Scalable Mismatch Correction for Time-interleaved Analog-to-Digital Converters in OFDM Reception[1]

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***Abstract*—Realization of all-digital baseband receiver processing for multi-Gigabit communication requires analog-to-digital converters (ADCs) of sufficient rate and output resolution. A promising architecture for this purpose is the time-interleaved ADC (TI-ADC), in which L "sub-ADCs" are employed in parallel. However, the gain, timing and voltage-offset mismatches between the sub-ADCs, if left uncompensated, lead to error floors in receiver performance. A standard technique for gain and timing mismatch correction is to use L FIR filters, with tap lengths increasing with the mismatch levels. In this paper, we investigate the use of TI-ADCs in OFDM receivers, and provide a scalable technique for mismatch compensation whose complexity is independent of L and the mismatch levels. We achieve this by decomposing the FFT operator that is at the core of the OFDM receiver into eigenmodes, and showing that, even for large values of L and mismatch levels as high as 25%, two eigenmodes suffice to provide an accurate description of the mismatch-perturbed FFT operator. We provide simulation results that show that the associated mismatch compensation algorithm is successful is eliminating the mismatch-induced error floor.**

I. Introduction

The analog-to-digital converter (ADC) is a critical component in modern digital communication receivers, enabling cost-effective, all-digital implementation of sophisticated baseband signal processing algorithms. However, as communication bandwidths increase, the availability of ADCs with sufficient speed and resolution becomes a concern: Gigahertz bandwidths are required for emerging ultrawideband [1] and millimeter wave [2] applications, while 8-12 bits of resolution are required for providing enough dynamic range when operating in multipath environments with large constellations. The technology of choice at GHz speeds is "one shot" flash ADC, but it becomes unmanageable beyond 5 bits resolution, due to exponentially (in number of bits) increasing power consumption and hardware complexity [3]. An attractive alternative [3] is a time-interleaved (TI) architecture, where high rate and high resolution can be achieved by employing several low rate, high resolution, sub-ADCs in parallel. An ideal TI-ADC formed by time-interleaving four sub-ADCs is shown in Fig. 1. However, an inherent problem with the TI-ADC architecture is mismatch between the sub-ADCs, which to first order can be assumed to be gain, timing and voltage-offset mismatch [4]. Left uncompensated, such mismatch leads to error floors when TI-ADCs are employed in communication receivers. In this paper, we investigate scalable (as the number of sub-ADCs and the mismatch levels increase) mismatch compensation techniques for removing mismatch-induced error floors due to TI-ADCs employed in OFDM reception.

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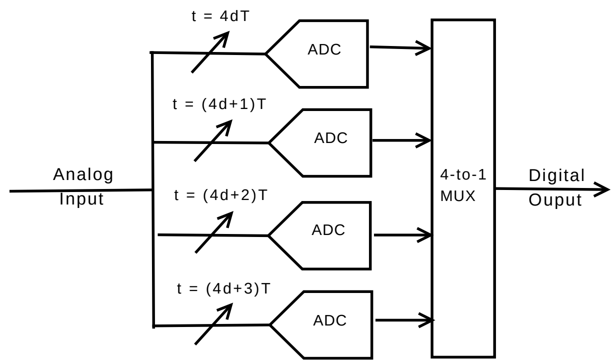

Fig. 1. Ideal time-interleaved ADC (d =integer)

Our approach is as follows. We show that the demodulator (FFT operator in the case of no mismatch) following the TI-ADC can be decomposed into eigenmodes, and that first two eigenmodes provide an excellent approximation, even as the number of sub-ADCs and the mismatch levels grow large. We show that, as long as the overall TI-ADC sampling rate is at least twice the Nyquist rate, mismatch compensation based on the two eigenmode approximation is effective in eliminating mismatch-induced error floors. Since the time variation of mismatch is very slow [7], we assume that the mismatch parameters can be accurately estimated [5], [7], [10]. Hence, the focus of this paper is only on compensating for mismatch given accurate estimates of mismatch parameters.

A. *Related work and Comparison*

Digital mismatch compensation for TI-ADCs has received a great deal of attention in the literature [7], [10], [11], [12], [13]. A standard time-domain approach to correct for gain and timing mismatch is to use L FIR filters [7], [11], [12], [13], where L denotes the number of sub-ADCs. Since the ideal correction filters are of infinite length, with slowly decaying taps, a large number of taps were needed when the resolution requirement and/or the mismatch range is large. Furthermore, the hardware complexity scales with L and the number of taps; the latter increases with the resolution requirement and the mismatch range (the ideal correction filters are of infinite length, and have slowly decaying taps). In our own prior work on TI-ADCs in OFDM receivers, we developed a frequency domain approach whose complexity scales with

L (regardless of the mismatch level) when L divides the number of subcarriers. The present paper therefore provides significant complexity reduction relative to the literature: the computational complexity is that of taking 2 oversampled FFTs (instead of taking the one FFT in an OFDM system), independent of the number of parallel sub-ADCs and the mismatch levels.

The problem of timing mismatch compensation can be thought of as the reconstruction a bandlimited signal from its non-uniform samples. Unlike prior work in non-uniform sampling theory [14] and [15], we consider the case where the sampling instants have small offsets (equivalently, the case of small timing mismatch) and hence obtain a simpler two-eigenmode approximation.

B. Organization

The paper is organized as follows. Section II provides a model for received samples in an OFDM receiver when sloppy TI-ADCs are used. Section III develops the two-eigenmode approximate model for a TI-ADC with gain and timing mismatches. Section IV presents a scalable compensation algorithm based on the model in Section III. Section V gives BER simulation results. Section VI provides our conclusions.

II. System Model

In this section, we present one of the widely used models for a non-ideal TI-ADC and then derive the model for the received signal when such an ADC is used in an OFDM communication system.

A. Mismatch model

A first order model for a non-ideal TI-ADC is to assume gain, timing and voltage-offset mismatches among the parallel sub-ADCs in Fig. 1. For clarity, we repeat the model in [4] for the output of a non-ideal TI-ADC as

$$r[m] = g_{m \text{mod} L} r(mT_o + \delta_{m \text{mod} L}) + \mu_{m \text{mod} L} + q[m] \qquad (1)$$

where L denotes the number of interleaved sub-ADCs. We denote the sampling rate in the case of no-mismatch as T_o^{-1} and refer to it as the *nominal sampling rate*. The m^{th} output sample, denoted as $r[m]$, is sampled by the sub-ADC with index $m\text{mod}L$, where $\text{mod}L$ indicates the remainder after division by L (From here on, the notation $\text{mod}L$ is understood and not explicitly written). The gain, timing and voltage-offset mismatches of the m^{th} sub-ADC are denoted as (g_m, δ_m, μ_m). Since the output precision is finite, the resulting quantization noise is denoted as $q[m]$.

We list the assumptions made related to the use of (1) in this paper:

- Since error term μ_m due to voltage-offset mismatch in (1) is additive and signal-independent, it can be easily corrected after estimation. Hence, we neglect offset mismatch and refer the reader to related literature for further details [5]. The gain and timing mismatches result in a more complicated interference structure and hence are the subject of this paper.
- In all our analyses, we assume that sufficient bits of precision are available at the ADC output and hence, we neglect the quantization noise in (1).
- For the passband transmission, two ADCs are needed for I and Q channels. We assume that sub-ADCs with index i for I and Q channels have the same mismatch parameters. In practice, this refers to the circuit layout where L low rate (I, Q) ADC pairs are interleaved and there is no mismatch within each (I,Q) pair of ADCs.

B. Communication Model

By performing scalar equalization in frequency domain, OFDM provides a scalable solution for high data-rate wireless systems in multi-path channels [9]. A model for the received signal in OFDM reception, employing the non-ideal TI-ADC modeled by (1), is derived in [10]. For completeness of this manuscript, we repeat with a more detailed and insightful presentation.

Fig. 2 depicts the typical block diagram of an OFDM transceiver [9]. The time samples $\{b[x]\}$ corresponding to the information-bearing symbols $\{B[y]\}$ are obtained using IFFT. These time samples are then transmit filtered and sent over the wireless channel. After the action of the channel and receive filters, the received signal $r(t)$ is sampled using a non-ideal TI-ADC.

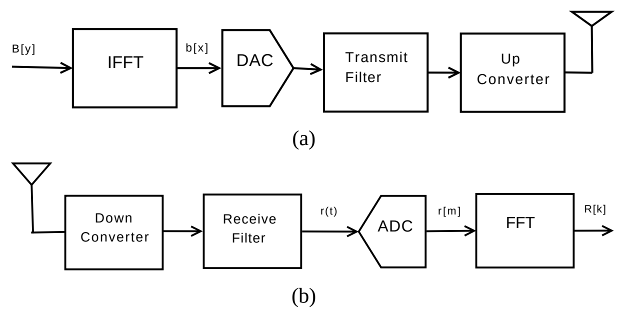

Fig. 2. (a) OFDM Transmitter (b) OFDM Receiver

We introduce some notation before arriving at an expression for the received signal samples $\{r[m]\}$. Let M denote the number of subcarriers and T^{-1}, the symbol rate. Assuming no excess bandwidth, baseband analysis uses the band $\left(-\frac{1}{2T}, \frac{1}{2T}\right)$ for analysis. For simplicity of representation, we use the band $\left(0, \frac{1}{T}\right)$. The transmit and receive filters are chosen as $I_{[0,W]}(f)$, where $I_{[0,W]}(f)$ represents the function that equals one on $[0, W]$ and zero elsewhere. The receiver input noise, $w(t)$, is assumed to be proper, complex, zero-mean, white, Gaussian process with a power spectral density N_o. Then, the receive-filter output noise $n(t)$ is a proper, complex, zero-mean, Gaussian process with power spectral density that is N_o in $[0, W]$. The impulse response of the cascade of the transmitter, channel and receive filters is denoted by $h(t)$ and it is assumed that $h(t)$ is almost zero outside $[0, NT]$. This implies the required length of cyclic prefix to be N. Now, considering an isolated OFDM frame, the input to the TI-ADC is given as

$$r(t) = \sum_{m=-N}^{M-1} b[m \bmod M] h(t - mT) + n(t) \tag{2}$$

Replacing the time-domain samples $\{b[m]\}$ with frequency domain symbols $\{B[y]\}$, which are related by the IFFT operation, we have

$$r(t) = \frac{1}{M} \sum_{y=0}^{M-1} B[y] \sum_{m=-N}^{M-1} h(t - mT) e^{j2\pi ym/M} + n(t) \tag{3}$$

All significant samples of h at rate T are covered in the summation over m in (3) for any t in $[0, (M-1)T]$. This is under the assumption that $h(\cdot) \approx 0$ outside $[0, NT]$. Hence, the summation over m in (3) can be converted to a summation over m in $(-\infty, \infty)$. This implies that (3) can be written as

$$r(t) = \frac{1}{M} \sum_{y=0}^{M-1} B[y] \phi_y(t) \sum_{m=-\infty}^{\infty} h(t - mT) e^{-\frac{j2\pi y}{MT}(t-mT)} + n(t) \tag{4}$$

where $\phi_y(t) = e^{\frac{j2\pi yt}{MT}}$. Let $H(f)$ denote the Fourier transform of $h(t)$. Then, $H(f) = 0$ for $f \notin [0, W]$. Now, we apply Poisson's summation formula for the summation over m in (4) with the function in time as $h(t)\phi_y^*(t)$ and the corresponding Fourier transform as $H(f + \frac{y}{MT})$. We have,

$$r(t) = \sum_{y=0}^{M-1} H[y] B[y] \phi_y(t) + n(t) \tag{5}$$

where $H[y] = \frac{1}{MT} H\left(\frac{y}{MT}\right)$.

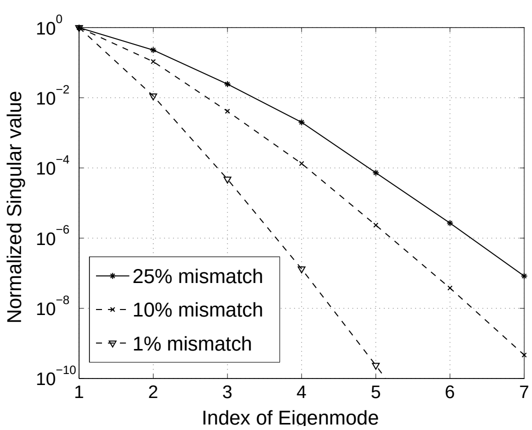

Fig. 3. Normalized singular values for 128 sub-carrier OFDM system employing a sloppy TI-ADC with 32 sub-ADCs

Now, we use (1) in (5) to obtain the digital samples of $r(t)$ taken at a nominal sampling rate of T_o^{-1} as

$$r[m] = g_m \sum_{y=0}^{M-1} H[y] B[y] e^{\frac{j2\pi y(m+\delta_m)}{M_o}} + n[m] \tag{6}$$

where the noise samples $n[m] = g_m n(mT + \delta_m)$ and m takes values in $\{0, \lceil M_o \rceil\}$ with $M_o = MT/T_o$. The oversampling rate is given as $(T_o/T)^{-1}$. The equation (5) is shown to be approximately true only for $t \in [0, (M-1)T]$. In the presence of oversampling and/or timing mismatch, some samples at the edges would not belong to this range. Assuming large M, we neglect this edge effect and assume that (6) holds true for all the samples.

We now proceed to derive an approximate structure for (6).

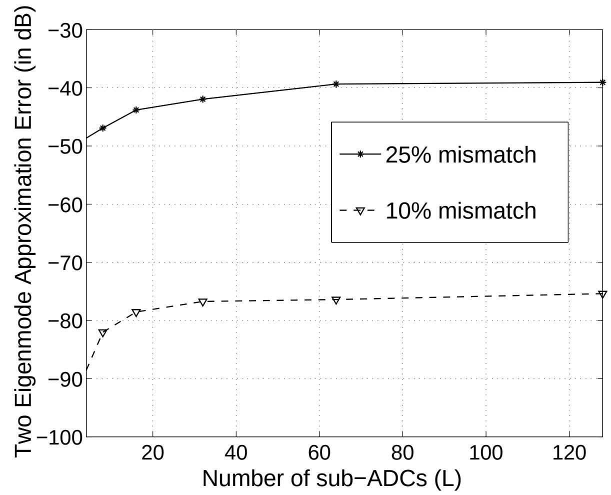

Fig. 4. Approximation error due to the use of two eigenmodes in 128 sub-carrier OFDM system as a function of number of sub-ADCs in the sloppy TI-ADC

III. Approximate Model for Mismatch-induced Interference

Firstly , equation (6) can be written in vector notation as

$$\mathbf{r} = A\mathbf{x} + \mathbf{n} \tag{7}$$

where we denote the receive vector by $\mathbf{r} = \{r[m]\}$, the signal vector by $\mathbf{x} = \{H[y]B[y]\}$ and the noise vector by $\mathbf{n} = \{n[m]\}$. The elements of the M_o-by-M matrix A are given as $A(m, y) = g_m e^{\frac{j2\pi y(m+\delta_m)}{M_o}}$. For integral M_o, it can be seen that A is a perturbation to the standard IFFT operator given by $F^*(m, y) = e^{\frac{j2\pi ym}{M_o}}$. We define the matrix characterizing the perturbation as $\Delta = A \cdot / F^*$, where $\cdot/$ denotes element-wise division. The singular-value decomposition of Δ can be obtained as

$$\Delta = \sum_{i=1}^{M} \lambda_i \mathbf{u_i} \mathbf{v_i^*} \tag{8}$$

Using (8), the operator A can be expanded as

$$A = \sum_{i=1}^{M} \lambda_i D(\mathbf{u_i}) F^* D(\mathbf{v_i^*}) \tag{9}$$

where $D(\mathbf{u})$ denotes the diagonal matrix with the vector $\mathbf{u}$ in the diagonal. We refer to the pair $(D(\mathbf{u_i}), D(\mathbf{v_i^*}))$ as the i^{th} **eigenmode** of A **w.r.t.** F^*. In the case of L sub-ADCs, the matrix Δ has at most L independent rows and hence there are a maximum of L eigenmodes. However, not all eigenmodes are necessarily dominant. To obtain a simpler model, we investigate the number of dominant eigenmodes.

TABLE I
MISMATCH PARAMETERS (IN % DEVIATION FROM NOMINAL VALUES, $g = 1$ AND $\delta = 0$) FOR THE TIME-INTERLEAVED ADC WITH 32 SUB-ADCS

g_m	{ 6, 1 , -4, -4 , 6, 1, -3 , 8 , 7 , 7, 5 , -4 , 4, 8 , 5 , -6 , -1 , 0 , 0, -1 , 9, 9 , 1, -3, 1, 8, -3 , -8 , -6 , 8 , -4 , 0 }
δ_m	{ -4, -8 , 0, 1 , -1, 7, 7 , 1 , 8 , 5 , -7 , -6, 4 , 1 , 3 , 6 , -3 , 1, -6 , -9, -9 , 9, -9, -8, 1, -9 , -1 , 8 , 6 , -9 , -9 , 9 }

We assume $M = 128$ subcarrier OFDM system employing a TI-ADC with $L = 32$ sub-ADCs. We consider Nyquist sampling, that is $M_o = M$ in (6). Fig. 3 depicts the fall of singular values in (8), with the increase in the index of the eigenmode. By $q\%$ mismatch, we refer to a gain and timing mismatches uniformly chosen in the ranges $[1-q, 1+q]$ and $[-qT, qT]$ respectively. Although shown for one mismatch set drawn from the uniform distribution, a similar exponential fall is observed for other instances. In many practical designs, the mismatch is typically within the range of 10%. Hence, we infer that two eigenmodes suffice to approximate A in (9). Let A_2 denote the two eigenmode approximation of A. Using A_2 instead of A in (7), the resulting error is given as $\mathbf{e} = (A - A_2)\mathbf{x}$. The normalized interference power due to approximation is given as $P_2 = ||(A - A_2)\mathbf{x}||^2/||(A\mathbf{x}||^2$. As P_2 is a function of $\mathbf{x}$, we consider the maximum of P_2 w.r.t $\mathbf{x}$ for the worst case scenario. This maximum value can be obtained as the squared, maximum, generalized singular value of the matrices $(A - A_2)$ and A [18]. Fig. 4 plots the worst case error power due to two eigenmode approximation as a function of the number of sub-ADCs of the TI-ADC. The results are averaged over 25 mismatch sets. Thus, it can be inferred that the two-eigenmode approximation is accurate for a wide range of mismatch values ranging till 25%, independent of the number of sub-ADCs. This is observed to hold true even when the ADC is oversampling at two times the Nyquist rate. Finally, we arrive at the approximate model of a sloppy TI-ADC with small mismatch in Fig. 5 (a).

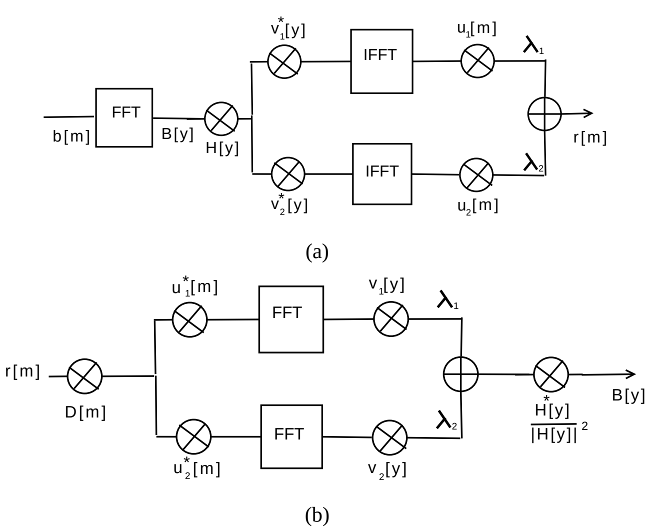

Fig. 5. {(a) Approximate model (b) Scalable correction scheme} for a non-ideal TI-ADC in OFDM system for small mismatch

IV. SCALABLE COMPENSATION OF MISMATCH

The aim of OFDM demodulator in Fig. 2 is to obtain the frequency domain symbols $\{B[y]\}$ from the time-domain samples $\{r[m]\}$. In the case of an ideal ADC at the receiver, this is typically accomplished using FFT and a scalar channel equalization (multiplication with $H^*[y]$). Ignoring mismatch between the sub-ADCs of the TI-ADC and using the same demodulator has been shown to result in interference between subcarriers [10]. This interference, when left uncorrected, implied an error floor.

Consider the standard Zero-forcing solution for $\mathbf{x}$ in (7) given by $Z\mathbf{r} = (A^*A)^{-1}A^*\mathbf{r}$. In the absence of mismatch, the zero-forcing operator Z is FFT operator F and in general, it is a perturbation to F. Although a model like that of (9) is possible for Z, the number of dominant eigenmodes w.r.t F was not observed to be small unlike those of A w.r.t F^*.

We make an interesting observation that for oversampling ratios of 2 or greater, a zero-forcing equalizer $Z^{'}$ exists of the form A^*D, where D is a Diagonal matrix. By definition of zero-forcing equalizer $Z^{'}$ for $\mathbf{x}$ in (7), $Z^{'}A\mathbf{x} = \mathbf{x}$. Suppose we prove the existence of D which satisfies $A^*DA = I$ (I = identity operator), then we have obtained a zero-forcing equalizer $Z^{'} = A^*D$. Using the expression of A from (7), we can write the k^{th} element of $Z^{'}\mathbf{r} = A^*DA\mathbf{x}$ as

$$(Z^{'}\mathbf{r})[k] = \sum_{y=0}^{M-1} x[y]P[k-y] \tag{10}$$

where $k \in \{0, \cdots, M-1\}$ and $P[k-y]$ is given as

$$P[k-y] = \sum_{m=0}^{M_o-1} D[m]g_m^2 e^{\frac{j2\pi(y-k)(m+\delta_m)}{M_o}} \tag{11}$$

We have $Z^{'}\mathbf{r} = \mathbf{x}$ when $P[k-y] = 1$ for $(k-y) = 0$ and 0 otherwise. The term $(k-y)$ can take values ranging between $\{-(M-1), \cdots, (M-1)\}$ as both k and y take values in $\{0, \cdots, M-1\}$. Using (11), this implies that the M_o coefficients $D[m]$ have to satisfy $2M-1$ linear equations for the conditions on P to be true. At least one solution is guaranteed when the number of variables is greater than the equations, that is when $M_o > 2M-1$. This implies that when the oversampling ratio is greater than 2, there exists a zero-forcing equalizer of the form A^*D.

Since A^* has the same singular values as A, we can write the zero-forcing solution for $\mathbf{x}$ in (7) as

$$Z^{'}\mathbf{r} = \sum_{i=1}^{M} \lambda_i D(\mathbf{v_i})FD(\mathbf{u_i^*})D\mathbf{r} \tag{12}$$

Taking the two eigenmode representation of A from Section III, an approximate form of zero-forcing equalizer $Z^{'}$ can be given as in Fig. 5 (b). We now calculate the complexity of the scheme in Fig. 5 (b). We denote the complexity of FFT as $O(MlogM)$. The complexity of taking oversampled FFT of size $2M$ for M frequency points is $O(2MlogM)$. Ignoring the complexity of multiplications by diagonal matrices, the computational complexity of the scheme in Fig. 5 (b) is $O(4MlogM)$, independent of L and the mismatch levels.

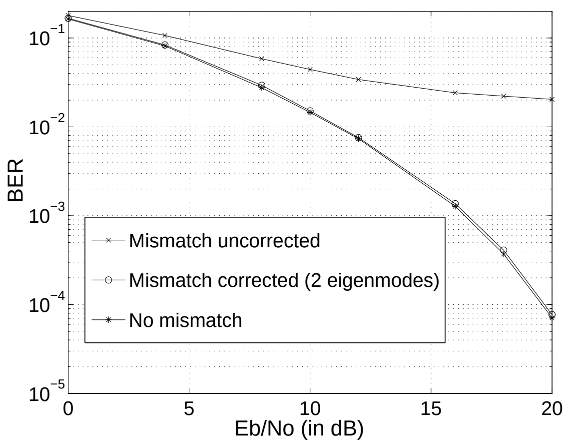

Fig. 6. BER in a 16-QAM, OFDM system (128 sub-carriers) employing a time-interleaved ADC (32 sub-ADCs) with a mismatch level of 10%.

V. Numerical Results

In this section, we present in the form of BER plots, how the negative effects of using a sloppy ADC get significantly compensated by applying the scalable two-eigenmode compensation developed in section IV.

First, we list the system parameters used in the simulation. The number of sub-ADCs is assumed to be 32. We use the same frequency selective channel as in [10] and use 128 subcarriers in the OFDM system. The mismatch due to gain and timing mismatches is assumed to be 10% and the exact values of mismatch are given in Table I. We neglect the effect of quantization noise in the simulations. Also, perfect estimates of channel and mismatch parameters are assumed to be available.

Fig. 6 plots the BER in the OFDM system using 16-QAM constellation for the cases of corrected and uncorrected mismatch between the sub-ADCs. It is assumed that perfect estimates are available for channel taps and the mismatch parameters. Mismatch when left uncorrected results in an error floor. As can be seen from Fig. 6, the two-eigenmode based correction algorithm eliminates the error floor.

VI. Conclusion

The scalable mismatch compensation scheme presented here, even though it requires oversampling, provides the flexibility of interleaving a large number of relatively slow, power-efficient, sub-ADCs with high resolution. For example, consider OFDM with 16-QAM over a communication bandwidth of 2 GHz, which yields an uncoded bit rate of 8 Gbps. For 2x oversampling, the I and Q components each require a TI-ADC operating at an aggregate sampling rate of 4 GHz. If we use 32 sub-ADCs, each sub-ADC must operate at 125 MHz with 8-10 bits resolution for accurate calculation of FFTs. Attractive low-power solutions for implementing such low rate sub-ADCs exist in Successive-Approximation Register (SAR) [16] or Pipelined architecture [17], resulting in reasonable overall power consumption.

A specific topic of ongoing research is the design of efficient algorithms for estimating the dominant FFT eigenmodes, using on-chip training or using the training information available in communication signals. A broader area of investigation is the design of scalable mismatch compensation techniques for singlecarrier communication, as well as for generic applications of TI-ADC.

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