Acquisition based capacity of a Synchronous Direct Sequence Spread Spectrum multiple access technique

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I. Introduction

In deriving the capacity of a direct sequence spread spectrum (DS/SS) multiple access system using the bit error rate (BER) [1], it is assumed that the receiver has already achieved chip synchronization with the spreading sequence used. In the absence of any other timing information [2] the *acquisition* based *capacity*, that is the maximum number of users that can be transmitting data while one or more users are achieving chip synchronization is also very important. It has been found that *asymptotically*, that is as the length N of the matched filter used to achieve the synchronization tends to infinity and p_{fa} the probability of acquisition failure tends to zero, the acquisition based capacity is upper bounded by $\frac{N}{2\ln N}$, hence forth denoted as J_{asymp}. However, this bound J_{asymp} is not tight for finite N [2]. In fact for a specified p_{fa} and N, the actual maximum number of users can be orders of magnitude lower than the predicted J_{asymp}. Thus, from a practical point of view the asymptotic bound is not very useful for a given problem at hand. In this paper, we formulate the acquisition based capacity for a *specified* p_{fa} and N.

II. Background

Consider the standard model [1, 2] for a DS/SS system with a total of $(J+1)$ users Without loss of generality say only one user, user one, is trying to achieve chip synchronization. For the simplicity of analysis, we assume that the frequency and phase of the carrier have been determined exactly and that the carriers of all the users and their chips are *synchronous*. The receiver is assumed to be a matched filter of length N, the filter coefficients being given by signature sequence of the user. *Synchronization* is said to be achieved if the receiver matched filter synchronizes with user's the signature sequence. Let τ denote the unknown delay by which the matched filter is out of sync with the signature sequence. For the purpose of our analysis, we consider that τ takes integer values between 0 and $(N-1)$. Let the matched filter output at time $0 \leq n \leq (N-1)$ be denoted by Y_n. Consider the one-shot acquisition scheme wherein the detector ignores the output of the matched filter once it exceeds a threshold $\eta < 1$.

III. Computation of the maximum number of users

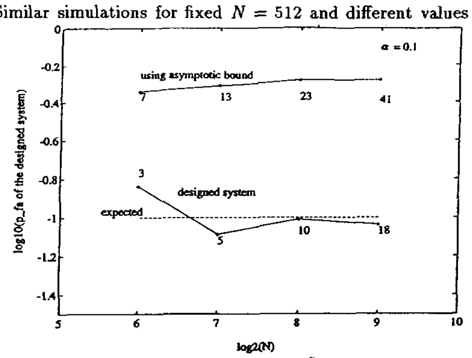
Define events $A = \{\cup_{n=0}^{\tau-1} Y_n > \eta\}$ and $B = \{Y_\tau < \eta\}$ and the probabilities; p_f = probability of false alarm = $\Pr\{A\}$, p_m = probability of miss = $\Pr\{B\}$ and p_{fa} = probability of acquisition failure = $\Pr\{A \cup B\}$. We assume that the signature sequence is composed of *Bernoulli* random variables taking the values +1 and -1 with an equal probability. This assumption is commonly made in the analysis of DS/SS multiple access systems [1, 2]. Now consider the problem of finding J_{max}, where $J_{max} = max J; \{p_{fa} \leq \alpha\}$ It is very difficult in general to compute the p_{fa} as a function of J since $\{Y_n\}_{n=0}^{n=\tau}$ are not independent. Hence, instead of computing J_{max}, we have considered the simpler problem of finding $\tilde{J}$, where

$$\tilde{J} = max J; \{p_{fa} \leq p_f + p_m \leq \alpha\} \tag{1}$$

Even though $\tilde{J} \leq J_{max}$, from the simulations it can be seen that $\tilde{J}$ does give a good estimate of the maximum capacity. Employing the union bounds for large J [2], p_f and p_m can be expressed as a function of J and η. Getting rid of η yields $J = N/(Q^{-1}(\frac{2p_f}{N-1}) + Q^{-1}(p_m))^2$; $Q(\cdot)$ being the error function. Now maximizing J with respect to p_f and p_m under the constraint in equation (1) yields for $\alpha \leq 0.1$, $p_f = p_m = \alpha/2$ and $\tilde{J} = N/(Q^{-1}(\frac{\alpha}{N-1}) + Q^{-1}(\frac{\alpha}{2}))^2$; $\eta = 1 - (\frac{J}{N})^{\frac{1}{2}} Q^{-1}(p_m)$. As $x \rightarrow 0$, $Q^{-1}(x) \rightarrow (-2\ln(x))^{\frac{1}{2}}$, hence as $N \rightarrow \infty$, $\tilde{J} \rightarrow J_{asymp}$, the asymptotic result obtained previously[2] for a synchronous system.

IV. Simulation Results

The $\tilde{J}$ is a function of N and α. Hence to verify whether $\tilde{J}$ is a good bound for J_{max}, simulations are performed by fixing one of the quantities and varying the other. In the figure below, the α is kept fixed and the p_{fa} obtained by simulations is plotted as a function of N for a system containing $\tilde{J}+1$ and $J_{asymp}+1$ number of users. As can be seen, in all the cases, the asymptotic bound predicts much larger numbers for J_{max}. Also, the false alarm probability for a system containing $\tilde{J}$ users is very close to the desired false alarm probability. Similar simulations for fixed $N = 512$ and different values of



α, show that the p_{fa} resulting from $\tilde{J}$ users is close to the required false alarm probability α. The $\tilde{J}$ thus gives a much tighter bound for J_{max} for finite N and non-zero α.

References

[1] Michael B. Pursley, "The Role of Spread Spectrum in Packet Radio Networks", *Proceedings of the IEEE*, vol.75, no. 1, pp. 116-134, 1987.

[2] Upamanyu Madhow & Michael B. Pursley, "Acquisition in Direct-Sequence Spread Spectrum Communication Networks: An Asymptotic Analysis", *IEEE Transactions on Information Theory*, vol.39, no. 3, pp. 903-912, 1993.

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