Scalable mismatch compensation for time-interleaved A/D converters in OFDM reception [1]

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***Abstract*—Realization of all-digital baseband receiver processing for multi-Gigabit communication requires analog-to-digital converters(ADCs) of sufficient rate and output resolution. A promising architecture for this purpose is the time-interleaved ADC (TI-ADC), in which several "sub-ADCs" are employed in parallel. However, the timing mismatch between the sub-ADCs, if left uncompensated, leads to error floors in receiver performance. Standard linear digital mismatch compensation (e.g., based on the zero-forcing criterion) requires a number of taps that increases with the desired resolution. In this paper, we show that oversampling provides a scalable (in the number of sub-ADCs and in the desired resolution) approach to mismatch compensation, allowing elimination of mismatch-induced error floors at reasonable complexity. While the structure of the interference due to mismatch is different from that due to a dispersive channel, there is a strong analogy between the role of oversampling for mismatch compensation and for channel equalization. We illustrate the efficacy of the proposed mismatch compensation techniques for an OFDM receiver.**

I. Introduction

The analog-to-digital converter (ADC) is a critical component in modern digital communication receivers, enabling cost-effective, all-digital implementation of sophisticated baseband signal processing algorithms. However, as communication bandwidths increase, the availability of ADCs with sufficient speed and resolution becomes a concern: Gigahertz bandwidths are required for emerging ultrawideband and millimeter wave [1] applications, while 8-12 bits of resolution are required for providing enough dynamic range when operating in multipath environments with large constellations. The technology of choice at GHz speeds is "one shot" Flash ADC, but it becomes unattractive beyond 5 bits resolution, due to exponentially (in number of bits) increasing power consumption and hardware complexity [5]. An attractive alternative is a time-interleaved (TI) architecture (Refer Fig. 1.), where several low rate, high resolution, "sub-ADCs" can be operated in parallel to obtain a high overall rate and resolution. However, an inherent problem with the TI-ADC architecture is mismatch between the sub-ADCs [7]. Left uncompensated, such mismatch leads to error floors when TI-ADCs are employed in communication receivers [13]. We consider a linear model for the TI-ADC mismatch by assuming that each sub-ADC is modeled as a linear, time-invariant channel. In addition, since effective algorithms for estimating the mismatch parameters are available [8], [10], [13], we assume that perfect estimates of these parameters are available.

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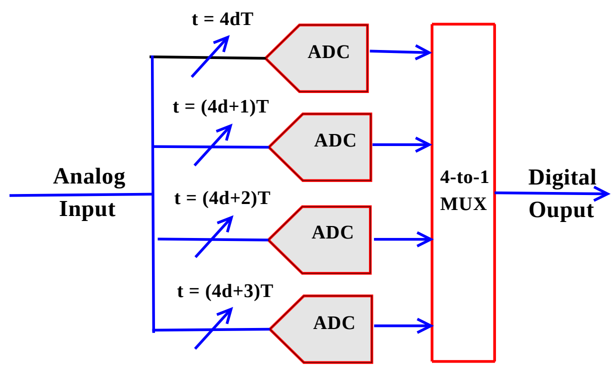

Fig. 1. Ideal time-interleaved ADC with 4 sub-ADCs. The sampling instants (d = integer, T_o = sampling period) of the sub-ADCs are staggered such that each sub-ADC operates at one-fourth of the net sampling rate of the TI-ADC

A. Contributions

In order to alleviate the performance floor due to mismatch, we consider the standard technique of linear mismatch compensation (which can be implemented digitally using the quantized samples at the output of the TI-ADC). When the overall TI-ADC operates at the desired sampling rate, the number of taps required scales up rapidly with the desired resolution. The main contribution of this paper is to show that the number of taps can be reduced significantly by the use of oversampling. While we find through our simulation results that oversampling by 25% is effective, for the special case of sampling at twice the symbol rate, we prove that a Bezout-like identity holds for mismatch compensation, so that perfect zero-forcing compensation can be guaranteed using a finite number of taps. This is analogous to results from fractionally spaced channel equalization, even though the detailed interference structure due to mismatch is different from that due to intersymbol interference. In practical terms, our results imply that, when the system bandwidth and the sampling rate of an individual sub-ADC are fixed, we can achieve better effective resolution at lower complexity by increasing the number of sub-ADCs beyond the minimum required.

B. Related work and Comparison

Digital mismatch compensation for TI-ADCs has received a great deal of attention in the literature [8]-[13]. Since the exact zero-forcing equalizers are of infinite length, with slowly decaying taps, truncated/least-squares solutions were employed in [8], [9], [10]. Even then, a large number of taps were needed when the resolution requirement and/or the mismatch range is large.

Oversampling was priorly used to aid in mismatch compensation [10], [11], [12]. Specifically in [10], the mismatch estimation is facilitated by the use of oversampling but the compensation needed 41-tap FIR filters. In [11], [12], FFT-based compensation is proposed for higher accuracy but the calculation of many FFTs (equal to the number of sub-ADCs) seems expensive for a typical communication receiver setting.

In our own prior work on OFDM-specific mismatch compensation for TI-ADCs, we developed a frequency domain approach whose complexity scales with L, the number of sub-ADCs (regardless of the mismatch level) when the number of subcarriers is a multiple of L.

C. Organization

The paper is organized as follows. In section II, we describe a z-domain discrete-time model for TI-ADC mismatch and zero-forcing mismatch compensation. Section III discusses how oversampling can reduce the number of taps required for mismatch compensation, including a proof that mismatch-induced interference can be eliminated using finite-length filters when we oversample by a factor of two. Section IV illustrates performance-complexity tradeoffs for an OFDM receiver, and Section V contains our conclusions.

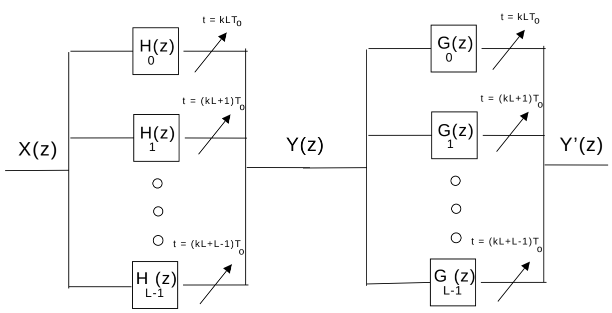

Fig. 2. Linear model for mismatch in a TI-ADC and zero-forcing based mismatch compensation (k=integer, T_o=sampling period). All symbols indicate discrete streams at the symbol rate T_o^{-1}.

II. System Model

We first elaborate on a linear mismatch model for the TI-ADC and then give details of the linear schemes (based on zero-forcing equalization) employed for mismatch compensation.

A. TI-ADC model

We consider the problem of sampling an analog signal $x(t)$ with the sampling period T_o^{-1}. The desired digital samples, denoted by $x[n] = x(nT_o)$, would be referred to as *symbols* and by *symbol rate sampling*, we mean sampling at the rate of T_o^{-1}. We assume that the values of the continuous signal $x(t)$ can be obtained by interpolating the symbols as

$$x(t) = \sum_{n=-\infty}^{\infty} x[n]h(t - nT_o) \quad (1)$$

where $h(t)$ represents the interpolating function. The class of signals in (1) is fairly general: for example, $x(t)$ could denote a general bandlimited signal, with $h(t)$ taken as the sinc function, or $\{x[n]\}$ could be interpreted as symbols transmitted in a linearly modulated communication system, with $h(t)$ taken as the impulse response of a cascade of the transmit, channel and receive filters.

The ADC has a time-interleaved architecture as in Fig. 1, with L sub-ADCs indexed by integers between 0 and $L-1$. We model the l^{th} sub-ADC by a linear, time-invariant channel response $\widetilde{h}_l(t)$: gain, timing and bandwidth mismatches are special cases of this model [8]. Thus, the variation of $\widetilde{h}_l(t)$ with l captures the mismatch among the sub-ADCs [8]. The l^{th} sub-ADC outputs nontrivial samples at times $(kL + l)T_o$ for integer k, and outputs zeros at all other times. Assuming high enough output resolution, we ignore quantization noise. The digital output of the l^{th} sub-ADC, $y_l[m]$, can then be written in terms of the analog input $x(t)$ as [8]

$$\begin{aligned} y_l[m] &= \sum_{n=-\infty}^{\infty} x[n]h_l[m-n], \qquad m \bmod L = l \\ &= 0, \qquad \text{otherwise} \end{aligned} \quad (2)$$

where mod denotes the modulo operation and the function $h_l(t)$, henceforth termed the *sub-ADC response*, is the convolution of the sub-ADC response $\widetilde{h}_l(t)$ with the interpolating function $h(t)$.

Consider $\widetilde{y}_l[m]$, defined as the convolution of $x[m]$ with $h_l[m]$. The corresponding z-transforms are related as $\widetilde{Y}_l(z) = X(z)H_l(z)$. We can now relate the transforms of $y_l[m]$ and $\widetilde{y}_l[m]$ to obtain [2]

$$Y_l(z) = \frac{1}{L}\sum_{i=0}^{L-1} w_L^{-li}\widetilde{Y}_l(w_L^i z) \quad (3)$$

where we have collected terms with degrees $l + kL$ for integer k from the polynomial $\widetilde{Y}_l(z)$ to obtain $Y_l(z)$, and where $w_L = e^{j2\pi/L}$ is an L^{th} root of unity. We now add the outputs of all sub-ADCs and use the linearity of the z-transform to obtain the transform of the TI-ADC output $y[m]$ as

$$Y(z) = \sum_{i=0}^{L-1} X({w_L}^i z)F_i(z) \quad (4)$$

where the terms $F_i(z)$ are given in terms of $H_l(z)$ as

$$F_i(z) = \frac{1}{L}\sum_{l=0}^{L-1} {w_L}^{-li}H_l(w_L^i z) \quad (5)$$

If $H_l(z) = 1$ for all l (no mismatch, ideal transfer functions for all sub-ADCs), then $Y(z) = X(z)$. In general, the expression for $Y(z)$ in (4) has a signal term $F_0(z)X(z)$, and interference terms $\{F_i(z)X({w_L}^i z)\}$ for $i \neq 0$. We now discuss conventional zero-forcing mismatch compensation for eliminating the interference terms.

B. Zero-forcing mismatch compensation

First consider a single sub-ADC ($L = 1$) with non-ideal response, in which case $Y(z) = H_0(z)X(z)$. In this case, the zero-forcing equalizer is given by $G_0(z) = [H_0(z)]^{-1}$. For L interleaved sub-ADCs, zero-forcing mismatch compensation (which also addresses non-idealities in the sub-ADC transfer functions) can be achieved using L equalizers in parallel, $\{G_l(z)\}$, as shown in Fig. 2. These equalizers operate on the TI-ADC output $y[m]$ such that the l^{th} equalizer output is calculated only for discrete time indices of the form $kL+l$ for integer k. Thus, in practice, the L parallel equalizers can be implemented as a single filter with periodically time-varying coefficients with period L. Owing to the similarity between the structures of the TI-ADC and the equalizer, we can use (4) for relating the equalizer output $Y^{'}(z)$ to the equalizer input $Y(z)$ as

$$Y^{'}(z) = \sum_{k=0}^{L-1} \phi_k(z) Y(w_L^k z) \tag{6}$$

where $\phi_k(z)$ is defined in terms of the equalizer filters $\{G_l(z)\}$ as

$$\phi_k(z) = \frac{1}{L} \sum_{l=0}^{L-1} {w_L}^{-lk} G_l({w_L}^k z) \tag{7}$$

We now substitute the expression for $Y(z)$ from (4) in (6) to simplify $Y^{'}(z)$ as

$$Y^{'}(z) = \sum_{i=0}^{L-1}\sum_{k=0}^{L-1} F_i({w_L}^k z)\phi_k(z) X({w_L}^{i+k} z) \tag{8}$$

We now collect the terms of the form $X({w_L}^{\alpha} z)$ in (8). Since, ${w_L}^{\alpha} = w_L^{\alpha \text{ mod } L}$, we can restrict the range of α to integers in $[0, L-1]$. The R. H. S of (8) can now be rearranged as

$$Y^{'}(z) = \sum_{\alpha=0}^{L-1} X({w_L}^{\alpha} z) \sum_{(i,k)\in S_\alpha} F_i({w_L}^k z)\phi_k(z) \tag{9}$$

where the set S_α includes all $0 \le k, i \le L-1$ that additionally satisfy $(i + k) \bmod L = \alpha$. Clearly, each S_α has only one element, which is $\alpha - k$. Now, we can replace the second summation in (9) by a single summation over k as

$$Y^{'}(z) = \sum_{\alpha=0}^{L-1} X({w_L}^{\alpha} z) \sum_{k} F_{\alpha-k}({w_L}^k z)\phi_k(z) \tag{10}$$

where we have used the fact that $F_i = F_{i \text{ mod } L}$. For zero-forcing the interference terms $\{X({w_L}^{\alpha} z)\}$ (for $i \neq 0$) in the expression for $Y^{'}(z)$ of (10) and to consequently obtain an undistorted signal term $X(z)$ (except for an integer delay d), we need to satisfy the following system of equations in $\underline{\phi}(z) = \{\phi_0(z), \cdots, \phi_{L-1}(z)\}^t$

$$A(z)\underline{\phi}(z) = z^{-d}(1, 0, \cdots, 0)^t_{L\times 1} \tag{11}$$

where the $(\alpha, k)^{\text{th}}$ entry of the $L \times L$ matrix $A(z)$ of (11) is given by

$$A_{\alpha,k} = F_{\alpha-k}({w_L}^k z) \tag{12}$$

Since the matrix $A(z)$ contains information about the mismatch responses $\{H_l(z)\}$, it characterizes the TI-ADC, hence we refer to it as the *system matrix*. Once we obtain $\{\phi_k(z)\}$ from (11), we can find $G_l(z)$ by obtaining the inverse relation to (7)

$$G_l(z) = \sum_{k=0}^{L-1} {w_L}^{lk} \phi_k({w_L}^{-k} z) \tag{13}$$

We now illustrate, through a running example, how linear equalizers can be obtained when there is timing mismatch among the sub-ADCs.

1) Running Example: Without loss of generality, we assume $T_o = 1$ and consider the sub-ADC response, $h_l(t)$, as $h(t + \delta_l)$, where the function $h(t)$ is chosen to be

$$h(t) = \begin{cases} 10(1 - |t|), & |t| \le \frac{3}{2} \\ 10(|t| - 2), & \frac{3}{2} \le |t| \le 2 \\ 0, & \text{otherwise} \end{cases} \tag{14}$$

We consider $L = 2$ (two sub-ADCs) and $\delta_0 = 1/10, \delta_1 = -1/10$. Hence, the timing mismatch (relative to T_o) is $\pm 10\%$. The z-domain responses of the sub-ADCs can be written as

$$H_0(z) = 9 - z^{-1} + z - z^2, H_1(z) = 9 + z^{-1} - z + z^2 \tag{15}$$

We can now obtain F_0 and F_1 using (15) in (5) and then find the system matrix A. We solve for $\underline{\phi}$ in (11) and then use the obtained values of ϕ_0 and ϕ_1 in (13) to give expressions for the zero-forcing equalizers as

$$G_0(z) = \frac{-z(z^3 - z^2 + 9z + 1)}{z^6 - z^4 - 79z^2 - 1}, G_1(z) = \frac{-z(z^3 + z^2 - 1)}{z^6 - z^4 - 79z^2 - 1} \tag{16}$$

From (16), it can be seen that $G_0(z)$ and $G_1(z)$ possess an infinite power series expansion. This implies that the corresponding time-domain functions $g_0[n]$ and $g_1[n]$ cannot be implemented as FIR filters. It is easy to see that this observation holds in general. For finite-length mismatch responses $\{h_l[n]\}$, the z-transforms $\{H_l(z)\}$, and hence the entries of the matrix $A(z)$, are finite-length polynomials. The solution $\underline{\phi}$ to (11), when it exists, is, in general, a *rational function.* Consequently, the zero-forcing choices of $G_l(z)$ are rational functions with infinite-length time domain responses in general. In the next section, we show that, under certain conditions, we can obtain FIR equalizers for mismatch compensation by the use of oversampling.

III. Oversampling for Scalable Mismatch Compensation

For ease of exposition, we first consider oversampling for $L = 1$. We consider a rational oversampling ratio of p/q, where p and q are relatively prime positive integers such that $p \ge q$. From (1), the m^{th} output sample of the p/q-oversampling TI-ADC can be obtained as

$$y[m] = \sum_n x[n] h\left(\left(\frac{mq}{p} - n\right) T_o\right) \tag{17}$$

In order to find the output z-transform, we first consider the following discrete signals:

$$\begin{aligned}\widetilde{y}[m] &= y[qm]\\ \widetilde{x}[m] &= \begin{cases} x[m/p], & m \bmod p = 0\\ 0, & \text{otherwise}\end{cases}\\ \widetilde{h}[m] &= h\left(\frac{mT_o}{p}\right)\end{aligned} \tag{18}$$

Clearly, $\widetilde{y}[m]$ and $\widetilde{x}[m]$ represent the down-sampled (by q) and up-sampled (by p) versions of $y[m]$ and $x[m]$ respectively. Now, it can be deduced from (18) that $\widetilde{y}[m]$ is a convolution of $\widetilde{x}[m]$ and $\widetilde{h}[m]$ or equivalently, the corresponding z-transforms are related as $\widetilde{Y}(z) = \widetilde{H}(z)\widetilde{X}(z)$. In (18), we use the z-transform properties related to up/down sampling[2] to obtain

$$Y(z) = \frac{1}{q}\sum_{k=0}^{q-1} X(w_q^k z^{p/q})\widetilde{H}(w_q^k z^{1/q}) \tag{19}$$

where $w_q = e^{j2\pi/q}$. Note that when $p = q = 1$, the expression for $Y(z)$ in (19) reduces to $X(z)H(z)$, which agrees with the discussion in section II.A.

We now consider the general case of L interleaved sub-ADCs. As in (18), we can define, for each l, a discrete signal $\widetilde{h}_l[m]$ that is obtained by sampling the corresponding sub-ADC response $h_l(t)$ at p times the symbol rate. If the l^{th} sub-ADC were to obtain all the samples, that is at the rate of pT_o^{-1}/q, the output z-transform is obtained from (19) by replacing $\widetilde{H}(z)$ by $\widetilde{H}_l(z)$. In the time-interleaved architecture, we use (19) in (3) and (4) to obtain

$$Y(z) = \sum_{k=0}^{q-1}\sum_{i=0}^{L-1} X(w^{kL+ip} z^{p/q}) F_{i,k}(z^{1/q}) \tag{20}$$

where $w = e^{\frac{j2\pi}{qL}}$. Compared to the expression obtained for the $p = q = 1$ case in (4), the coefficients $F_{i,k}(z^{1/q})$ vary over two variables (i,k) and are defined in terms of the sub-ADC responses $\{\widetilde{H}_l(z)\}$ as

$$F_{i,k}(z) = \frac{1}{qL}\sum_{l=0}^{L-1} w^{-qli}\widetilde{H}_l(w^{kL+i}z) \tag{21}$$

We now analyze the special case of oversampling by 2 to obtain useful insights regarding the length of the zero-forcing equalizers. The analysis also applies to other integer oversampling factors, but in practice, we would probably be interested in rational oversampling factors between 1 and 2.

A. Oversampling by 2

Substituting $p = 2, q = 1$ in (20), we obtain the following expression for the TI-ADC output:

$$Y(z) = \sum_{i=0}^{L-1} X(w^{2i}z^2)F_i(z) \tag{22}$$

where $F_i(z)$ is now defined as

$$F_i(z) = \frac{1}{L}\sum_{l=0}^{L-1} w^{-li}\widetilde{H}_l(w^i z) \tag{23}$$

where we used the fact $w_L = w$ for $q = 1$. For zero-forcing equalization, we consider L filters $\{G_l(z)\}$ as in (6) such that successive outputs are obtained from different filters operating in succession. Using (22) in (6), the output of the equalizer can be written in the z-domain as

$$Y'(z) = \sum_{i=0}^{L-1}\sum_{k=0}^{L-1} F_i(w^k z)\phi_k(z)X(w^{2i+k}z^2) \tag{24}$$

The equalizer output in (24) refers to a discrete signal at twice the symbol sampling rate. To obtain the “symbols” we down-sample (by 2) the signal represented by $Y'(z)$ in (24) and then, give conditions for zero-forcing the interference terms. The transform of the down-sampled version is given by [2]

$$\begin{aligned}Y_d'(z) &= \frac{1}{2}\sum_{i=0}^{L-1}\sum_{k=0}^{L-1}(F_i(w^k u)\phi_k(u) + F_i(-w^k u)\phi_k(-u))\\ &\quad X(w^{2i+k}z)\end{aligned} \tag{25}$$

where $u = \sqrt{z}$. We realize that the functions $\phi_k(u)$ and $\phi_k(-u)$ are dependent on each other. To obtain an unconstrained zero-forcing problem formulation, we define two transformed variables $\phi_{k,e}(u)$ and $\phi_{k,o}(u)$ as

$$2\phi_{k,e}(u) = \phi_k(u)+\phi_k(-u), 2\phi_{k,o}(u) = u^{-1}(\phi_k(u)-\phi_k(-u)) \tag{26}$$

Using the power series expansion for $\phi_k(u)$, we can infer that $\phi_{k,e}(u)$ and $\phi_{k,o}(u)$ contain different coefficients of the expansion and thus, can be chosen independent of each other. Now, the zero-forcing conditions (with a delay d) for the 2-times oversampling case are given by (11) with $\underline{\phi} = \{\phi_{0,e}, \cdots, \phi_{L-1,e}, \phi_{0,o}, \cdots, \phi_{L-1,o}\}$ and in this case, the $L \times 2L$ system matrix $A(z)$ has its entries as

$$A_{\alpha,k} = \begin{cases} \sum_{i\in S_{\alpha-k}} F_{i,e}(w^k u), & 0 \le k \le L-1\\ u^2\sum_{i\in S_{\alpha-k}} F_{i,o}(w^k u), & L \le k \le 2L-1\end{cases} \tag{27}$$

where $u = \sqrt{z}$. The functions $F_{i,e}$ and $F_{i,o}$ are defined as in (26) by replacing ϕ_k by F_i. The set S_a, for an integer a, is defined as $S_a = \{i : (2i) \bmod L = a\}$. After solving the equation (11) using (27), the solution $\underline{\phi}$ can be used in (13) to obtain the equalizers $\{G_l(z)\}$. We now revisit the running example to show how oversampling can help to simplify the equalizer design.

1) Running Example: We consider the same case of $L = 2$ sub-ADCs but assume that the net sampling rate is two times the symbol rate. The sub-ADC responses, sampled at twice the symbol rate, are given by

$$\begin{aligned}z^4\widetilde{H}_0(z) &= -z^8 - 4z^7 + z^6 + 6z^5 + 9z^4 + 4z^3 - z^2 - 4z\\ z^4\widetilde{H}_1(z) &= -4z^7 - z^6 + 4z^5 + 9z^4 + 6z^3 + z^2 - 4z - 1\end{aligned} \tag{28}$$

To determine the system matrix A, we calculate F_0 and F_1 using (23) and determine S_a of (27) for the allowed values of $a = \{0,1\}$. We obtain $S_0 = \{0,1\}$ and S_1 is empty. Now, we can write the system of equations from (11) using (27)

$$\begin{aligned}b(z)\phi_{0,e} + zc(z)\phi_{0,o} &= z^{-d+2}\\ b(z)\phi_{1,e} + zc(z)\phi_{1,o} &= 0\end{aligned} \tag{29}$$

where we replaced u with $\sqrt{z}$. The functions $b(z)$ and $c(z)$ in (29) are given as $b(z) = -z^4+z^3+9z^2-z$ and $c(z) = -4z^3+4z^2+6z-4$. Due to the greater number of variables, several solutions are possible the two equations in (29) and we are particularly interested in polynomial solutions. For the second equation, we have a trivial solution: $\phi_{1,e}(u) = \phi_{1,o}(u) = 0$. For the first equation, the application of the standard Bezout's identity to polynomials with no common zeros, $b(z)$ and $c(z)$, implies the existence of polynomial solutions for both $\phi_{0,e}$ and $\phi_{0,o}$. These solutions can be found by using the extended Euclidean algorithm [3]. The obtained solutions are used in (13) to obtain finite length equalizers $G_0(z)$ and $G_1(z)$.

The existence of finite length equalizers can be generalized for a two-times oversampling TI-ADC with L sub-ADCs. From (27), we can decompose the $L \times 2L$ matrix A into two $L \times L$ matrices B and C, such that B consists of the first L columns of A and C has the next L columns. Now, we can rewrite (11) used with A obtained from (27) as

$$B(z)\underline{\phi}_b(z) + C(z)\underline{\phi}_c(z) = z^{-d}(1, 0, \cdots, 0)^t_{L\times 1} \qquad (30)$$

where $\underline{\phi}_b(z) = \{\phi_{0,e}(z), \cdots, \phi_{L-1,e}(z)\}$ and $\underline{\phi}_c(z) = \{\phi_{0,o}(z), \cdots, \phi_{L-1,o}(z)\}$. We now state the following lemma regarding the existence of finite length zero-forcing equalizers expressed in terms of the matrices B and C.

Let $\det$ denote the determinant of a matrix (this is a polynomial when the matrix has polynomial entries).

Lemma 1. *Finite length zero-forcing equalizers exist for mismatch compensation in the two-times oversampling case, when the polynomials* $\det B$ *and* $\det C$*, corresponding to the two effective system matrices* B *and* C*, have no zeros in common.*

Proof: We first realize that (30) is a system of linear equations in $\underline{\phi}_b(z)$ and $\underline{\phi}_c(z)$. The coefficient matrix $U = [B(z) \;\; C(z)]$ is a $L \times 2L$ matrix with polynomial entries. We can form the augmented matrix U_a by appending the column vector on the R. H. S of (30) to the matrix U. From [4], polynomial solutions exist for all the entries of $\underline{\phi}_b(z)$ and $\underline{\phi}_c(z)$, when the greatest common divisor (gcd) of all the $L \times L$ determinants is same for both U and U_a. (Actually, [4] provides results for when the variables and coefficients in the linear system of equations are integers, but this result extends to polynomials). By hypothesis, $\det B(z)$ and $\det C(z)$ have no common zeros and hence the gcd is 1. These two determinants constitute two of all the $L \times L$ determinants calculated for both the matrices U and U_a. Since the gcd of any other polynomial with 1 is also 1, we conclude the required gcd s are same (equal 1) for both U and U_a, implying the existence of a polynomial solution to (30). ∎

Referring to the running example, we have, from (29), that $\det B = b^2(z)$ and $\det C = z^2c^2(z)$ and the determinants can be verified to have no common zeros, except for at $z = 0$, which anyways can be traded off with a delay in (30). We give further illustration of the relation between the zeros of $\det B$ and $\det C$ in Fig. 3, where we consider $L = 4$ sub-ADCs and assume different levels of timing mismatch. The mismatch parameters $\{\delta_l\}$ relative to the sampling period T_o, for each case on Y-axis, are chosen as $\{0.6, 0.8, -0.7, 0.8\}$, scaled by the corresponding relative mismatch value. We observed no common zeros between the determinants and thus, the existence of a finite length equalizer is guaranteed.

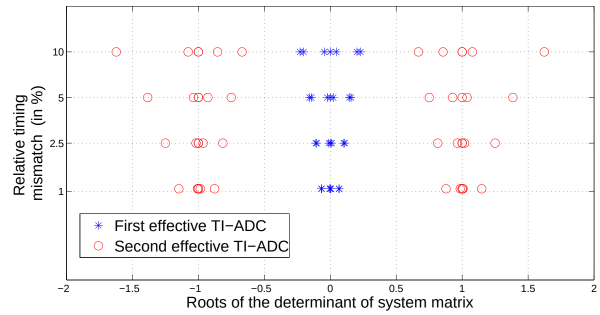

Fig. 3. Zeros of the determinants of the matrices B and C (corresponding to the two effective TI-ADCs in 2x oversampling) as a function of relative timing mismatch. The zeros are all real and there are no zeros of $\det C$ beyond the depicted range. We ignored any common zeros at $z = 0$.

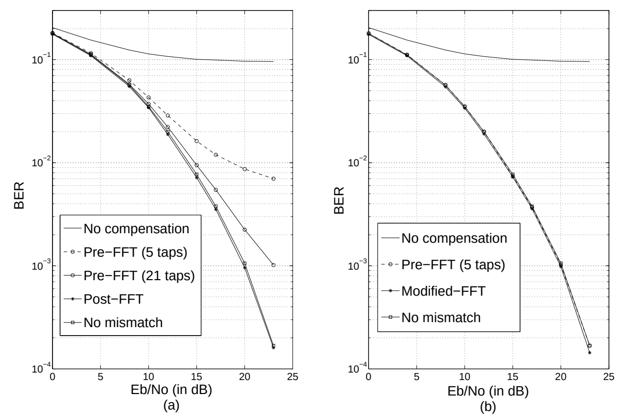

Fig. 4. BER in a 64-QAM, 128-subcarrier OFDM system employing a sloppy TI-ADC with 10% timing mismatch. For the left subfigure (a), Nyquist rate sampling is performed and the TI-ADC interleaving factor L is 8. On the other hand, we assume two times oversampling and $L = 32$ in the right subfigure (b). The Modified-FFT approach corresponds to an OFDM-specific mismatch compensation approach given in [14].

IV. Application to an OFDM receiver employing a TI-ADC

We now illustrate the use of oversampling for mismatch compensation by considering a communication link using 128-subcarrier OFDM with 64-QAM signaling on each subcarrier, transmitted (with no excess bandwidth) over a frequency selective communication channel. In our numerical results, we use a channel impulse response obtained as a realization of the near Line-of-Sight (LOS) channel model defined in the UWB standardization process [1]. For the TI-ADC, we consider a 10% relative timing mismatch for each sub-ADC. Details of the channel and mismatch parameters are omitted here due to lack of space, but are available at [13], [14].

Following the discussion in Sections II and III, ideal zero-forcing equalizers for mismatch compensation can have an infinite number of taps. In this case, we can employ Minimum Mean-Squared Error (MMSE) mismatch compensation, minimizing the total residual interference power with a finite number of taps. When there exists a finite length zero-forcing (ZF) solution (as in the two times oversampling case under certain conditions), a ZF equalizer would be obtained as the MMSE solution for a sufficient number of taps. In other cases, as our numerical results illustrate, the equalizer length must increase with the desired resolution in order to limit the residual interference to an acceptable level.

Zero-forcing time domain mismatch compensation is of general applicability, but given our focus on OFDM in this section, we also evaluate the performance of a frequency-domain mismatch compensation scheme that we proposed in [13], which is specifically designed for OFDM receivers. It was shown in [13] that, regardless of the desired resolution, we can compensate for mismatch *after* the FFT using L-tap frequency domain equalizers operating on groups of subcarriers of size L, when the number of subcarriers is a multiple of the number of sub-ADCs L. We refer to this scheme as **Post-FFT** compensation, and to the general zero-forcing mismatch compensation solution here as **Pre-FFT** compensation. For large constellations, we desire a high ADC resolution: in this case, post-FFT compensation works well for small L, but that pre-FFT compensation with oversampling (to limit complexity as the desired resolution increases) becomes attractive for large L.

We first consider a Nyquist sampling TI-ADC with a moderate interleaving factor of $L = 8$. BER results, depicted in Fig. 4 (a), indicate that the mismatch, when left uncorrected, leads to significant error floors. Also, pre-FFT compensation, even with as many as 21 taps, could not completely eliminate the mismatch-induced interference. On the other hand, post-FFT compensation approaches the performance without mismatch at a much smaller complexity of $L = 8$ taps. When we consider increasing the interleaving factor of the TI-ADC to increase the net sampling rate, the complexity of the post-FFT scheme increases and beyond a point, we resort to oversampling to enable low-complexity mismatch compensation.

Now, consider a TI-ADC with $L = 32$, for which post-FFT compensation is less attractive. We consider sampling at twice the Nyquist rate. For a given technology, the absolute mismatch remains fairly constant. Assuming 10% relative mismatch for Nyquist sampling, we have 20% relative mismatch for 2x oversampling. From Fig. 4 (b), we observe that the Pre-FFT scheme requires only 5 taps to achieve the performance without mismatch for BERs as low as 10^{-4}. For the same range of BERs, when the oversampling factor is decreased to 5/4 (corresponds to 25%), the number of taps increased to 9 to approach the performance without mismatch.

V. Conclusion

To summarize, oversampling proves valuable in limiting the complexity of mismatch compensation with increasing interleaving factor and resolution. Thus, it provides the flexibility of obtaining a high-resolution, high-rate ADC by interleaving a large number of relatively slow, power-efficient, sub-ADCs with high resolution. For example, consider an OFDM transceiver employing 64-QAM over a communication bandwidth of 1 GHz (uncoded bit rate of 6 Gbps). For 2x oversampling, each of the I and Q components require a TI-ADC operating at an aggregate sampling rate of 2 GHz with 8-10 bits resolution. If we use 32 sub-ADCs with the same resolution, each sub-ADC must operate at 62.5 MHz. Attractive low-power solutions for implementing such low rate sub-ADCs exist in Pipelined or Successive-Approximation Register (SAR) architecture [6], resulting in reasonable overall power consumption. Of course, detailed circuit design and evaluation are required to determine the efficacy of such system-level designs.

A specific topic of ongoing research is to extend the ideas presented in this paper to more general mismatch models and to design efficient algorithms for estimating the mismatch parameters either by using specialized on-chip training or by using the training information available in communication signals. A broader area of investigation is the design of scalable mismatch compensation techniques for generic applications of TI-ADC.

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