Wideband Distributed Transmit Beamforming using Channel Reciprocity and Relative Calibration

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***Abstract*—This paper describes a technique for transmit beamforming from the nodes in a distributed radio network to a distant target node across a frequency-selective channel. The approach exploits signals transmitted by the target node and channel reciprocity to avoid requiring explicit channel state feedback from the target. Channel estimates between network radios are used for relative calibration to address non-reciprocal effects due to independent clocks and electronic component variability. Variants of the technique allow wideband coherent beamforming to the target when the target signal is known or when it is unknown.**

***Index Terms*—distributed beamforming, channel reciprocity, relative calibration, synchronization**

I. Introduction

It is well-known that channel state information at the transmitters (CSIT) can improve the efficiency of wireless communication and can facilitate coherent communication techniques including distributed beamforming (see [1] and the references therein) and distributed nullforming, e.g., [2]–[6]. The diversity and power growth associated with distributed beamforming allows a network of low-power radios to communicate more reliably and over much longer distances than single radios. In the context of the system model shown in Fig. 1, each node $j \in \{1, \ldots, N\}$ in the transmit cluster needs an estimate of the forward link (uplink) channel $H_{j\to 0}(f)$ to facilitate coherent transmission to the target node. One common approach is "feedback-based" techniques, e.g., [7]–[13], where the target node estimates the channels and feeds back quantized versions of these estimates to the transmit cluster. Feedback techniques suffer from overhead and latency which can be prohibitive, especially in large-scale "massive MIMO" [14] systems. Direct estimation of the uplink channel by the target quickly becomes SNR-limited as network sizes and communication ranges increase since isotropic channel sounding doesn't benefit from beamforming gain.

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An alternative approach are "reciprocity-based" techniques, e.g., [15], [16], where the transmit cluster estimates the uplink channels from signals emitted by the target node on the reverse link (downlink). These techniques assume the uplink and downlink are time division duplexed (TDD) and accessed on the same frequency. Reciprocity-based techniques are especially attractive in asymmetric links such as cellular systems where the transmit power available at the target node is considerably higher than the individual radios in the transmit cluster and downlink channels can be accurately estimated at much longer ranges.

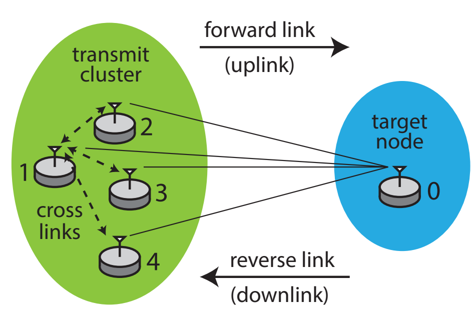

Fig. 1. System model with and N node transmit cluster and a single target node. Each node is assumed to have a single antenna. Node 0 corresponding to the target and node 1 corresponding to the master transmit node assumed to have a direct link to transmit nodes $j \in \{2, \ldots, N\}$.

Basic electromagnetic principles have long established that channel reciprocity holds at the antennas when the channel is accessed at the same frequency in both directions [17]. While propagation is inherently reciprocal, the radio frequency chains in each transceiver are generally not reciprocal and may vary with temperature, aging, and other effects. These effects can sometimes be mitigated with specialized transceiver architectures such as a reciprocal transceiver architecture, e.g., [18]. To compensate for time-varying effects, however, it is usually preferred to perform some type of transceiver calibration, e.g., [19]. A particularly interesting approach to compensating for non-reciprocal transceivers is *relative calibration* [20]. To summarize this approach, let $H_{j\to 0}(f)$ and $H_{0\to j}(f)$ denote the uplink and downlink channels, respectively, including the effects of the transceivers and clock offset. By exchanging messages between node j and node 0 and receiving feedback[1] from node 0 regarding the $j \to 0$ channel, node j can estimate the relative calibration function

$$\frac{H_{j\to 0}(f)}{H_{0\to j}(f)} \tag{1}$$

using, for example, a structured total least squares solver [21]. Then, given a downlink signal from node 0 to node j, node j can simply estimate the downlink channel $H_{0\to j}(f)$ multiply this estimate by the relative calibration function in (1) to generate a corresponding estimate of the uplink channel for subsequent use during coherent transmission.

The main contribution of this paper is an explicit description of a techniques for implementing distributed beamforming using channel reciprocity in a scenario where the nodes in the transmit cluster can not perform relative calibration with the target node. The techniques described in this paper can be used in scenarios where the target node is unable to exchange calibration messages with the transmit cluster. For example, the target node may not have the capability to estimate the uplink channels and provide feedback for relative calibration. Our techniques are based on *indirect relative calibration* where the nodes in the transmit cluster exchange messages only among themselves and pre-calibrate prior to the target node emitting a signal. The two variants of this approach, distinguished by whether the signal emitted by the target node is known or unknown, are described. In both cases, we show that the transmit cluster can compute appropriate precoder filters to pre-compensate for clock offsets, electronics and propagation delays and achieve wideband coherent combining at the target node. Numerical results characterize the performance of the variants in terms of the uplink channel capacity when compared to the capacity when perfect CSIT is available.

II. System Model

We consider the system shown in Figure 1 with nodes numbered $\{0, \dots, N\}$. Each node is assumed to have a single antenna. Node 0 corresponds to the intended target. Nodes $j \in \{1, \dots, N\}$ comprise the transmit cluster and node 1 corresponds to the "master" transmit node. It is assumed that node 1 can directly communicate with each of the other transmit nodes $j \in \{2, \dots, N\}$ in the transmit cluster.

The effective channel between any pair of nodes $i \to j$ can be expressed as

$$\mathsf{CLK}_i \to \mathsf{TX}_i \to \mathsf{PROP}_{i\to j} \to \mathsf{RX}_j \to \mathsf{CLK}_j^{-1}. \tag{2}$$

For simplicity, we assume all of these effects are linear and slowly time varying so that the frequency response over time intervals of interest is approximately constant. We can also lump the effect of the clock and transmit electronics at node i as $T_i(f)$ and, similarly, the effect of the clock and receive electronics at node j as $R_j(f)$. Then, in the frequency domain, we have

$$H_{i\to j}(f) = \underbrace{T_i(f)}_{\text{transmitter}}\underbrace{G_{i\leftrightarrow j}(f)}_{\text{propagation}}\underbrace{R_j(f)}_{\text{receiver}}. \tag{3}$$

In general, while propagation is reciprocal such that $G_{i\leftrightarrow j}(f) = G_{j\leftrightarrow i}(f)$, the transmitter and receiver functions are non-reciprocal and $H_{j\to i}(f) \neq H_{i\to j}(f)$. From (3) it follows immediately that for any sequence of nodes, the clock-wise and counter-clockwise concatenation of their channels are equal [15], [16]. In particular, for every node $j \in \{2, \dots, N\}$

$$H_{j\to 0}(f)H_{0\to 1}(f)H_{1\to j}(f) = H_{1\to 0}(f)H_{j\to 1}(f)H_{0\to j}(f) \tag{4}$$

which can be rewritten in two ways as

$$\begin{aligned} H_{j\to 0}(f) &= \frac{H_{1\to 0}(f)}{H_{0\to 1}(f)} \qquad \times \frac{H_{j\to 1}(f)H_{0\to j}(f)}{H_{1\to j}(f)} && \text{(5a)} \\ &= H_{1\to 0}(f) \qquad \times \frac{H_{j\to 1}(f)H_{0\to j}(f)}{H_{1\to j}(f)H_{0\to 1}(f)}, && \text{(5b)} \end{aligned}$$

showing that the uplink channels are products of a term that is common across all the uplink channels (left term) times a term involving only the downlink and crosslink channels (right term). The two key insights behind our beamforming approach are

1) Beamforming coherence only requires precoding to correct for the *relative* uplink channels; distortions common to all uplink channels (left terms) can be equalized by the target node.
2) In addition to the downlink channels, we are also able to measure the crosslink channels via node-to-node channel sounding (right terms).

III. Reciprocal Beamforming Protocols

In this section, we develop reciprocal beamforming protocols for two cases: (i) known downlink signals and (ii) unknown downlink signals. In the former case, the transmit cluster can use the known downlink signal to directly estimate the downlink effective channels $H_{0\to j}(f)$ for $j \in \{1, \dots, N\}$. In the latter case, we assume the transmit cluster can not directly estimate the downlink effective channel due to the fact that the downlink signal is unknown.

Both protocols can be described in four phases:

- Phase I: Transmit cluster pre-calibration
- Phase II: Transmit cluster reception of waveform from target
- Phase III: Precoder estimation
- Phase IV: Transmit cluster beamforming

Both cases have have essentially the same transmit cluster pre-calibration phase (Phase I) but differ in the remaining phases. The following sections provide details on the four phases for the cases with known and unknown downlink signals.

[1]Note that the relative calibration feedback from node 0 is infrequent since it is only used to compensate for transceiver non-reciprocity.

A. Known Downlink Signals

1) Phase I: Transmit cluster pre-calibration: In this phase, each node $j \in \{2, \ldots, N\}$ exchanges known messages with the master node (node 1) to synchronize their clock frequencies and estimate the relative calibration function $\frac{H_{1\to j}(f)}{H_{j\to 1}(f)}$. Specifically, each node $j \in \{2, \ldots, N\}$ sends a known message to the master node and the master node estimates the effective channels $H_{j\to 1}(f)$. Node 1 then broadcasts one or more known messages and each node $j \in \{2, \ldots, N\}$ estimates the effective channel $H_{1\to j}(f)$. By also feeding back estimates of $H_{j\to 1}(f)$ from the master node, node j can then estimate the relative calibration function

$$\frac{H_{j\to 1}(f)}{H_{1\to j}(f)} = \frac{T_j(f)R_1(f)}{T_1(f)R_j(f)}, \tag{6}$$

In the presence of noise, these estimates can be generated with a structured total least squares (STLS) solver as discussed in [20]. Accurate frequency synchronization can also be achieved during this phase since each node $j \in \{2, \ldots, N\}$ can adjust its clock rate according to the observed frequency offset in messages received from node 1.

2) Phase II: Transmit cluster reception of waveform from target: The target (node 0) transmits the *known* signal $X_0(f)$. Nodes $j \in \{1, \ldots, N\}$ then receive

$$Y_j(f) = H_{0\to j}(f)X_0(f). \tag{7}$$

We assume $X_j(f)$ is non-zero on all f. In this case, node j can directly estimate the quantity

$$\frac{Y_j(f)}{X_0(f)} = H_{0\to j}(f). \tag{8}$$

3) Phase III: Precoder selection: From (5a) we see that the uplink channel $H_{j\to 0}(f)$ is, up to a common term, the product of (6) and (8). We a priori assume the unknown common term is the identity $\frac{H_{1\to 0}(f)}{H_{0\to 1}(f)} = 1$ which represents the an information-less prior. The nodes can now compute precoders for the resulting uplink channels:

$$W^{\mathrm{KD}}_{j\to 0} = \begin{cases} H_{0\to 1}(f) & \text{if } j = 1 \\ \frac{H_{j\to 1}(f)}{H_{1\to j}(f)} H_{0\to j}(f) & \text{if } j = 2 \ldots N \end{cases} \tag{9}$$

In a distributed application, the transmit power at each node will be fixed and thus we will have a unity-gain constraint on *each* of the precoders. The capacity-maximizing precoder formulation in this scenario is derived in [22]. A sub-optimal but effective alternative, with a closed form, is to select the precoder to be the scaled conjugate of the relative uplink channel, i.e.

$$P^{\mathrm{KD}}_j(f) = \frac{\overline{W}^{\mathrm{KD}}_{j\to 0}(f)}{\sqrt{\int_f |W^{\mathrm{KD}}_{j\to 0}(f)|^2 df}} \tag{10}$$

A practical precoding scheme must introduce a bulk beamforming delay across all the nodes to insure the resulting precoders are causal.

4) Phase IV: Transmit cluster beamforming: Upon some local clock trigger, each node $j \in \{1, \ldots, N\}$ transmits the common signal $X(f)$ to the target with precoder filter $P^{\mathrm{KD}}_j(f)$ to form a beam at the target. Note that the precoder filters align the signals so that they combine coherently at node 0, including compensating for clock offsets among the transmit nodes. For the precoders defined in (10), the aggregate SISO channel seen at node 0 is

$$\begin{aligned} H^{\mathrm{KD}}_{\mathrm{BF}\to 0}(f) &= \sum_{j=1}^{N} P^{\mathrm{KD}}_j(f) H_{j\to 0}(f) \\ &= \frac{H_{1\to 0}(f)}{H_{0\to 1}(f)} \sum_{j=1}^{N} \frac{|W^{\mathrm{KD}}_{j\to 0}(f)|^2}{\sqrt{\int_f |W^{\mathrm{KD}}_{j\to 0}(f)|^2 df}} \end{aligned} \tag{11}$$

For reasonable transceiver designs, $\frac{H_{1\to 0}(f)}{H_{0\to 1}(f)} = \frac{R_0(f)T_1(f)}{T_0(f)R_1(f)}$ is approximately flat and should not typically introduce deep nulls or other undesirable artifacts into the received signal at node 0. Hence, in the case with known downlink signals, the transmit cluster effectively equalizes the uplink channels to node 0. Residual amplitude variation can be accommodated via receive-side equalization.

B. Unknown Downlink Signals

1) Phase I: Transmit cluster pre-calibration: This phase is identical to the case with known downlink received signals. One minor difference is that nodes $j \in \{2, \ldots, N\}$ do not need to estimate the relative calibration function as in (6) but, rather, as will be seen in Phase III, only need to form an estimate of $H_{j\to 1}(f)$ which can be obtained via feedback from node 1.

2) Phase II: Transmit cluster reception of waveform from target: The target (node 0) transmits the *unknown* signal $X_0(f)$. Nodes $j \in \{1, \ldots, N\}$ then receive $Y_j(f) = H_{0\to j}(f)X_0(f)$. Since $X_0(f)$ is unknown, node j can not directly separate $H_{0\to j}(f)$ from $X_0(f)$.

Node 1 now rebroadcasts its received signal in Phase II to nodes $j \in \{2, \ldots, N\}$. The delay between target signal reception and rebroadcast is known to all nodes and ignored here. Node j then receives

$$\begin{aligned} Z_j(f) &= H_{1\to j} Y_1(f) \\ &= H_{1\to j}(f) H_{0\to 1}(f) X_0(f) \end{aligned} \tag{12}$$

Node $j \in \{2, \ldots, N\}$ now computes the *quotient channel*

$$\begin{aligned} Q_j(f) &= \frac{Y_j(f)}{Z_j(f)} \\ &= \frac{H_{0\to j}(f)}{H_{1\to j}(f) H_{0\to 1}(f)}. \end{aligned} \tag{13}$$

3) Phase III: Precoder selection: From (5b) we see that the uplink channel $H_{j\to 0}(f)$ is, up to the common term $H_{1\to 0}(f)$, the product of $H_{j\to 1}(f)$ and (13). As before we assume the information-less prior $H_{j\to 0}(f) = 1$ and compute precoders for the resulting uplink channels:

$$W^{\mathrm{UD}}_{j\to 0} = \begin{cases} 1 & \text{if } j = 1 \\ Q_j(f) H_{j\to 1}(f) & \text{if } j = 2 \ldots N \end{cases} \tag{14}$$

The same precoder approach as in the known-downlink signal case can be used here as well. The simple but sub-optimal scaled conjugate precoder is

$$P_j^{\mathrm{UD}}(f) = \frac{\overline{W}_{j\to 0}^{\mathrm{UD}}(f)}{\sqrt{\int_f |W_{j\to 0}^{\mathrm{UD}}(f)|^2 df}} \tag{15}$$

4) Phase IV: Transmit cluster beamforming: This phase is also identical to the case with known downlink signals. For the precoders defined in (15), the aggregate SISO channel seen at node 0 is

$$\begin{aligned} H_{\mathrm{BF}\to 0}^{\mathrm{UD}}(f) &= \sum_{j=1}^{N} P_j^{\mathrm{UD}}(f) H_{j\to 0}(f) \\ &= H_{1\to 0}(f) \sum_{j=1}^{N} \frac{|W_{j\to 0}^{\mathrm{UD}}(f)|^2}{\sqrt{\int_f |W_{j\to 0}^{\mathrm{UD}}(f)|^2 df}} \end{aligned} \tag{16}$$

Unlike the case with known downlink signal, we observe that this aggregate channel includes the effect of the propagation channel $G_{0\leftrightarrow 1}(f)$. The uplink channels are not equalized (except to match the $1 \to 0$ effective channel). Hence, the performance of this technique may be sensitive to the choice of the master node (node 1).

C. Remarks

Most practical channels have a maximum possible delay spread which imposes a smoothness constraint on their transfer functions. This smoothness constraint can be imposed to insure the stability of estimators (8) and (13) which involve the ratio of noisy samples.

IV. Numerical Results

The beamforming performance of our techniques can be quantified in terms of the resultant ergodic capacities of the beamforming channels (11) and (16). This ergodic capacity [23] is given by:

$$\mathbb{E}_{H_{i\to j}}\left(\int_f \log_2\left(1 + |H_{\mathrm{BF}\to 0}(f)|^2\right) df\right) \tag{17}$$

Fig. 2 shows the ergodic capacity vs. per-transmitter SNR for known and unknown downlink signals for a $N = 10$ node network with $L = 8$ independent frequency subcarriers. The transmitter and receiver transfer functions $T_j(f)$ and $R_j(f)$ were chosen to be unity gain and to have independent random phases between nodes and subcarriers. The propagation channels $G_{i\leftrightarrow j}(f)$ were modeled as independent, identically distributed complex Gaussian random variables with variance SNR/L. In this simple scenario, we model the propagation channels as IID across subcarriers and between nodes.

For this scenario we note that there is less than a 0.25 bits/s/Hz reduction in capacity associated with not knowing the downlink signal. The results highlight the potential benefit of distributed transmit beamforming for extending communications range: spectral efficiencies greater than 1 bit/s/Hz can theoretically be achieved even for per-transmitter SNRs below -10dB.

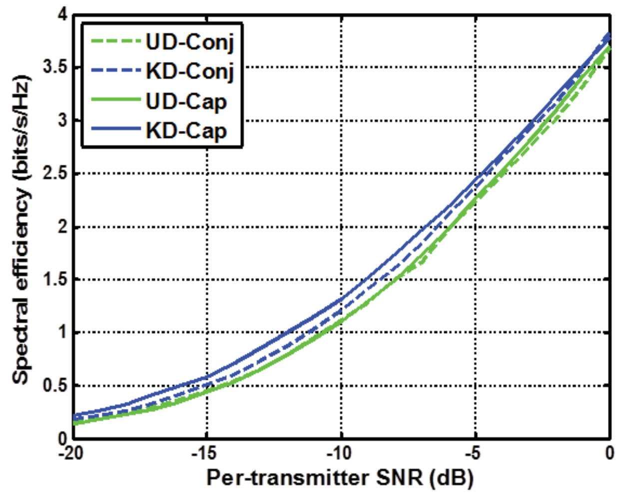

Fig. 2. Ergodic capacities of 10-node frequency-selective beamforming channels for known downlink (KD=blue) and unknown downlink (UD=green) signals. Solid line shows capacity-maximizing precoder [22], dashed line shows simple scaled-conjugate precoder (10, 15).

V. Conclusion

The proliferation of networked wireless devices enables the possibility of distributed coherent communications which can provide much longer uplink ranges and better quality-of-service than possible with single devices. While much of the early work in this area has looked at feedback-based techniques, direct uplink CSI estimation at the target node becomes SNR-limited as communications ranges increase. Reciprocity-based techniques that estimate downlink CSI potentially benefit from higher transmit powers at the target node, but independent clocks and electronic components introduce non-reciprocal components which must be addressed. The contribution of this paper is a practical method of using crosslink channel estimates to provide an indirect relative calibration for these effects. We show that this technique is applicable whether or not the downlink signal from the target node is known.

The technique in this paper relies on a star-topology within the radio network; future work could address the generalization of this technique to more arbitrary network topologies.

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