.

2) Model for the Desired Signal Vector: Consider the response to a single chip (i.e., a given element of the spreading sequence for the desired user) at the output of the chip matched filter: given an ideal channel, this response is given by (a shift of) $\phi(t) = \psi(t) * \psi(-t)$. For the rectangular chip waveform considered in Section II-A, $\phi(t)$ is a triangular pulse of width $2T_c$ ($T_c = 1$), as shown in Fig. 4.

The discrete-time response to a single chip depends on the relation between the chip timing of the user and the receiver sampling clock. For chip rate sampling, the overall response to a single chip for the sampling times shown in Fig. 4 is $(\cdots, 0, 0, \frac{1}{4}, \frac{3}{4}, 0, 0, \cdots)$. This implies that the discrete-time response to the spreading sequence is a linear combination of two adjacent shifts of the spreading sequence weighted by $\frac{1}{4}$ and $\frac{3}{4}$, respectively. Restriction to an observation interval simply corresponds to windowing this response. Thus, for chip-rate sampling and an ideal channel, each signal vector in (12) corresponds to (a possibly truncated version of) a linear combination of two adjacent shifts of the spreading sequence for the associated user.

When the chip response $\phi(t)$ has duration larger than $2T_c$ (e.g., for band-limited chip waveforms or due to channel distortions), the discrete-time chip response after sampling may have more than two nonzero elements. In this case, the response to the spreading sequence is a linear combination of several shifted versions of it. Define the desired symbol $b_0[n]$ in the nth observation interval to be the symbol of the desired user with the largest energy in that interval. We may write the desired signal vector as

$$\mathbf{u}_0 = \sum_{m=m_0}^{m_0+M-1} h_m \mathcal{T}^m \mathbf{s}_1 \tag{13}$$

where $m_0 T_c$ is the delay of the signal modulating the desired symbol with respect to the left edge of the observation interval, and M and the channel coefficients $\{h_m\}$ depend on the channel and the chip waveform. The receiver might have an estimate of M, but m_0 and $\{h_m\}$ are assumed to be unknown by receivers in category C2). In many applications of interest, the channel delay spread is small compared to the symbol interval, so that $M \ll N$.

3) Example: For a rectangular chip waveform, a single path channel, a $2T$ observation interval, and chip rate sampling (13) specializes to

$$\mathbf{u}_0 = \beta\big[(1-\delta)\mathcal{T}^m \mathbf{s}_1 + \delta \mathcal{T}^{m+1}\mathbf{s}_1\big] \tag{14}$$

where $\tau = (m+\delta)T_c$ (m an integer, $0 \le \delta < 1$) is the (unknown) delay of the desired symbol from the left edge of the observation interval, and β is an unknown complex gain. This will be the example used in our numerical results.

III. Blind Reception Based on Second-Order Statistics

Assume that the observation interval is large enough such that most of the energy from at least one symbol of the desired user is contained in it. This is achieved, for instance, by choosing an observation interval of length $2T$, plus the anticipated channel delay spread. Thus, the delay spread due to the channel causes little additional complexity in terms of number of correlator taps if it is small compared to the symbol interval T. We consider the equivalent synchronous model (12), modeling the desired signal vector as in (13), where m_0 and the coefficients $\{h_m\}$ are unknown, and where the choice of M depends on the anticipated delay spread of the channel. Since blind reception cannot resolve phase ambiguity, it is assumed that the desired user's data is differentially encoded.

Under the model (13) for the desired vector, a conventional differentially coherent RAKE receiver (see [46] for the original formulation of the RAKE receiver) is given as follows. For a received vector in observation interval n, let

$$X_{m,n} = \langle \mathcal{T}^m \mathbf{s}_1, \mathbf{r}_n \rangle \tag{15}$$

denote the output of a correlator matched to the contribution of the mth shift of the spreading sequence. The output of the correlator matched to the desired signal vector $\mathbf{u}_0$ given by (13) is therefore $Y_n = \sum_{m=m_0}^{m_0+M-1} h_m^* X_{m,n}$. The decision statistic for differential encoding is given by $Z_n = Y_{n-1}^* Y_n$. Assuming that the shifts involved in (13) are approximately orthogonal, we may write

$$Z_n \approx \sum_{m=m_0}^{m_0+M-1} |h_m|^2 X_{m,n-1}^* X_{m,n}. \tag{16}$$

Our objective in this section is to estimate $|h_m|^2$ and m_0 so as to be able to apply (16), while replacing the matched filter in (15) by an interference suppressing correlator.

In Section III-A, we discuss preliminary concepts in linear interference suppression. A blind receiver algorithm based on the CMOE version of the MMSE receiver is given in Section III-B. Another algorithm based on subspace decompositions is given in Section III-C. Typical numerical results for a particular CMOE-based algorithm are presented in Section III-D.

A. Linear Interference Suppression

For the equivalent synchronous model (12), let $\mathcal{S}_I$ denote the subspace spanned by the interference vectors

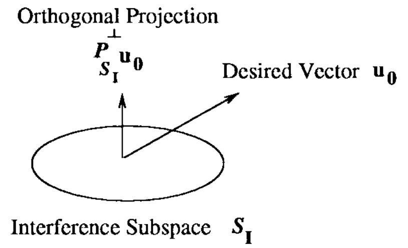

Fig. 5. Geometry underlying zero-forcing receiver.

$\mathbf{u}_1, \cdots, \mathbf{u}_J$. Let $\mathcal{P}^{\perp}{}_{\mathcal{S}_I}\mathbf{u}_0$ denote the projection of $\mathbf{u}_0$ orthogonal to the interference subspace, as illustrated in Fig. 5.

1) Decorrelating (Zero-Forcing) Detector: If the projection $\mathcal{P}^{\perp}{}_{\mathcal{S}_I}\mathbf{u}_0$ is nonzero, then a linear correlator chosen along this direction forces the contribution of the interference to zero while preserving some of the energy of the desired vector. Thus, such a receiver can be written as

$$\mathbf{c}_{ZF} = \alpha \mathcal{P}^{\perp}{}_{\mathcal{S}_I}\mathbf{u}_0 \tag{17}$$

where α is an arbitrary scale factor. Application of this concept to ISI suppression results in the well-known zero-forcing equalizer [47], while its application to MAI suppression results in the decorrelating detector studied by Lupas and Verdu [30], [31]. The adjectives decorrelating and zero-forcing will be used interchangeably in the sequel.

2) Near–Far Resistance: Choosing the correlator along the orthogonal projection $\mathcal{P}^{\perp}{}_{\mathcal{S}_I}\mathbf{u}_0$ rather than along the desired vector implies some loss of signal energy relative to the noise energy at the output of the detector. This in turn implies, for Gaussian noise, that the exponent of decay of error probability with E_b/N_0 is reduced by the factor by which the signal energy is reduced. For any multiuser detector, such a factor is called its asymptotic efficiency [69]. The worst-case value of the asymptotic efficiency over all possible interference amplitudes is called the near–far resistance [31] of the detector. Since the decorrelating detector projects the received vector orthogonal to the interference subspace, its performance is independent of the interference amplitudes. Thus, the near–far resistance η equals its asymptotic efficiency and has a geometric interpretation given by the following expression:

$$\eta = \frac{||\mathcal{P}^{\perp}{}_{\mathcal{S}_I}\mathbf{u}_0||^2}{||\mathbf{u}_0||^2}. \tag{18}$$

The near–far resistance is therefore the energy of the component of the desired vector orthogonal to the interference subspace relative to the energy of the desired vector.

Alternatively, by choosing the scale factor α in (17) so that the size of the desired vector's contribution to the detector output is fixed, $1/\eta$ may be thought of as the noise enhancement factor. This is the commonly used interpretation in equalization applications, where the interference vectors are due only to ISI.

Remark III.1—Linear Independence Condition: The zero-forcing detector exists if and only if the orthogonal projection $\mathcal{P}^{\perp}{}_{\mathcal{S}_I}\mathbf{u}_0$ is nonzero, which in turn is true if and only if the desired signal vector $\mathbf{u}_0$ is linearly independent of the interference vectors $\mathbf{u}_1, \cdots, \mathbf{u}_J$.

3) Linear MMSE Detector: The MMSE detector minimizes the MSE between the decision statistic and the desired symbol, given by $E[|\langle \mathbf{c}, \mathbf{r}_n\rangle - b_0[n]|^2]$ in the context of the model (12). The MMSE receiver $\mathbf{c}_{\text{MMSE}}$ is closely related to the zero-forcing receiver, and satisfies the following key properties [35], as described in Remarks III.2–III.4.

Remark III.2—Maximization of SIR: The MMSE receiver maximizes the output SIR and is superior to the zero-forcing receiver in that respect. Moreover, the MMSE solution is well defined even when the zero-forcing solution does not exist (i.e., when the linear independence condition is not satisfied).

Remark III.3—Performance at High SNR: As the noise level goes to zero, the MMSE receiver tends to the zero-forcing receiver (if the latter exists). This means that the asymptotic efficiency and near–far resistance of the MMSE and zero-forcing receivers are the same.

Remark III.4—Performance in Near–Far Regime [35], [44]: Even for nonzero noise levels, the MMSE receiver forces to zero the contribution of any interference vector $\mathbf{u}_j$ whose energy is large compared to that of the desired vector. That is, $\langle \mathbf{c}_{\text{MMSE}}, \mathbf{u}_j\rangle \to 0$ as $(||\mathbf{u}_j||^2/||\mathbf{u}_0||^2) \to \infty$. Thus, if all interference vectors are strong, the MMSE receiver again degenerates to the zero-forcing solution. Otherwise it forces interference contributions to zero to different extents to optimize the tradeoff between noise enhancement and interference suppression.

3) Computation of the MMSE Solution: The MSE is a quadratic function of $\mathbf{c}$, and the minimizing solution is readily obtained as [16, Ch. 5]

$$\mathbf{c}_{\text{MMSE}} = \mathbf{R}^{-1}\tilde{\mathbf{u}}_0 \tag{19}$$

where $\mathbf{R} = E[\mathbf{r}_n \mathbf{r}_n^H]$ is the statistical correlation matrix for the received vector and $\tilde{\mathbf{u}}_0 = E[b_0^*[n]\mathbf{r}_n]$ is the correlation of the desired symbol with the received vector.[8] Assuming (as in Remark II.2) that the symbols $\{b_j[n]\}$ are uncorrelated, we have $\tilde{\mathbf{u}}_0 = \mathbf{u}_0$.

4) Training-Based Implementation: For an adaptive implementation, the statistical average $\mathbf{R}$ can be replaced by an empirical average $\hat{\mathbf{R}}$, for example

$$\hat{\mathbf{R}} = \frac{1}{M_{\text{LS}}} \sum_{n=1}^{M_{\text{LS}}} \mathbf{r}_n \mathbf{r}_n^H \tag{20}$$

(the factor $1/M_{\text{LS}}$ is unnecessary, but is inserted to make the averaging interpretation transparent). Furthermore, if a training sequence $\{b_0[n]\}$ is available, then we may replace the statistical average $\tilde{\mathbf{u}}_0$ by an empirical average $\hat{\mathbf{u}}_0$, for example

$$\hat{\mathbf{u}}_0 = \frac{1}{M_{\text{LS}}} \sum_{n=1}^{M_{\text{LS}}} b_0^*[n]\mathbf{r}_n. \tag{21}$$

[8] Note that our use of the notation $\mathbf{R}$ differs from that in much of the literature on multiuser detection (e.g., see [70] and [71]), wherein $\mathbf{R}$ denotes the matrix of cross correlations between the spreading waveforms of the different users.

Plugging (20) and (21) into (19) comprises the LS implementation of the MMSE receiver, and $M_{\rm LS}$ denotes the number of symbol-rate iterations. For time-varying channels, it may be more appropriate to use an RLS implementation [16] in which the arithmetic averages in (20) and (21) are replaced by exponential averages that gradually forget the past. A lower complexity adaptive mechanism is the LMS algorithm resulting from stochastic gradient descent based on the MSE [16], but we have found its convergence to be too slow for wireless applications, especially in near–far regimes. For further results on LS and RLS implementations in the context of CDMA, see [21] and [45].

Remark III.5—Toward a Blind Adaptive Implementation: The estimate $\hat{\mathbf{R}}$ used in the preceding LS implementation does not require any knowledge regarding either the desired or interfering users. The only function of the training sequence is to provide the estimate (21) of the desired signal vector. Thus, a blind adaptive implementation of the MMSE receiver only requires that we can obtain an estimate of the desired signal vector in some other fashion. This leads to the blind receiver algorithms in the next two subsections.

B. CMOE-Based Blind Receiver

In Section III-B1, we present background material regarding the category C1) receiver in [18], which assumes that the channel seen by the desired user is known to the receiver. In Section III-B2 this is extended, as in [33], to a category C2) receiver, which estimates the propagation channel of the desired user.

1) Category C1) CMOE Receiver: From (13) or (14) it is clear that if the receiver knows the spreading waveform and has an estimate of the propagation channel of the desired user, it can obtain an estimate $\hat{\mathbf{u}}_0$ (henceforth termed the nominal signal vector) of the desired signal vector $\mathbf{u}_0$. This provides a method for computing an estimate of the MMSE correlator without a training sequence as follows:

$$\hat{\mathbf{c}}_{\rm MMSE} = \hat{\mathbf{R}}^{-1}\hat{\mathbf{u}}_0 \tag{22}$$

where $\hat{\mathbf{R}}$ is computed as in (20).

Remark III.6: Since any scalar multiple of the MMSE correlator leads to the same SIR, the nominal vector $\hat{\mathbf{u}}_0$ can be an arbitrary scalar multiple of the true vector $\mathbf{u}_0$. The resulting scaling of the correlator output (possibly by a complex number) can be handled using differential encoding of the desired symbol sequence. Thus, for a single path channel, (14) indicates that it is only necessary to know the timing and spreading waveform of the desired user to obtain the nominal vector.

The category C1) receiver (22) forms the starting point for obtaining receivers in category C2) that estimate the propagation channel. In order to see this, it is necessary to view (22) as the solution to an optimization problem, as described in the following.

Any linear receiver that does not null out the desired vector $\mathbf{u}_0$ satisfies $\langle \mathbf{c}, \mathbf{u}_0\rangle \neq 0$. Thus, if an estimate $\hat{\mathbf{u}}_0$ of the direction of the desired signal vector were available, a

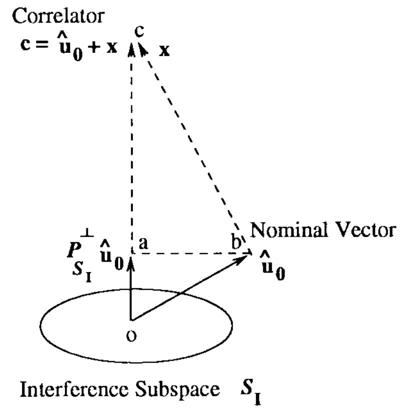

Fig. 6. Geometry of interference suppression using CMOE detector.

scaled version of this receiver could be chosen to satisfy

$$\langle \mathbf{c}, \hat{\mathbf{u}}_0\rangle = 1. \tag{23}$$

This constraint can be interpreted geometrically as follows. Any $\mathbf{c}$ satisfying (23) can be decomposed into two orthogonal components

$$\mathbf{c} = \hat{\mathbf{u}}_0 + \mathbf{x},\ \mathbf{x} \text{ orthogonal to } \hat{\mathbf{u}}_0 \tag{24}$$

where we assume, for simplicity, that $\hat{\mathbf{u}}_0$ is normalized to have unit norm. Clearly, if the nominal $\hat{\mathbf{u}}_0$ is proportional to the desired vector $\mathbf{u}_0$, the contribution of the desired vector to the receiver output is not affected by the choice of $\mathbf{x}$. On the other hand, Fig. 6 shows that, provided $\hat{\mathbf{u}}_0$ has a nonzero projection orthogonal to the interference subspace $\mathcal{S}_I$, it is possible to choose $\mathbf{x}$ so as to synthesize a zero-forcing receiver $\mathbf{c}$, which nulls out the contribution of the interference vectors.

The CMOE receiver is chosen to minimize the average output energy $E[|\langle \mathbf{c}, \mathbf{r}_n\rangle|^2]$, subject to the constraint (23). Since (23), or equivalently (24), freezes the contribution of the desired vector to the output, the CMOE receiver can only suppress the sum of the noise and interference energies at the output. This is precisely the quantity being minimized by the MMSE receiver, except that the latter also optimizes the scaling of the contribution of the desired vector to the output so as to track the desired symbol $b_0[n]$, rather than any arbitrary scalar multiple of $b_0[n]$. This implies that the CMOE receiver is proportional to the MMSE receiver (see [18] for a formal proof).

Using the similarity of the triangles oab and obc, the length $||\mathbf{c}||$ of the correlator in Fig. 6 can be calculated as $1/(||\mathcal{P}^{\perp}{}_{\mathcal{S}_I}\hat{\mathbf{u}}_0||)$ (the length of the nominal vector $||\hat{\mathbf{u}}_0|| = 1$). Thus, the smaller the projection of the nominal vector orthogonal to the interference subspace the more the noise enhancement (which is proportional to $||\mathbf{c}||^2$) associated with suppressing the interference.

2) Effect of Mismatch: In order to extend the CMOE receiver to settings in which the propagation channel of the desired user, and hence the desired signal vector, is unknown, it is first necessary to address the issue of mismatch, i.e., when the receiver does not have exact knowledge of the direction of the desired signal vector $\mathbf{u}_0$. We provide a brief discussion of this issue here, referring the reader to [18] and [33] for details. Let $\mathcal{S}_{\mathbf{u}_0}$ denote the one-dimensional subspace spanned by $\mathbf{u}_0$. Replacing $\mathcal{S}_I$ by $\mathcal{S}_{\mathbf{u}_0}$ in Fig. 6, we see that suppression of the desired signal vector is possible if the orthogonal projection $\mathcal{P}^{\perp}{}_{\mathcal{S}_{\mathbf{u}_0}}\hat{\mathbf{u}}_0$ is nonzero (i.e., if the nominal $\hat{\mathbf{u}}_0$ is not exactly proportional to $\mathbf{u}_0$). However, if the mismatch (and hence the length of the orthogonal projection) is small, then reasoning as before, the length of $\mathbf{c}$ required to achieve this is large, which leads to more noise enhancement, which in turn leads to increased output energy. Thus, the MOE criterion in itself can prevent excessive signal suppression by virtue of the tradeoff between signal suppression and noise enhancement, provided that the noise levels are moderate. If the noise levels are low, the amount of signal suppression due to mismatch may be excessive unless it is prevented by some explicit constraint. One possibility, explored in [18] and [33], is to put a constraint on $||\mathbf{x}||$: $||\mathbf{x}||$ is allowed to be large enough to permit the interference suppression depicted in Fig. 6, but not the analogous suppression of the desired signal. Mathematically, incorporating the constraint on $||\mathbf{x}||^2$ can be interpreted as adding fictitious noise to the system, since the constraint results in $\hat{\mathbf{R}}$ being replaced by $\hat{\mathbf{R}} + \nu\mathbf{I}$, where ν is the Lagrange multiplier corresponding to the norm constraint. See [18] for details of this approach to dealing with mismatch.

When the mismatch can be modeled accurately, the signal degradation due to mismatch can be completely eliminated without the need for a norm constraint. Consider the expression (13) for $\mathbf{u}_0$, and suppose that the receiver has a coarse timing estimate, so that m_0 is known. If the receiver knows the multipath spread M but does not know the channel coefficients $\{h_m\}$, then it only knows that $\mathbf{u}_0$ lies in the subspace spanned by $\mathcal{T}^{m_0}\mathbf{s}_1, \cdots, \mathcal{T}^{m_0+M-1}\mathbf{s}_1$. In general, suppose that the desired signal vector is not known perfectly but is known to lie in a subspace $\mathcal{S}_{\mathbf{u}_0}$ of dimension possibly larger than one. Any nominal $\hat{\mathbf{u}}_0$ chosen in this subspace is vulnerable to mismatch. However, the CMOE algorithm can be modified to get around this problem by constraining the correlator to be of the form (24), where $\mathbf{x}$ is chosen to be orthogonal to the entire subspace $\mathcal{S}_{\mathbf{u}_0}$. Thus, if the desired vector is indeed in this subspace, it is unaffected by the choice of $\mathbf{x}$. This idea, which was proposed independently by several authors, including [22], [64], and others, is depicted in Fig. 7. Note that constraining $\mathbf{x}$ in this fashion reduces the interference suppression capability of the detector. However, if the dimension of $\mathcal{S}_{\mathbf{u}_0}$ is small, this reduction is expected to be small. Finally, note that mismatch occurs again if the knowledge of the subspace $\mathcal{S}_{\mathbf{u}_0}$ in which the desired vector lies is imperfect (e.g., due to timing errors or larger than anticipated delay spreads), so that it may be necessary to impose a norm constraint in addition to subspace constraints.

Remark III.7—Extending CMOE to Obtain Category C2) Receivers: Now that we can handle mismatch, the basic idea behind estimating the propagation channel of the desired user can be stated. Run the CMOE algorithm

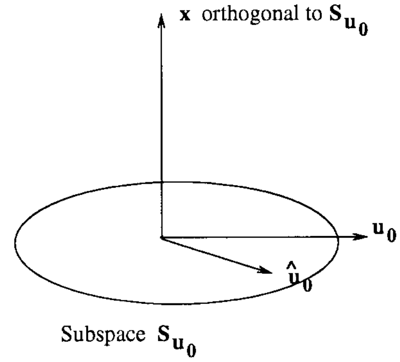

Fig. 7. Geometry of additional constraints in the CMOE formulation for handling mismatch. Ideally, $\mathcal{S}_{\mathbf{u}_0}$ is chosen large enough so as to contain both the nominal and the desired vector.

(modified using the methods mentioned above to deal with a controlled amount of mismatch) under different hypotheses regarding the propagation channel of the desired user. The hypotheses that are grossly incorrect lead to large mismatch, which results in suppression of both the desired and interference vectors without much noise enhancement, thus giving small values of MOE. The good hypotheses, on the other hand, have less mismatch with the true signal vector, and can be identified because of their larger MOE due to the desired signal energy at the output and the noise enhancement due to suppression of the desired signal. The CMOE solutions under the good hypotheses are then used to obtain an interference suppressing receiver. One possible algorithm based on this idea is given next.

3) Category C2) CMOE Receiver: While there are many possible variations of CMOE based reception, we give one possible algorithm that is based on the assumption that the multipath delay spread of the channel is small compared to the symbol interval. The algorithm is new, being a generalization and improvement of the algorithm in [33], in the sense that it uses subspace constraints to handle mismatch more effectively. Note that more complicated algorithms might be needed for channels with significant delay spread.

The uncertainty regarding the propagation channel is expressed in terms of the following hypotheses.

Hypothesis H_m: The desired vector $\mathbf{u}_0$ has a significant component along $\mathcal{T}^m\mathbf{s}_1$, the mth shift of the spreading sequence. Under this hypothesis, the desired vector $\mathbf{u}_0$ lies in the subspace $\mathcal{S}_m$ spanned by the shifts $m-(M-1)$ through $m+(M-1)$ of the spreading sequence $\mathbf{s}_1$, since the maximum multipath spread is assumed to be M.

An algorithm for identifying the significant multipath components consists of solving a binary hypothesis testing problem for each m, namely, testing hypothesis H_m against the hypothesis that the mth shift is not present. The algorithm can be decomposed into the following steps, where we omit the detailed computations involved.

Step 1) Computation of CMOE Solution Under Hypothesis H_m: Under the hypothesis H_m, set the nominal $\hat{\mathbf{u}}_0^{(m)} = \mathcal{T}^m\mathbf{s}_1$, consider a correlator of the form

$$\mathbf{c} = \hat{\mathbf{u}}_0^{(m)} + \mathbf{x},\ \mathbf{x} \text{ orthogonal to } \mathcal{S}_m \qquad (25)$$

and minimize the output energy subject to (25). Denote the correlator thus obtained by $\mathbf{c}_m$ and the corresponding MOE by ζ_m.

Assuming that the CMOE solution under H_m suppresses the interference and does not suppress the desired signal if the hypothesis is true, the MOE ζ_m gives a coarse estimate of $|h_m|^2$, where h_m is the unknown channel coefficient. If the hypothesis is not true, i.e., $h_m \approx 0$, then the MOE will be small, and again gives a rough estimate of $|h_m|^2$. Furthermore, the CMOE solution $\mathbf{c}_m$ under H_m is simply a correlator matched to the mth shift, plus an interference suppressing orthogonal component $\mathbf{x}$. Thus, one simple method of combining the CMOE solutions under the different hypotheses is to mimic (15) and (16), with $\mathbf{c}_m$ replacing $\mathcal{T}^m\mathbf{s}_1$, and ζ_m replacing $|h_m|^2$.

Step 2) Combining the CMOE Solutions: The decision statistic for the nth symbol is given by

$$\hat{Z}_n = \sum_{m=0}^{N-1} \zeta_m \langle \mathbf{c}_m, \mathbf{r}_{n-1}\rangle^* \langle \mathbf{c}_m, \mathbf{r}_n\rangle. \tag{26}$$

Remark III.8—Improving the Combining Rule: A better decision rule might be to discard hypotheses corresponding to small values of MOE ζ_m rather than rely on the MOE weighting to deemphasize them. Other heuristics include considering only hypotheses within the maximum delay spread of the hypothesis with the highest MOE, or to consider only a cluster of M contiguous hypotheses in the summation (26), with the cluster chosen to maximize the net estimated energy, say ζ_m.

Remark III.9—Relation to Previous Work: The solution to the original CMOE problem with constraint (24) and norm constraint $||\mathbf{x}||^2 \le \chi$ is given by $\mathbf{c} = (\hat{\mathbf{R}} + \nu\mathbf{I})^{-1}\hat{\mathbf{u}}_0$, where $\nu \ge 0$ is adjusted so as to satisfy the norm constraint [18]. The idea behind the category C2) receiver in [33] is to use a fixed small value of ν to compute a CMOE-based receiver under each hypothesis as follows: $\mathbf{p}_m = (\hat{\mathbf{R}} + \nu\mathbf{I})^{-1}\hat{\mathbf{u}}_0^{(m)}$. The reasoning is as follows: the small value of ν provides robustness against mismatch for the good hypotheses, but is not enough to prevent mismatch under the bad hypotheses. For heuristics on the choice of ν, see [33]. This approach has the disadvantages that, despite the norm constraint, some desired signal degradation still occurs under the good hypotheses, and that the choice of ν relies on heuristics. Note that heuristics are not needed for the results in this paper, since we assume that the mismatch can be modeled accurately and handled using subspace constraints as in (25), and can therefore set $\nu = 0$.

Remark III.10—Computations Required in the Algorithm: In either an LS or RLS formulation, it is necessary to compute $\hat{\mathbf{R}}^{-1}$ [or $(\hat{\mathbf{R}} + \nu\mathbf{I})^{-1}$ if a norm constraint is being imposed], where $\hat{\mathbf{R}}$ is the empirical correlation matrix. The correlators $\mathbf{c}_m$ solving the CMOE problem with constraints (25) under the hypotheses H_m can be shown to be linear combinations of the $\{\mathbf{p}_l = \hat{\mathbf{R}}^{-1}\hat{\mathbf{u}}_0^{(l)}\}$. For obtaining the coefficients of the linear combinations, we need computation of the correlations $\langle \hat{\mathbf{u}}_0^{(m)}, \hat{\mathbf{u}}_0^{(l)}\rangle$ and $\langle \mathbf{p}_m, \hat{\mathbf{u}}_0^{(l)}\rangle$ for all pairs (m, l).

Remark III.11—Improved Algorithm for a Single Path Channel: For a single path channel, the form of the desired signal vector is given by (14). Assuming an observation interval of $2T$, m can range from 0 to $N-1$. In this case, it is possible to obtain better performance using a refinement of the preceding algorithm. Let H_m denote the composite hypothesis corresponding to (m, δ) in (14). Under H_m, the desired vector lies in the subspace $\mathcal{S}_m$ spanned by two consecutive shifts $\mathcal{T}^{(m)}\mathbf{s}_1$ and $\mathcal{T}^{(m+1)}\mathbf{s}_1$ of the spreading sequence. Running Step 1) of the previous algorithm for $\hat{\mathbf{u}}_0^{(m,\,\delta)}$ equal to a normalized version of (14), we can obtain a closed form expression for the CMOE as a function of m and δ. Maximization over $0 \le \delta \le 1$ for each m, and then maximization over m, gives a delay estimate $(\hat{m}, \hat{\delta})$. The corresponding CMOE solution $\hat{\mathbf{c}}$ is then given by the solution to (25) and can be used for differential detection as before.

Remark III.12—Faster than Chip Rate Sampling: In this case, in order to avoid slowing down the adaptation (i.e., estimation and tracking of $\hat{\mathbf{R}}$), parallel versions of the algorithm could be run on chip rate subsamples, as mentioned in Section II, and the MOE solutions for all hypotheses for all versions could be combined.

Remark III.13—Ambiguity in Symbol Timing: Depending on the multipath spread, some of the correlators in (26) may correspond to earlier or later symbols of the desired user. This ambiguity can be resolved by correlating (over a number of observation intervals) the noncoherent outputs $\langle \mathbf{c}_m, \mathbf{r}_{n-1}\rangle^* \langle \mathbf{c}_m, \mathbf{r}_n\rangle$ of a reference correlator m against shifts of other outputs $\langle \mathbf{c}_l, \mathbf{r}_{n-1-\Delta}\rangle^* \langle \mathbf{c}_l, \mathbf{r}_{n-\Delta}\rangle$, where the range of $\Delta = 0, \pm 1, \cdots$ considered depends on the anticipated channel delay spread as a multiple of the symbol interval. Choose for each l the value of Δ that provides a peak and use that value in the summation (26).

C. Subspace-Based Blind Receiver

We begin with background on signal and noise subspaces, followed by an algorithm which is an extension of the timing acquisition algorithms presented in [5] and [55]. More recently, subspace-based representations of multiuser detectors have been considered in [73].

D. Basic Subspace Notions

For the equivalent synchronous model (12), the $L \times L$ correlation matrix $\mathbf{R}$ for this model (recall that L denotes the dimension of the received vector $\mathbf{r}_n$ in this generic model) is given by

$$\mathbf{R} = \sum_{j=0}^{J} \mathbf{u}_j \mathbf{u}_j^H + \sigma^2 I \tag{27}$$

assuming that the noise is white with variance σ^2 per component. Consider an eigendecomposition of $\mathbf{R}$ into orthonormal eigenvectors $\mathbf{v}_1, \cdots, \mathbf{v}_L$ with corresponding eigenvalues $\lambda_1, \cdots, \lambda_L$. The eigenvalue λ_j can be interpreted as the expected (desired and interference) signal plus noise energy in the direction $\mathbf{v}_j$. Thus, the minimum value of each λ_j is σ^2 since the latter is the noise energy is any direction. The signal vectors $\mathbf{u}_0, \cdots, \mathbf{u}_J$ span a signal subspace of dimension $d_s \le J+1$. Since the signal vectors contribute energy along the eigenvectors spanning the signal subspace, the associated eigenvalues are larger than σ^2. On the other hand, the eigenvectors spanning the $L-d_s$ dimensional noise subspace all have eigenvalues σ^2.

The preceding arguments imply that an eigendecomposition of an estimate $\hat{\mathbf{R}}$ of the correlation matrix can be used to estimate the signal and noise subspaces. If the number of signal vectors $J+1$ is known, and if the vectors may be assumed to be linearly independent,[9] then we would set $d_s = J+1$. If d_s is not known *a priori*, then it can be estimated using information-theoretic criteria [75]. In any case, the eigenvectors $\hat{\mathbf{v}}_1, \cdots, \hat{\mathbf{v}}_{d_s}$ associated with the d_s largest eigenvalues are assumed to span the signal subspace, and the remaining eigenvectors provide an estimate of the noise subspace. In practice, eigendecomposition may be replaced by subspace tracking algorithms such as [6], [54], and [78] for time-varying channels.

Remark III.14—Using Subspace Methods to Obtain Category C2) Receivers: Having estimated the signal and noise subspaces, the problem reduces to one of finding good fits among different hypothesized propagation channels (or equivalently, among hypothesized desired signal vectors) with the estimated signal subspace. One example is the MUSIC algorithm, which minimizes the projection of the hypothesized signal vectors onto the estimated noise subspace (or equivalently, maximizes the projection onto the estimated signal subspace). A version of this algorithm is described next.

Remark III.15—Subspace-Based Computation of MMSE Receiver: The spectral decomposition of $\mathbf{R}$ gives

$$\mathbf{R} = \sum_{j=1}^{L} \lambda_j \mathbf{v}_j \mathbf{v}_j^H \quad \text{and} \quad \mathbf{R}^{-1} = \sum_{j=1}^{L} \frac{1}{\lambda_j} \mathbf{v}_j \mathbf{v}_j^H.$$

Applying the formula (19), and noting that the desired signal vector is orthogonal to the eigenvectors $\mathbf{v}_{d_s+1}, \cdots, \mathbf{v}_L$ spanning the noise subspace, we obtain that [73]

$$\mathbf{c}_{\text{MMSE}} = \sum_{j=1}^{L} \frac{1}{\lambda_j} \mathbf{v}_j \mathbf{v}_j^H \mathbf{u}_0 = \sum_{j=1}^{d_s} \frac{1}{\lambda_j} \langle \mathbf{v}_j, \mathbf{u}_0 \rangle \mathbf{v}_j. \quad (28)$$

Only the signal subspace eigenvectors appear on the extreme right-hand side of (28), which gives explicit expression to the intuition that any correlator that has a component in the noise subspace is suboptimal, since it only adds noise to the output. The formula is particularly useful in the presence of mismatch, since restriction to the signal subspace makes suppression of the desired signal more difficult. See [73] for details.

E. Category C2) Subspace-Based Receiver

The receiver is based on the MUSIC algorithm [50], originally invented for array processing applications. Application of MUSIC for CDMA timing acquisition was independently proposed by Bensley and Aazhang [5], and by Strom *et al.* [55]. The algorithms in [5] and [55] consider an observation interval of length T, so that one complete symbol of the desired user is not guaranteed to fall in the observation interval. While this is enough to provide a timing estimate, direct computation of a demodulator is not possible in the framework of [5] and [55]. The algorithm presented here is a simple generalization, applying essentially the same algorithm to an observation interval of length $2T$. This yields a receiver in category C2), i.e., provides a demodulator in addition to a timing estimate. As in [5], [55], we consider only the case of a single path channel, which corresponds to the model (14) for the desired signal vector. However, we comment on possible extensions for multipath channels in Remark III.18.

The algorithm is described in the following steps.

Step 1) Estimation of Signal and Noise Subspaces: Compute estimates of the signal and noise subspaces using eigendecomposition of $\hat{\mathbf{R}}$, as indicated in Section III-D. This yields a matrix $\hat{\mathbf{V}}_s$ whose columns are the orthonormal eigenvectors spanning the estimated signal subspace, and a corresponding matrix $\hat{\mathbf{V}}_n$ of orthonormal eigenvectors spanning the estimated noise subspace.

Step 2) Minimization of Projection onto Noise Subspace: For each nominal parameterized by (m, δ) as in (14) and normalized to unit energy, the energy of the projection onto the noise subspace is given by [5], [55]

$$\xi(m, \delta) = \frac{\left(\hat{\mathbf{u}}_0^{(m,\delta)}\right)^H \hat{\mathbf{V}}_n \hat{\mathbf{V}}_n^H \hat{\mathbf{u}}_0^{(m,\delta)}}{\left(\hat{\mathbf{u}}_0^{(m,\delta)}\right)^H \hat{\mathbf{u}}_0^{(m,\delta)}}. \quad (29)$$

Minimize $\xi(m, \delta)$ over $0 \le \delta \le 1$ for each m, and then minimize over m. This gives the timing estimate $(\hat{m}, \hat{\delta})$.

Step 3) Computation of CMOE Solution: Compute the CMOE solution (25) for $\mathbf{c}$ with $\hat{\mathbf{u}}_0^{(m)}$ replaced by $\hat{\mathbf{u}}_0^{(\hat{m},\hat{\delta})}$. This can now be used for differential detection.

Alternative to Step 3) Signal Subspace Receiver: Once a subspace decomposition has been obtained, attention may be restricted to the signal subspace when computing a correlator based on the timing estimate obtained in Step 2. In particular, we may use the formula (28), with $\mathbf{u}_0$ replaced by $\hat{\mathbf{u}}_0^{(\hat{m},\hat{\delta})}$, and $\mathbf{v}_j$, λ_j replaced by their estimates.

Remark III.16—Effect of Errors in Subspace Estimation: If the dimension of the signal subspace is underestimated, the true desired signal vector may have a significant component in the estimated noise subspace, thereby leading to possible errors in timing estimation due to a "miss" of the correct timing values. On the other hand, overestimation of the signal subspace dimension means that nominals corresponding to incorrect timing have a larger component in the signal subspace, thereby making "false alarms" more likely. The effect of estimation errors on subspace based algorithms for CDMA therefore deserves detailed investigation.

Remark III.17—Relation of Subspace Methods to CMOE: The CMOE and subspace methods are closely related. For a nominal $\hat{\mathbf{u}}_0$, application of the spectral decomposition of

[9] Linear independence might be a reasonable assumption for CDMA systems in which the signal vectors are shifts of spreading sequences with good auto- and cross correlation properties.

$\mathbf{R}$ yields that the MOE with constraint (24) is given by

$$1/\text{MOE} = \hat{\mathbf{u}}_0 \mathbf{R}^{-1} \hat{\mathbf{u}}_0 = \sum_{j=1}^{L} \frac{1}{\lambda_j} \frac{|\langle \mathbf{v}_j, \hat{\mathbf{u}}_0 \rangle|^2}{\lambda_j} = \sum_{j=1}^{d_s} \frac{1}{\lambda_j} \frac{|\langle \mathbf{v}_j, \hat{\mathbf{u}}_0 \rangle|^2}{\lambda_j} + \frac{\|P_{\mathcal{S}_n} \hat{\mathbf{u}}_0\|^2}{\sigma^2}$$

where $P_{\mathcal{S}_n} \hat{\mathbf{u}}_0$ is the component of the nominal in the noise subspace $\mathcal{S}_n$. In a high SNR regime, the signal subspace eigenvalues $\{\lambda_j, 0 \leq j \leq d_s\}$ are much larger than the noise variance σ^2, so that 1/MOE is dominated by $\|P_{\mathcal{S}_n} \hat{\mathbf{u}}_0\|^2$. Choosing the largest MOE, or smallest 1/MOE, among timing hypotheses corresponding to different nominals is therefore roughly equivalent to a subspace algorithm that chooses the nominal(s) with the smallest projection in the noise subspace, where this equivalence becomes exact as $\sigma^2 \to 0$.

Remark III.18—Extensions to Multipath Channels: For large delay spreads (of the order of a symbol duration), one approach is to find the shifts $\mathcal{T}^m \mathbf{s}_1$ of the desired spreading sequence with small projections onto the noise subspace, and to then apply a suitable CMOE formulation based on the assumption that these good shifts span the space in which the true desired signal vector $\mathbf{u}_0$ lies. This approach has not been explored in detail in the literature. For small delay spreads, it is possible to hypothesize desired signal vectors of the form (13), where M is small, and to try to estimate the channel coefficients $\{h_m\}$ using an LS fit within the signal subspace. This approach has been explored in [28] for synchronous CDMA and in [61] for asynchronous CDMA (the latter employs subspace decomposition for the received signal corresponding to an observation interval several times larger than the symbol interval, unlike the earlier work on subspace based timing acquisition in [5], [55]).

F. Numerical Results

Extensive numerical comparisons of different receiver algorithms in realistic settings are beyond the scope of this paper, since our primary purpose here is to explain the conceptual framework for obtaining blind algorithms. Hence, we only give a few numerical results to indicate the potential of the methods discussed here. We consider a category C2) blind CMOE receiver optimized, as indicated in Remark III.10, for an asynchronous CDMA system with a single path channel for each user. The model is similar to that in [33], with processing gain $N = 15$. The algorithm in [33] uses a norm constraint to deal with mismatch, so that the CMOE criterion concentrates on suppressing the desired signal (the extent of signal degradation depends on the amount of mismatch and on the strictness of the norm constraint) when the interference is weak, since this strategy minimizes the output energy. For strong interference, on the other hand, suppressing interference is a more effective way of reducing the output energy. This leads to the algorithm in [33] actually performing worse for weak interference. This problem is eliminated in the new algorithm, since it explicitly models the mismatch, and uses subspace constraints of the form (25) to eliminate it.[10] Since the performance of the new algorithm improves as the interference gets weaker (as it should), we consider only the following two systems: a system with $K = 10$ equal power users (perfect power control), and a system with $K = 6$ users, with the five interfering users each 20 dB stronger than the desired user (severe near–far problem).

A fixed, but random, choice of K spreading sequences is used in each case; results for other choices are qualitatively similar. The observation interval is of length $2T$. For the kth user, denote the delay τ_k of this symbol from the left edge of the observation interval by τ_k. The delays τ_k are chosen randomly in $[0, T]$ and then kept fixed. Chip-spaced sampling is used, so that the received vectors $\mathbf{r}_n$, and hence the CMOE correlator, are of length $2N = 30$. Two or more parallel chip-spaced versions of the algorithms could be run to handle the signal loss due to chip asynchronism, but we do not consider that option here.

An evaluation of the steady-state performance of the algorithm can be obtained by running it using the statistical correlation matrix $\mathbf{R}$. For both systems considered, this yields timing estimates within $0.02T_c$ of the true values. The SIR attained by demodulators based on these steady-state estimates is within a dB of that of the ideal MMSE receiver, where the loss in SIR occurs due to the subspace constraints imposed to deal with mismatch.

Next, we consider an LS adaptive implementation based on an estimate $\hat{\mathbf{R}}$ of the correlation matrix as in (20). We plot in Figs. 8 and 9 the SIR, averaged over 100 runs, achieved by the CMOE receiver as a function of the number M_{LS} of (symbol rate) LS iterations. As other benchmarks, we indicate the SIR of an LS version of a training based MMSE receiver (averaged over the same realization as our blind algorithm), and the SIR of the LS version of the [category C1)] blind receiver in [18] with perfect timing information. Finally, we plot the largest achievable SIR, that of the ideal MMSE receiver.

Some key observations from the numerical results are as follows. Further discussion is given in Section V.

1) The conventional matched filter (whose performance is not plotted in the figures) is useless in the near–far regime, attaining an SIR of −16 dB. Even with perfect power control it provides poorer performance than the receivers considered here, obtaining an SIR of about 5 dB (which is about 10 dB worse than the ideal MMSE receiver). Note that the matched filter has the advantage of perfect timing information.
2) Fig. 9 shows that both the blind- and training-based receivers are near–far resistant.
3) For both systems, the category C2) receiver (which uses a timing estimate to compute the correlator)

[10] Note that the mismatch problem (and the role of norm constraints in the algorithm) could resurface if there were too much uncertainty regarding the desired signal vector to be completely captured using constraints such as (25), e.g., for channels with large multipath spread.

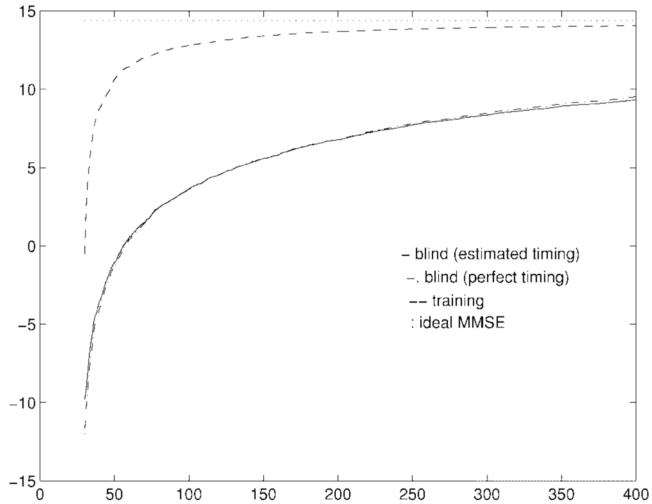

Fig. 8. SIR (dB) for LS versions of category C1) and C2) blind receivers and a training based receiver, compared with that of the ideal MMSE receiver. The correlators have 30 taps. There are nine equal power interfering users.

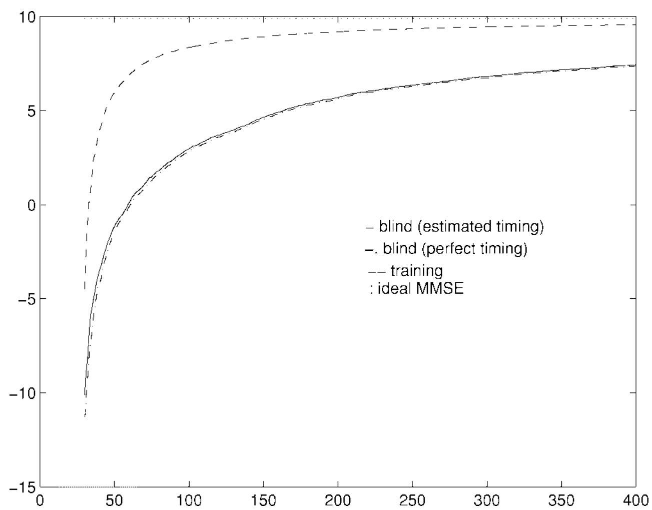

Fig. 9. SIR (dB) for LS versions of category C1) and C2) blind receivers and a training based receiver, compared with that of the ideal MMSE receiver. The correlators have 30 taps. There are five interfering users, each 20 dB stronger than the desired user.

incurs no performance loss relative to the category C1) receiver (which has perfect timing information). Thus, the timing estimate provided by the algorithm in Remark III.10 is accurate even with a severe near–far problem.

4) The training-based receiver converges much faster than either blind receiver. It appears that this is because the errors in the LS estimates (20) and (21) of the correlation matrix and desired vector, respectively, somehow conspire to give a better approximation to the steady-state MMSE correlator than in a blind algorithm which does not use a LS estimate of $\mathbf{u}_0$. The blind receivers use the LS estimate (20) of the correlation matrix, but the category C1) receiver uses perfect knowledge of the desired vector, while the category C2) receiver uses its timing estimate to obtain a near-perfect estimate of the desired vector.[11] In view of these remarks, we recommend switching from blind adaptation to decision-directed MMSE adaptation as soon as possible.

G. Category C3) Receivers

Since the same correlation matrix $\mathbf{R}$ can correspond to infinitely many possibilities for the choice of signal vectors $\{\mathbf{u}_j\}$ in the equivalent synchronous model (12), it is clear that second-order statistics alone cannot be used to discriminate between different users without some prior knowledge about the user(s) of interest. However, for a system with only one user, it is possible to use second-order statistics for category C3) reception by exploiting the relationship between the symbols and the vectors (corresponding to desired signal and ISI) appearing in successive received vectors $\mathbf{r}_n$ and $\mathbf{r}_{n+1}$. To the best of our knowledge, the first authors to consider blind equalization and channel identification using second-order statistics were Tong *et al.* in [59]. Slock [52] applies second-order methods to a multiuser system, using a linear prediction framework to partially identify a multi-input, multi-output system (Slock points out that the remaining uncertainties would need to be resolved by higher order techniques). For a survey of further work along these lines see [60], and for applications of similar ideas for blind channel identification and demodulation for a system with a single DS signal see [65].

IV. Comments on Higher Order Statistics Methods

Consider the equivalent synchronous model (12). We first give a (new) limiting result which makes evident the theoretical potential of HOS methods for separating digitally modulated sources. We then comment on the relevance of two specific HOS methods, source separation based on fourth-order cumulants and CMA, to CDMA applications.

A. A Limiting Result

Assume that the symbols $b_j[n]$ in (24) are independent random variables chosen from a known discrete alphabet. It is possible to identify an arbitrarily large number of signal vectors $\{\mathbf{u}_j\}$ using HOS methods (allowing the orders of the statistics used to be arbitrarily large). That is, the number $J+1$ of identifiable signal vectors tends to infinity even though the dimension L of the received vectors $\{\mathbf{r}_n\}$ is fixed.

For an informal proof of this result, see the Appendix.

Remark IV.1—Caveats: The preceding result is purely theoretical, in that practical algorithms to identify arbitrarily many signal vectors with reasonable complexity and convergence speeds (i.e., number of received vector

[11] Conversely, we have checked that using a perfect estimate of $\mathbf{R}$ and a noisy LS estimate (21) of $\mathbf{u}_0$ also performs worse than the training based implementation using noisy LS estimates of both $\mathbf{R}$ and $\mathbf{u}_0$, giving further support to our claim of a benevolent interaction between the errors in the LS estimates of $\mathbf{R}$ and $\mathbf{u}_0$.

samples $\{\mathbf{r}_n\}$ required for accurate estimation) are probably infeasible. Furthermore, even if arbitrarily many signal vectors can be identified, reliable demodulation using linear receivers is not possible if the linear independence condition stated in Section III is not satisfied.[12] Note also that the convergence of HOS-based algorithms is more sensitive to noise than SOS-based algorithms, especially as the order of the statistics increases. Algorithms that specifically exploit the finite alphabet property for source separation have been proposed in [58].

B. Fourth-Order Cumulant-Based Methods

In theory, the blind source separation methods based on fourth-order cumulants (see [7]), although originally intended for antenna arrays, apply directly to the model (12). Indeed, these methods have been shown to be able to identify "more sources than sensors," which translates in the context of (12) to the number $J+1$ of signal vectors exceeding the dimension L of the received vectors $\{\mathbf{r}_n\}$. This is a practical corroboration of the limiting result stated previously. Fourth-order methods (possibly enhanced by prior knowledge regarding spreading waveforms) deserve further investigation for CDMA applications, especially for training antenna arrays when precise array calibration is not available, or when the angular spread of the multipath for a given transmission is large.

C. CMA

CMA was first proposed in the context of equalization (i.e., suppression of ISI) [15], [62], and has been studied extensively since then (see [25] in this special issue for a review). In view of the analogy between ISI and MAI, it is natural to raise the question of the applicability of CMA to multiuser applications since it is perhaps the simplest HOS-based receiver algorithm. To this end, consider (12), which applies to either intersymbol interference or multiple-access interference. Assume that the symbols $b_j[n]$ are constant modulus, i.e., $|b_j[n]| = 1$ (although CMA is known to work for constellations that do not satisfy this property as well).

CMA chooses a linear receiver $\mathbf{c}$ that minimizes the deviation of the receiver output from a constant modulus; in particular, consider the cost function $E\{[|\langle \mathbf{c}, \mathbf{r}_n\rangle|^2 - 1]^2\}$. The hope is that this cost function causes "locking on" to the contribution of a particular (constant modulus) desired symbol while making the contribution of interfering symbols (ISI and MAI) as small as possible. However, the cost function does not distinguish between desired and interfering symbols, which leads to a number of local minima.

For the equivalent synchronous model (12), for $0 \le j \le J$, let $\tilde{\mathcal{S}}_j$ denote the subspace spanned by all signal vectors except for $\mathbf{u}_j$, i.e., $\tilde{\mathcal{S}}_j$ is spanned by $\mathbf{u}_0, \cdots, \mathbf{u}_{j-1}, \mathbf{u}_{j+1}, \cdots, \mathbf{u}_J$. Let $\mathbf{v}_j = \mathcal{P}^\perp{}_{\tilde{\mathcal{S}}_j}\mathbf{u}_j$ denote the projection of $\mathbf{u}_j$ orthogonal to the space spanned by the remaining signal vectors. If $\mathbf{v}_j \neq 0$, then any scalar multiple of it provides a zero-forcing receiver for demodulation of $b_j[n]$. In particular, in the absence of noise, it is possible to choose a correlator $\mathbf{c}_j$ along $\mathbf{v}_j$ such that $\langle \mathbf{c}_j, \mathbf{r}_n\rangle = b_j[n]$, which is constant modulus. Thus, the zero-forcing receivers for each symbol $b_j[n]$ (when they exist, i.e., when $\mathbf{v}_j \neq 0$) are local minima of the CMA cost function. In the presence of noise, the MMSE receivers corresponding to the different signal vectors approximate the local minima of the CMA cost function (see [25] in this issue for a discussion of how the CMA cost function approximates the MSE locally). See [10], [27], and [51] for approaches to characterizing the local minima of CMA and other blind equalizers.

Remark IV.2—Global Minima for Infinite Length Equalizers: For a single digital source, an infinite observation interval implies that the (infinitely many) signal vectors $\mathbf{u}_j$ are simply shifted versions of each other, as are the corresponding zero-forcing receivers $\{\mathbf{v}_j\}$. Thus, any of the zero-forcing receivers constituting the local minima in the preceding result are equally good from the point of view of performance. This reasoning does not apply either for a finite observation interval or for a multiuser system, since the zero-forcing solutions for different ISI and MAI symbols in an observation interval are not equivalent.

Remark IV.3—Discriminating Between Local Minima: In a multiuser context, we would like to arrive at the zero-forcing receiver for a particular symbol of the particular transmission of interest. A preliminary attempt to apply blind equalization techniques to a multiuser context appeared in [43], but this paper did not satisfactorily address the issue of discriminating between different zero-forcing receivers. Since then, several methods have been proposed in the literature for this purpose. One is to allow the algorithm to lock on to one user, and to then subtract or project out its contribution to the received vector [26]. The result is fed to a second algorithm, which is expected to lock on to a second user, and so on. Another approach is to run parallel versions of CMA, coupled by additional constraints that are designed to make the outputs of different versions uncorrelated [3], [8]. The objective is to use the uncorrelatedness of the symbols $b_j[n]$ to force different versions to converge to different zero-forcing receivers. Of course, the number of versions needed might be reduced because of the fact that signal vectors $\mathbf{u}_j$ with small orthogonal projections $\mathbf{v}_j$ correspond to zero-forcing receivers which constitute shallow minima (especially at moderate noise levels, due to the noise enhancement associated with small $\mathbf{v}_j$) of the CMA cost function. Disadvantages of both approaches include slow convergence and complexity, due to the large number of parallel versions needed for multiuser applications (many MAI symbols) or for highly dispersive channels (many ISI symbols). As an overall comment, searching for the zero-forcing receiver for the desired symbol using such general techniques may be impractical for CDMA applications, where the number of signal vectors as well as the number of correlator taps is large.

[12] On the other hand, for nonlinear receivers, the idea of "stripping," or successive interference cancellation, might lead to reliable demodulation even without linear independence if the energies of the signal vectors are sufficiently disparate.

Remark IV.4—More on Constraints: If the propagation channel and spreading sequence of the desired transmission is known, it is possible to use a nominal desired vector to constrain the correlator $\mathbf{c}$ as in the CMOE algorithm. Since the CMA cost function prevents the output from being forced to zero, CMA has the advantage of being more robust to mismatch than the CMOE algorithm. Preliminary results in this direction were reported in [19], where constrained optimization of the Sato cost function $E[|\langle \mathbf{c}, \mathbf{r}_n\rangle - \text{sgn}(\langle \mathbf{c}, \mathbf{r}_n\rangle)|^2]$ was shown to be more robust to mismatch than the CMOE algorithm. Thus, the constrained optimization of cost functions other than the output energy under multiple hypothesized nominals is an interesting option to explore in the context of blind signal detection for CDMA. The practicality of algorithms based on fourth-order cost functions such as CMA, however, may be limited by their poorer convergence compared to algorithms such as CMOE, which are based on second-order cost functions.

V. Conclusions

We conclude with the following comments.

1) The equivalent synchronous model provides a simple tool for design and performance analysis of interference suppression techniques, for CDMA as well as for any system with ISI and/or MAI. In particular, it provides geometric insights into both SOS and HOS methods.
2) For single path channels, blind SOS algorithms that give accurate timing estimates for all ranges of interference amplitudes (including a severe near–far problem) are now available. These timing estimates can then be used to compute a near–far resistant demodulator. Further research is needed for obtaining and evaluating the performance of extensions of these algorithms for multipath channels.
3) While adaptive equalizers are often implemented using the relatively low-complexity LMS algorithm, the latter may be inappropriate for rapidly time-varying systems and near–far settings. Thus, adaptive receivers for CDMA applications will probably need to use more complex LS or subspace-based implementations, which is a challenge.
4) The blind adaptive receiver in 2) converges slower than training-based adaptation based on the MMSE criterion. Thus, whenever possible, blind CMOE adaptation should be switched to decision-directed MMSE adaptation. While standard decision-directed adaptation does not work for fading channels [80], modified decision-directed mechanisms that are more robust are now available [20], [80], [81]. Robust receiver design using an appropriate combination of training, decision-directed adaptation, blind CMOE adaptation, and subspace-based methods, remains an important topic for future research. The performance of any such receiver design must be validated in severely time-varying environments, including fading, shadowing, and time variations due to the arrival and departure of interfering transmissions.
5) While the blind interference suppression techniques considered here naturally extend to systems with antenna arrays, optimizing the complexity and convergence associated with such extensions requires further study.
6) Blind HOS-based reception techniques developed in the context of equalization or source separation apply directly to the equivalent synchronous model presented here. As such, the applicability of source separation methods based on fourth-order cumulants, or of constrained versions of CMA, to the context of CDMA deserves further investigation. A limiting result stated in Section IV also leads to the following intriguing concept: HOS-based methods can (in theory) identify a very large number of signal vectors, and nonlinear successive interference cancellation methods can (in theory) reliably demodulate them in the presence of sufficient power disparity, so that the combination could potentially lead to blind reception techniques for very high-capacity systems. Of course, this is purely theoretical speculation, since there are difficulties in the practical implementation of both HOS-based methods and nonlinear interference cancellation.

Appendix

We sketch here an informal proof of the result stated in Section IV-A regarding the ultimate limits of HOS methods applied to the equivalent synchronous model (12). For simplicity, consider binary signaling, i.e., the symbols $b_j[n] \in \{-1, 1\}$. In the limit of many observations and arbitrarily complicated estimates, HOS methods give us the distribution of the received vectors $\{\mathbf{r}_n\}$. In this limit noise can be ignored, so that the received vectors $\{\mathbf{r}_n\}$ have a discrete distribution given as follows. Since the $J+1$ symbols occurring in (12) are independent, $\mathbf{r}_n$ can take 2^{J+1} values, given by

$$\mathbf{v}_i = \pm\mathbf{u}_0 \pm \mathbf{u}_1 \pm \cdots \pm \mathbf{u}_{J+1}, \qquad i = 0, 1, \cdots, 2^{J+1} - 1. \tag{30}$$

If the vectors $\{\mathbf{v}_i\}$ are distinct, then each has probability $1/2^{J+1}$ of occurrence. If m of the $\mathbf{v}_i$ coincide, the corresponding vector occurs with probability $m/2^{J+1}$.

In the limit, the vectors $\{\mathbf{v}_i\}$ where the discrete distribution places its probability mass, as well the probabilities of occurrence of the $\mathbf{v}_i$, are perfectly known. The smallest possible probability of occurrence of a given $\mathbf{v}_i$ is $1/2^{J+1}$, which immediately yields the number of signal vectors $J+1$ (if all such probabilities are larger than this, then the number of nonzero signal vectors must be less than $J+1$).

Suppose first that the $\mathbf{v}_i$ in (30) are all distinct, so that there are 2^{J+1} distinct vectors $\mathbf{v}_i$. Now, take all $2^{J+1}(2^{J+1}-1)$ possible differences $\mathbf{v}_i - \mathbf{v}_j$ for $i \neq j$. It can be checked that, for each $0 \leq j \leq J$, the term $2\mathbf{u}_j$ occurs 2^J times, and the term $-2\mathbf{u}_j$ occurs 2^J times. Other distinct linear combinations of the signal vectors occur less frequently. Thus, the signal vectors $\mathbf{u}_j$ are identified (upto sign) as the most frequently occurring differences.

If the number of vectors $\mathbf{v}_i$ is smaller than 2^{J+1}, then replicate m times any vector which has probability mass $m/2^{J+1}$ before applying the argument in the previous paragraph. This concludes the proof.

Remark A.1—More Caveats: Elaborating further on remark IV.1 on the limited practical applicability of the result, note that estimating the distribution of the $\{\mathbf{r}_n\}$ accurately would require an exorbitant number of samples, and would be impossible in time-varying settings. In the presence of noise, the cluster points used in the proof are the local maxima of a continuous distribution which is a mixture of Gaussian distributions, so that identification of the cluster points and their probabilities is an issue. Finally, the "algorithm" itself, involving all possible differences of the cluster points, is too complex for practical applications.

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