Space-Time Precoding with Mean and Covariance Feedback: Implications for Wideband Systems

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The focus of this paper is on optimizing the capacity of a cellular "downlink" in which the base station (BS) is equipped with multiple antennas, while the mobile has a single antenna. The BS has access to both the first and second order statistics of the channel, obtained from *mean feedback* and *covariance feedback*, respectively. Our results apply to wideband Orthogonal Frequency Division Multiplexing (OFDM) systems, where accurate covariance feedback can be obtained without overhead by averaging uplink measurements [2]. Mean feedback can be obtained from uplink measurements using reciprocity for Time Division Duplex (TDD) systems, and would require explicit feedback for Frequency Division Duplex (FDD) systems. Since mean feedback degrades rapidly with feedback delay for mobile channels, and requires feedback overhead for FDD systems, our purpose is to quantify the tradeoffs in using covariance feedback alone (which is obtained for free for both TDD and FDD systems), versus using both covariance and mean feedback. We restrict attention to input distributions that are white across frequency (but possibly spatially colored), so that it suffices to consider a single narrowband frequency bin for characterizing the ergodic capacity of a wideband system. Prior work in this setting focuses on optimizing the ergodic capacity given mean feedback alone, or covariance feedback alone [1].

For outdoor channels with narrow spatial spread, beamforming is often an optimal strategy [2] when covariance feedback is available. Beamforming is also optimal when accurate mean feedback is available [1]. Thus, we focus on finding a good beamforming strategy when both mean and covariance feedback are available. While the optimal beamformer for maximizing ergodic capacity is difficult to find, we show that the "maximum SNR" beamformer, which is easy to compute, is a practical approach that yields near-optimal results. This is done by providing an upper bound to the optimal ergodic capacity achievable by beamforming, and showing by numerical computation that the maximum SNR beamforming capacity is close to the bound. We also consider how accurate the mean feedback needs to be in order to be beneficial if the BS already has covariance information. We provide quantitative measures that validate the following intuition: the accuracy requirements for mean feedback are less stringent for channels with larger spatial spread, or for a larger number of transmit elements.

Model: The received signal for a particular frequency bin in an OFDM system can be written as $y = \mathbf{h}^H \mathbf{x} + n$, where $\mathbf{x}$ is the $N_T \times 1$ transmitted symbol vector, $\mathbf{h}$ is the $N_T \times 1$ channel response, and n is complex Gaussian noise. The channel, $\mathbf{h}$, is modeled as zero mean complex Gaussian with covariance $\mathbf{C}$. It is assumed that the BS knows the channel covariance, and also has some direct feedback, or mean feedback, $\mathbf{f}$, regarding the channel realization. We model $\mathbf{f}$ and $\mathbf{h}$ as jointly Gaussian with identical marginals and correlation ρ. Thus, the distribution of the channel when feedback is available is given as $\mathbf{h} \sim CN(\rho \mathbf{f}, (1-\rho^2)\mathbf{C})$.

Beamforming capacity: The ergodic capacity when the transmitter employs a beamforming direction ν (normalized to unit norm) is given by $C_{bf} = \max_\nu E[\log(1+SNR\mathbf{h}^H \nu \nu^H \mathbf{h})]$, where the expectation is taken over $\mathbf{h}$. While optimizing C_{bf} over ν is difficult, we can state the following upper bound on the optimal capacity:

Theorem 1: An upper bound on beamforming capacity is given as follows: $C_{bf} \leq E[\log(1+SNR|z^\star|^2)]$, where $z^\star$ is a complex Gaussian random variable with $E[z^\star] = \rho||f||$, and $var[z^\star] = (1-\rho^2)\lambda_1$. λ_1 is the largest eigenvalue of $\mathbf{C}$.

As an example, consider a system with $N_T = 6$ transmit antenna elements and a Laplacian power-angle profile of zero mean and angular spread 10^o. Figure 1 shows that $\bar{I}(\nu_0)$, the average mutual information when sending along the direction that maximizes receive SNR, is close to the upper bound on beamforming capacity for the entire range of ρ. Also displayed are $\bar{I}(\mathbf{u}_1)$, which is the average beamforming capacity with covariance feedback alone [1], and $\bar{I}(\mathbf{f}/||\mathbf{f}||)$, which is the average beamforming capacity with mean feedback alone.

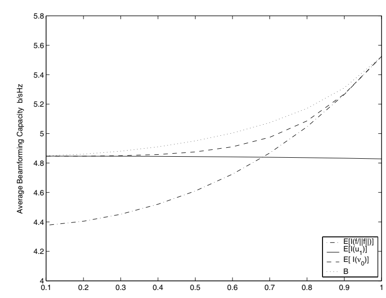

Fig. 1: The upper bound to average beamforming capacity, B, vs ρ, along with $\bar{I}(\mathbf{u}_1)$, $\bar{I}(\mathbf{f}/||\mathbf{f}||)$and $\bar{I}(\nu_0)$. $SNR = 10$.

References

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